When duopolists do not restrict supply[1]

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Abstract

The paper examines a duopoly in which buyers with unit demands come to market at different times. Sellers choose quantities before competing in prices, but are able to change prices between buyer arrivals. Under ideal conditions, the duopolists do not restrict supply, and the outcome is almost efficient. This links the Cournotian incentive to restrict supply with price inflexibility.

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[1] This paper supercedes Dudey [2000]. The previous paper examined a related model with an equilibrium selection problem.

I. Introduction

This paper relates the Cournotian incentive to restrict supply and price inflexibility.

The modern reformulation of Cournot's [1838] duopoly model, introduced by Kreps and Scheinkman [1983] in this *Journal*, is a two-stage game in which sellers may restrict supply before engaging in static price competition. In equilibrium, the duopolists produce less than the efficient level of output.

Why does this happen? Kreps and Scheinkman emphasized the order in which quantities and prices are chosen. If the duopolists chose prices and produced to order, an efficient, zero profit outcome would emerge. But a seller who is able to make a quantity precommitment can induce her rival to price above marginal cost. This allows the seller who restricted supply to raise her own price and earn positive profit.

The question at the center of this paper is whether duopolists have a similar incentive to restrict supply when "price competition is dynamic." I assume each buyer has unit demand and comes to market at a different time, and sellers can change prices between buyer arrivals.

The main result is that under this assumption and complementary ideal conditions, duopolists do not restrict their supplies and the outcome is approximately efficient. This links the modern version of Cournot's idea and price stickiness.[2] (The result will also draw attention to non-Cournotian incentives to restrict supply, which may be important when prices are flexible and

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[2] This paper is not concerned with the particular outcome predicted by Cournot. The focus is on supply restriction, which is a general feature of two-stage model equilibria. Two-stage models of quantity precommitment and price competition do not, in general, yield the particular outcome predicted by Cournot; see Davidson and Deneckere [1986].

other ideal conditions are violated.)

A variant of Cournot's mineral water parable can be used to illustrate the model. Imagine that the duopolists are vendors of grade one, honeydew melons in an open air market. The vendors are in adjacent stalls and meet buyers who arrive at different times during the market session. (See Balkin, Morales, and Cross [2004] and Cassady [1974] for excellent background on open air markets.)

Each buyer is willing to purchase a melon at a price that is not greater than his value. The number of buyers is given, but their values are private information. Vendors believe these values are independently drawn from the same distribution with support [0, V]. Melons cannot be stored until the next market session, and they have no consumption value for the vendors. A vendor's objective is to maximize her expected profit during the market session.

Each vendor is initially endowed with enough melons to serve all buyers, but can reduce her supply by any amount before the buyers arrive.[3]

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[3] To reduce her supply, a vendor could damage some melons and take them to a disposal site. An intuitive conjecture is that overproduction could occur in a more complicated model where sellers are uncertain about the number of buyers when making production decisions. This is a standard problem type in the inventory management literature, although that literature generally assumes fixed prices (e.g. "the newsboy problem") or monopoly. An exception is Van Miegham and Dada [1999], who study "price postponement with hold-back" in monopoly and oligopoly settings, motivating their analysis with the problem of a farmer who harvests a crop of fresh produce before demand is known; "[d]epending on the yield and market conditions, either all product is brought to the market, which sets the price, or a portion of the harvest is destroyed to influence the market-clearing price." Various examples of oversupply can be found in Cassady [1957], [1963], [1974] and Scherer [1971]. In the context of an open air market, Cassady [1974] notes that "if general supplies are *very* heavy, prices may go to zero." p.71.

(In Kreps and Scheinkman's [1983] model, firms make quantity precommitments by choosing their capacities. For now, I shall abstract from the cost-saving incentive to restrict supply, which is not in Cournot's model. However, upfront costs of production are discussed in this section and section VI, as a particular departure from the ideal conditions.)

When a buyer arrives, vendors call out prices. First, they offer initial asking prices, which the vendors cannot renege on. Vendors may then offer discounts from their initial asking prices. This assumption is reasonable. As Cassady [1974] observed, "it was crucial in early markets, as now, that the seller who would maximize gains would give considerable thought to the initial asking price. Since reneging on a commitment was taboo, the price a would-be seller enunciated to a potential buyer was obviously the ceiling above which he could not go in the transaction." (p. 42) [4]

Buyers have most preferred arrival times, and large deviations from these are too costly relative to V. Each buyer is a price-taker and, in case of a price tie, he purchases from each seller with probability one-half. (There can be bargaining if there is a chance that overpricing by vendors at the beginning of negotiations will cause the buyer to leave without attempting to negotiate further. [5])

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[4] More importantly, the analysis will show when an initial offer matters, the seller wants the buyer to be able to enforce it. A seller could make an initial offer verifiable by handing the buyer a written advertisement. Schechter [1933] discusses enforcement mechanisms used in early markets.

[5] See fn. 6. For certain individuals, the cost of bargaining may be quite high. For instance, Beals [1975] notes "the feeling of many people aspiring to higher social status in the city and larger towns that bargaining is demeaning."

Finally, I will need two technical assumptions. Namely, the price set is a discrete grid and there is a slight cost of disposal in the quantity precommitment stage. The disposal cost is a measure of the vendors' incentive to restrict supply. I will show that it can be made arbitrarily small (zero in the limit) by shrinking the mesh of the price grid. The discrete price set is used to solve an open set problem, which is explained below.

The main result is that the vendors have no incentive to restrict supply in any (Markov) equilibrium. Consequently, duopoly profits converge to zero as the mesh of the price grid approaches zero.

For intuition, consider the case of two buyers. Each vendor has two melons. If neither vendor reduces supply, there will be cutthroat competition for both buyers. But suppose one vendor disposes of a melon. If she fails to make a sale to the first buyer, there will be intense competition for the second. But if she makes a sale to the first buyer, the rival will be a monopolist when the second buyer arrives. This suggests that the supply-constrained vendor will set a lower price when the first buyer arrives.

The problem is that the vendors do not agree about the price at which the supply-constrained vendor should try to make the sale. The unconstrained vendor would like her rival to set a price of zero and sell out right away. (Her preference depends on the absence of a gap between the vendors' value of zero and the minimum buyer value. This assumption is critical but intuitive: vendors do not rule out small values.) The rival would like to post a higher first period price, because this would increase her expected profit.

If initial prices could be retracted, there would be equilibria in which vendors price at or near zero in the first period, as well as other equilibria in which they set first period prices that are much greater than zero. But when initial prices are enforceable, the unconstrained vendor can set a price ceiling that is near zero, thereby forcing the constrained rival to price even lower.[6] (The discrete price grid makes it possible for the constrained seller to slightly undercut the rival.)

This is why supply restriction does not occur in the case of two buyers. No supply restriction means that the two vendors will engage in cutthroat competition for both buyers and earn approximately zero profit.

The example points toward a few simple extensions. For instance, it should be clear that the above reasoning does not depend on vendors being certain about the existence of a second buyer (arrivals could follow a Bernoulli process). Vendors could maximize expected discounted profit. Disposal might take place at more than one point in time. The distribution of values could be a function of some observable characteristic of the buyer, such as whether the buyer looks like a tourist.

Further review of the example shows how supply restriction can occur even when price flexibility negates the modern version of Cournot's argument. Of course, vendors may restrict supply if they incur upfront costs of production. Quantity precommitment is also possible when

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[6] The same idea can be applied when there are multiple rounds of negotiation with the buyer. For example, suppose sellers simultaneously make offers after which a buyer accepts, rejects and stays, or rejects and leaves; in the case of reject and stay, the sellers simultaneously make new offers, etc. A buyer might reject and leave if his own cost of bargaining is too high; see fn. 5. On the enforceability of initial offers, see fn. 4.

initial offers are unenforceable, or when there is a gap between the sellers' value and the minimum buyer value.

The formal arguments are complicated by the assumption of a discrete price set. This assumption is needed since when buyers break price ties randomly, a seller may want to undercut the rival by an arbitrarily small amount. The problem can be avoided with an asymmetric tie-breaking rule, or with a discrete price set. I make the latter assumption, which provides a foundation for the former. (Note that in the two buyer example, there is no open set problem if buyers asymmetrically break ties in favor of the seller holding a smaller inventory. But, without justification, asymmetric price tie-breaking provokes the criticism that vendors would not expect indifferent buyers to behave this way.)

Interestingly, discrete prices alone can create a strategic incentive to restrict supply when disposal is free. The small disposal cost eliminates this side effect of discrete prices. (The disposal cost plays one other role: in its absence, and with an asymmetric price tie-breaking rule instead of discrete prices, vendors would be indifferent about restricting supply.)

Related literature is discussed in section II. The model and efficiency result are given in sections III and IV. Price inflexibility is discussed in section V and upfront costs of production are the subject of section VI. Price inflexibility creates the usual strategic incentive to restrict supply. Upfront costs, which are present in Kreps and Scheinkman's model but not in Cournot's, create an entirely nonstrategic incentive to restrict supply. Section VII shows that unenforceable initial asking prices or a gap between the duopolists' value and the minimum buyer value can, but do not necessarily, create a strategic incentive to restrict supply.

In subgames with one supply constrained duopolist, the unconstrained seller acts as a monopolist after the smaller rival sells out at a low price. Section VIII examines the surprising effect of a minimum mark up requirement (price floor). It increases surplus in subgames with one constrained seller, but can lead to supply restriction when imposed before the quantity precommitment decision. Section IX concludes and suggests several directions for further work.

II. Links to the literature

Edgeworth [1897] showed how supply-constrained duopolists could earn positive profits from static price competition. While his model does not generally have a pure strategy equilibrium, the existence of a mixed strategy equilibrium with positive profits for both sellers was established by Beckman and Hochstadter [1965], Levitan and Shubik [1972], and DasGupta and Maskin [1986]. In his path-breaking criticism of Cournot, Bertrand [1883] observed that static price competition between duopolists who are not supply-constrained would drive profits to zero. These results imply that duopolists will make quantity precommitments in the two-stage games that were formulated and analyzed by Kreps and Scheinkman [1983], Davidson and Deneckere [1986], and others.[7]

Nichol [1929] wrote "According to Edgeworth, competitors consider profits only of the immediate present, and not over a period of time. The latter goal is much more sensible."

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[7] Judd [1996] presents a related, multiperiod resolution of Cournot and Bertrand. In his model, duopolists choose both prices and quantities, but incur quadratic output adjustments costs. When these costs are large (small), the outcome resembles the one predicted by Cournot (Bertrand).

Griesmer and Shubik [1963] were the first to formally analyze multi-period price competition between supply-constrained duopolists. They studied an example in which two capacity-constrained firms place nonnegative, nonrandom bids on a sequence of six “jobs” with the same value. Capacity constraints are exogenous, but the duopolists simultaneously set nonnegative prices, chosen from a discrete set, between consumer arrivals. Griesmer and Shubik found that if at least one seller is capacity-constrained and if sellers do not have the same capacity constraint, there is a Markov equilibrium of the six period example in which sellers maximize joint profits. From the standpoint of the sellers, this Pareto dominates other equilibria.

When prices can take any real value, this result holds for any number of buyers (jobs) with the same value, and no other outcomes are supportable as subgame perfect equilibria in pure strategies; see Dudey [1992].[8] When sellers choose their supplies, each chooses to supply about half of the buyers.

Ghemawat [1997] and Ghemawat and McGahan [1998] drop the one customer per period assumption in a model with exogenous supplies. They characterize mixed strategy equilibria in a two-period duopoly where sellers initially hold supplies of K and 2K, and a mass of Q buyers with the same value arrives in each period. A result similar to Griesmer and Shubik’s emerges when K equals Q. In other cases, equilibrium involves random pricing.

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[8] Bester [1988] independently presented some results for a model with discounting.

Bhaskar [2001] examines a version of the one customer per period model in which supply-constrained sellers face a single buyer with demand for one unit in each period. He shows that the buyer can extract a significant amount of surplus, by recognizing that his current purchases affect the intensity of future competition.

Biglaiser and Vettas [2003] consider duopolists facing one or more buyers with multi-unit demands for a durable good in each of two periods. Their paper develops the idea that buyers may divide their orders between sellers in the first period, in order to increase competition in the second period.

The model of the present paper follows Griesmer and Shubik in assuming one customer with unit demand per period and a discrete price set. Basically, the result that sellers will not restrict supply is made possible by two reasonably natural departures from the earlier work: that buyers have random values, and a seller cannot renege on a price offer once it is made.

III. Model

Two vendors, Alpha and Beta, sell units of the same good to $n \geq 2$ buyers. Buyers have unit demands. They know their own values, but vendors never observe a buyer's value. From the vendors' perspective, each buyer's value is independently and identically distributed according to the distribution function F. This function is continuous and strictly increasing on its support, [0, V]. In addition, $F(0) = 0$ and $F(x) \geq kx$, for some positive k. The vendors have a common value of zero and maximize expected profit.

The unit of account, d, is small in a sense to be explained below. The set of prices is **P** = {0, d, 2 d, 3 d, ... L d}, where $d \in \mathbb{R}_{++}$ and L d is the smallest multiple of d satisfying L d > V. The function [1 - F(x)][x] gives expected monopoly profit at a price of x with one buyer. This is strictly increasing up to some $p^M \in \mathbb{R}_{++}$ and strictly decreasing thereafter. Let Π^M denote the monopoly profit, $[1 - F(p^M)][p^M]$.

The buyers come to market at n different times, $t_n < t_{n-1} < ... < t_1$, where $t_i \in \mathbb{R}_{++}$. Each buyer has enough time to visit the market once. Buyer i is the next to arrive when there are i buyers who have not yet arrived. "Period i" is an interval (a_i, b_i), that includes buyer i's arrival time, t_i, but no other buyer's arrival time.

Each vendor arrives at the market with an endowment of n units, and can dispose of any number of units. There is a small disposal cost of c per unit. Quantity decisions are made at t_{n+1}, which precedes t_n. Production or delivery lags prevent immediate restocking within a market session.

In any period after t_{n+1}, there are two stages in which the vendors post prices. In the first, each vendor i chooses an initial asking price, p*(i). In the second stage, vendor i may offer a discount from the initial asking price. This final price, p(i), must satisfy $p(i) \leq p^*(i)$. (In other words, *there is costless enforcement of initial offers by buyers*. As explained in footnote 4, whenever an initial offer matters, the vendor who makes it wants the buyer to be able to enforce it. A vendor can make an initial offer verifiable, e.g. by handing the buyer a written advertisement.) Initial and final prices are elements of **P**.

A buyer makes a purchasing decision after observing the final prices. If the vendors are setting *different* final prices and the buyer's value is not less than the smaller final price, the buyer makes a purchase from the vendor setting the smaller final price. If the vendors are setting the *same* final price, and this does not exceed the buyer's value, then each vendor attracts the buyer with probability 1/2. Finally, if the buyer's value is less than the minimum final price set by the vendors, the buyer leaves and does not return.

Past play can matter only through its effect on the current environment. Specifically, the vendors' initial asking prices in period t can depend only on period t inventory levels up to t units, and t. Their final offers in period t can depend only on the period t inventory levels up to t units, the initial offers in period t, and t. (For example, period 2 prices cannot determine whether vendors play the final prices (d,d) or the final prices (0,0) in period 1, when both vendors hold one unit and initial prices are at least d.)

Mixed strategies that satisfy this restriction and form a subgame perfect equilibrium will be called a Markov equilibrium.[9] In addition to eliminating the type of complication described at the end of the last paragraph, the Markov refinement prevents tacit collusion across different market sessions, as in the repeated games literature.

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[9] See Fudenberg and Tirole [1991] for a more extensive discussion of the Markov concept and its application to examples from economics.

IV. Equilibrium

Applying backward induction and Nash's [1950] theorem, the model has at least one Markov equilibrium.

Consider subgames in which Alpha and Beta hold inventories of x and y units, there are z buyers remaining, no initial asking prices have yet been chosen, and the unit of account is d. Let $EP^A(x, y, z, d)$ and $EP^B(x, y, z, d)$ denote the sets of Markov equilibrium payoffs for Alpha and Beta in these subgames.

The first result has three parts. The first and third parts say that when one of the vendors, say Beta, can supply all z buyers, and the other vendor, Alpha, holds $x \leq z$ units, then Beta earns not less than about $(z - x)\Pi^M$ and Alpha earns about zero, when d is small. According to the second part, Beta cannot earn more than about $(z - x)\Pi^M$ by any restriction of its supply, when Alpha's supply is $x \leq z$ and d is small.

Proposition 1. The following statements - $S_1(z)$, $S_2(z)$, and $S_3(z)$ - hold for $z \geq 1$, and for x, y satisfying $0 \leq x, y \leq z$.

$S_1(z)$: For all ϵ, there exists $d_1(z,\epsilon)$ such that

$$(z - x)\Pi^M - \epsilon \leq \inf EP^B(x, z, z, d)$$

for $d \leq d_1(z,\epsilon)$.

$S_2(z)$: For all ϵ, there exists $d_2(z,\epsilon)$ such that

$$\sup EP^B(x, y, z, d) \leq (z - x)\Pi^M + \epsilon$$

for $d \leq d_2(z,\epsilon)$.

$S_3(z)$: For all ϵ, there exists $d_3(z,\epsilon)$ such that

$$\sup EP^A(x, z, z, d) \leq \epsilon$$

for $d \leq d_3(z,\epsilon)$.

This sharpens and extends to n buyers the analysis of the two buyer case in the introduction. There, it was suggested that when $x = 1$ and $y = z = 2$ and d is small, Alpha earns approximately zero and Beta earns approximately the monopoly profit. With $x = 1$ and $y = z = 2$, $S_3(2)$ says Alpha will earn about zero, while $S_1(2)$ and $S_2(2)$ say that Beta will earn about Π^M, when d is small.

The central result is that duopolists do not choose quantities less than n. Thus, they engage in repeated Bertrand competition for the remainder of the game.

Proposition 2. If d is sufficiently small, there is no equilibrium in which one or both vendors ever choose quantities less than n. It follows that in any equilibrium, vendors do not restrict supply below n, and price near zero.

As explained below, proposition 2 depends critically on a nonzero cost of disposal. I view this as a reasonable assumption. Moreover, since the statement of proposition 2 does not depend on the size of c, the disposal cost needed to prevent supply restriction can be made arbitrarily small by shrinking the mesh of the price grid.

There are two reasons for assuming $c > 0$. The first can be seen in a two buyer example in which buyer values are distributed on [0,1]. There is an equilibrium in which both vendors earn 0 if Alpha does not restrict supply. Now suppose Alpha restricts its supply to one unit. In the second stage of period 1, if both vendors hold at least one unit, one equilibrium price pair is (0, 0) and each vendor's profit is zero. Assume the vendors always play this second stage equilibrium. Next consider period 2, where Alpha has one unit and Beta has two. Beta does not want to make the sale at a low price, and so would never set an initial or final price of zero. Alpha will therefore set a positive price as well and earn positive expected profit. Thus, Alpha does better by restricting supply.

The second reason for assuming $c > 0$ may be understood with a slightly modified version of the example just presented. To sidestep the problem described in the last paragraph, suppose that if Alpha restricts supply to one unit, the first buyer would make a purchase from Alpha with probability one when both vendors price at zero (i.e., asymmetric price tie-breaking). If $c = 0$, there would be an equilibrium in which Alpha restricts supply to one unit and earns zero, as well as an equilibrium in which both vendors earn zero because neither restricts supply.

V. Price inflexibility

There will be a strategic incentive to restrict supply when prices are inflexible. A cost of changing prices (“menu costs”) offer one possible, economic explanation of price inflexibility, which may be of particular importance when the time between buyer arrivals is short. In this section, duopolists simultaneously post prices that are entirely inflexible; that is, price competition is static as in two-stage games such as Kreps and Scheinkman [1983].

With static price competition, if neither vendor reduces supply, each will earn close to zero. Now suppose one vendor, say Alpha, reduces supply to n - 1 units at a cost of c. If Beta prices near p^M, at hd, Beta will earn at least $[1 - F(hd)]^{n-1} [1 - F(hd)] [hd]$.[10] If Beta sets a smaller price of p, then Beta earns no more than $n[1 - F(p)][p]$. It follows that Beta will not price below q, where q satisfies $n[1 - F(q)][q] = [1 - F(hd)]^n[hd]$, and therefore lies between 0 and hd. If d is small enough, Alpha can price at q - d without being undercut. Thus, Alpha’s supply restriction is profitable when c and d are small enough.

Proposition 3. The Cournotian incentive to restrict supply emerges when prices are inflexible relative to the rate at which buyers arrive, and c and d are sufficiently small.

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[10] Beta earns at least this when Alpha undercuts or prices at p^M. Beta earns more if Alpha overcuts.

VI. Upfront costs

Upfront production costs, such as a cost of building capacity, create a nonstrategic incentive to restrict supply.

For example, suppose the vendors build capacity at a cost of .15 per unit before meeting two buyers, and that the marginal cost of production is zero. Assume each buyer's value is independently and uniformly distributed on [0,1].

A monopolist who chooses a production capacity of two will earn about (.50) - 2(.15) = .20. A monopolist who chooses a production capacity of one will choose a first period price s to maximize (1 - s)(s) + s(monopoly profit). With small d, the monopoly profit is about .25, the optimal s is about 5/8 and the seller's profit is about .39 - .15 = .24. Thus, a monopolist prefers a production capacity of one.

Now if each of two vendors has one unit, there is a price p at which each vendor is indifferent between undercutting and overcutting. This price is close to the solution of (1 - x)x = (1 - x)(.25), so p is about .25. Thus, in period 1, each vendor will price near 1/4 and earn about 3/16 - .15 = .04 each.

Using proposition 1, if one vendor has one unit and the other has two, the first vendor earns about 0 - .15 and the second earns about .25 - 2(.15) = -.05. The approximate payoffs are summarized below:

			Beta	
		0	1	2
	0	(0, 0)	(0,.24)	(0,.20)
Alpha	1	(.24, 0)	(.04, .04)	(-.15, -.05)
	2	(.20,0)	(-.05, -.15)	(-.30, -.30)

First notice that the quantity 2 is strictly dominated in the reduced game. It is then easy to see that the unique equilibrium quantities are (1,1), where duopolists each earn positive profit.

The model with upfront production costs does not necessarily lead to a duopoly outcome. With a smaller cost of .08 per unit, the reduced quantity game changes to

		Beta		
		0	1	2
Alpha	0	(0, 0)	(0,.31)	(0,.34)
	1	(.31, 0)	(.11, .11)	(-.08, .09)
	2	(.34, 0)	(.09, -.08)	(-.16, -.16)

The duopoly outcome (1,1) can still occur. However, the quantity 2 is no longer strictly dominated, and in fact monopoly -- (2, 0) or (0, 2) -- is now also possible.

The strategic incentive to restrict supply does not appear in either example. Given the other's supply, a vendor does not increase her revenue by restricting her supply. Each vendor is reducing supply only to lower her own costs, given the other vendor's choice of quantity.

(The last example can be used to show that total surplus in equilibrium may be greater or less than when each duopolist required to produces enough units to serve the entire market. In other words, the cost-saving incentive has an ambiguous effect on total surplus.)

VII. Other strategic incentives to restrict supply

This section examines two model assumptions, aside from quantity choice, that do not appear in Griesmer and Shubik's original example. First, I study a version of the model in which there are no initial offers, i.e., sellers can back away from initial offers, perhaps because it is too costly to enforce them. The second part of this section introduces a gap between the vendors' value and the buyers' minimum value. It will be shown by example that these variations create strategic incentives to restrict supply.

A. Unenforceable initial offers

This subsection considers a case in which initial offers are too costly to enforce, so only final offers matter. In particular, suppose two buyers have values that are independently and uniformly distributed on [0, 1], and d is small. Assume that Alpha restricts its supply and sets its first period (final) price near $\Pi^M/2$. If Beta undercuts, Beta can earn only about $(1 - F(\Pi^M/2))(\Pi^M/2)$. But Beta can earn about $(1 - F(\Pi^M/2))(\Pi^M)$ by setting its price slightly above Alpha's and waiting for the next buyer. Thus, Alpha can profit from supply restriction.

There is also an equilibrium in which Alpha would undercut Beta at a price near zero, if Alpha restricts supply. In this equilibrium, there is no quantity precommitment and both vendors earn approximately zero profit. However, this equilibrium is Pareto-dominated from the standpoint of the vendors. When initial offers are unenforceable, it seems more likely that vendors would focus on an equilibrium that yields positive profits.

B. Gap case

Now consider a two-buyer example in which initial offers are binding, but the support of the buyers' values is [m d, 1 + md], where 0 < < m. If Alpha restricts supply to one unit, Beta can still price at less than m d, near zero, and "force" Alpha to accept a profit close to zero. However, Beta can do as well with an initial asking price and final price of (m + 1) d in the first period. Alpha can undercut to get md. If Alpha matches (m + 1)d, it earns no more than (m + 1)d/2 + d, which is not greater than m d. Alpha clearly does worse by overcutting, because this leads to Bertrand competition in the next period. Thus, Alpha will undercut to md. Beta has no incentive to reduce its final price, since Beta can get the monopoly profit from the next buyer. Hence supply restriction can be profitable.

Another equilibrium has Alpha undercutting Beta at a price near zero, if Alpha restricts supply. This equilibrium yields approximately zero profits and is therefore Pareto-dominated from the standpoint of the sellers.

VIII. Surplus-increasing price floors

In subgames with one supply-constrained duopolist, the unconstrained seller acts as a monopolist after the smaller rival sells out at a low price. This section explains why a minimum mark-up law increases surplus in such a subgame but can lead to supply restriction when imposed before the quantity precommitment decision.

Again, consider a two buyer example. Suppose the first vendor, Alpha, holds an inventory of one unit and rival Beta holds an inventory of two units. Intuitively, a small price floor raises Alpha's first price to a little more than zero. One effect is to block trades with low value buyers; that is, buyers with values close to zero. But a blocked trade in the first period means that Alpha will carry its unit of inventory into the next period. In the next period, the buyer may have a value that is much higher than zero. Thus, the benefit of a small price floor is that it shifts competition from a period and state of low demand to a period where competition is, on average, more valuable.

On the other hand, a small price floor imposed before sellers choose quantities may have a larger, negative effect on total surplus by causing supply restriction. Notice that in the two buyer example, a minimum price of m means that when Alpha reduces its supply to one unit, it earns $[1 - F(m)][m] + F(m)[(1 - F(m))(m/2)] - c$, which will be greater than its two unit profit of $[1 - F(m)][m/2][2]$ when c is sufficiently small.

IX. Conclusion

For more than 100 years, a conflict between two theories of duopoly has occupied a central place in the literature on imperfect competition. In Cournot's [1838] model, two firms choose output levels before an auctioneer chooses a market-clearing price. The sellers restrict supply to raise the auctioneer's price above marginal cost. The outcome is less efficient than perfect competition. Cournot's analysis motivated Bertrand [1883] to present a model of duopoly in which the firms choose prices, rather than quantities. His model predicts the perfectly competitive outcome.[11]

The Bertrand outcome is sometimes loosely described as a paradox (Tirole [1988], pp. 210-211], Wolfstetter [1999], pp. 116-117), but case studies show that the emergence of excessive inventories can trigger cutthroat competition (Cassady [1957], [1963], [1974] and Scherer [1971]). In the context of an open air market, Cassady [1974], p. 71, noted that competition may be "apathetic" in the early part of a market session, but can become much more intense if sellers become aware of heavy inventories. In cases where "general supplies are *very* heavy, prices may go to zero."

Kreps and Scheinkman [1983] showed that the Cournot (Bertrand) model can be reformulated as a two-stage game, in which firms choose quantities (prices) in the first stage and prices (quantities) in the second. The standard intepretation is that the debate over "who was

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[11] Since Cournot's model predicts the perfectly competitive outcome when the number of firms is large, the two models provide complementary, noncooperative foundations of Walrasian equilibrium.

right" can be recast as a question about which strategic variable, price or quantity, is more flexible. In Cabral's [2000] words, "suppose that firms must make capacity (or output) decisions in addition to pricing decisions ... If capacity and output can be easily adjusted, then the Bertrand model is a better approximation of duopoly competition. If, by contrast, output and capacity are difficult to adjust, then the Cournot model is a good approximation of duopoly competition. Most real-world industries seem closer to the case when capacity is difficult to adjust. In other words, capacity or output decisions are normally the long-run variable, prices being set in the short run."[12]

I have argued that when buyers with unit demands come to market at different times, the standard intepretation of Kreps and Scheinkman's paper depends on price stickiness. Of course, just as Kreps and Scheinkman's theory does not imply that "Cournot was right," the main result of this paper (proposition 2) does not mean that "Bertrand was right."[13]

In addition to linking Cournot-style inefficiency with price stickiness, the main result brings into focus *other* sources of supply restriction that may (or may not) be important when prices are relatively flexible. These include upfront costs of production, discreteness of the price set,

---

[12] Although the particular outcome will depend on the rule that is used to ration demand (Davidson and Deneckere [1986]), there is no argument about whether supply restriction will occur in a two-stage game where quantities are chosen first. As Daughety [1988] pointed out, "[i]n general, however, the basic result stands that the two-stage game in homogeneous products leads to price above marginal cost."

[13] Kreps and Scheinkman warned that "[i]t is witless to argue in the abstract whether Cournot or Bertrand was correct; this is an empirical question or one that is resolved only by looking at the details of the context within which the competitive interactions takes place." Clearly, empirical observations are at the heart of the contentions that quantities are normally the long-run variable, or that the Bertrand outcome is a paradox.

unenforceability of initial offers, and a gap between the sellers' value and the buyers' minimum value.

Griesmer and Shubik's [1963] study of multiperiod price competition between supply-constrained sellers focused on a similar setting. In their model, buyers have the same value and, if allowed to choose quantities, sellers would restrict supply. My approach differs in that buyer values are at least slightly random, and sellers cannot renege on price offers. (An enforceable price offer could take the form of a legal contract; however, this is not necessary on the equilibrium path, because sellers do not restrict supply.)

Upfront costs can lead to monopoly in a version of Bertrand's model with entry; see, for example, Mas-Colell, Whinston and Green [1995], pp. 405-411. Here, such costs can lead to monopoly or even a positive profit duopoly. Examples of a pure, cost-saving model were presented above. A more general analysis is left for future research.

It might also be quite interesting to study versions of the model in which products are not physically the same, or in which buyer arrival times are endogenous. I suspect that some forms of these assumptions could lead to supply restriction. On the other hand, allowing buyers to choose their arrival times will not matter if buyers have most preferred arrival times, and the maximum buyer value is relatively small. One can show that sellers will not restrict supply if buyers are willing to pay their values plus a certain premium for one vendor's good, because that vendor's good is of higher quality.

Appendix A

Proof of proposition 1. For small enough d, a monopolist facing one buyer earns at least Π^M - ϵ and static Bertrand competitors earn no more than ϵ.[14] Thus, $S_1(z)$, $S_2(z)$, and $S_3(z)$ hold for z = 1. The rest of the proof shows, in three steps, that $S_1(z)$, $S_2(z)$, and $S_3(z)$ imply $S_1(z + 1)$, $S_2(z + 1)$, and $S_3(z + 1)$.

Step 1. $S_1(z + 1)$

Basically, $\inf EP^B(x, z + 1, z + 1, d) \geq (z + 1 - x)\Pi^M - \epsilon^*$ is proved by showing that when $x < z + 1$ and d is small enough, Beta is capable of selecting an initial offer that will induce Alpha to make a sale with probability close to one in the first period (period z + 1). This means Beta can't earn less than about $\inf EP^B(x - 1, z, z, d)$, which, by $S_1(z)$, is not less than about $(z + 1 - x)\Pi^M$.

Formally, pick any γ satisfying

$$\gamma < \Pi^M/4 \qquad \text{and} \qquad F(\gamma) < \epsilon^*/[2(z + 1 - x)\Pi^M] \qquad [1]$$

---

[14] Matching the highest price kd set by the rival with positive probability is dominated by undercutting it as long as $[1 - F[(k-1)d]][(k - 1)d] > [1 - F(kd)][(kd)/2]$, or $2(k - 1)/k > [1 - F(kd)]/[1 - F[(k-1)d]]$ which is satisfied when $k > 1$. It follows that the sellers will not price above d.

and assume d is much smaller than γ. Suppose that in period z + 1 Beta makes an initial offer, p*, that is the largest multiple of d that is not greater than γ. Since p* is a price ceiling, Beta will place positive probability weight only on final prices that are no greater than p*.

Now suppose Beta places positive probability weight on some final price $\beta d < p^*$, and Alpha's random price is p^{α}. Then when Beta is setting this price it earns

$$
\begin{aligned}
&\sum_{\{\alpha:\, \alpha < \beta\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F(\alpha d)][v_1] + F(\alpha d)[v_2] \} \\
&+ \Pr(p^{\alpha} = \beta d)\{[1 - F(\beta d)][(\beta d + v_2)/2 + (v_1)/2] + F(\beta d)[v_2]\} \qquad [2] \\
&+ \sum_{\{\alpha:\, \alpha > \beta\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F(\beta d)][\beta d + v_2] + F(\beta d)[v_2]\}
\end{aligned}
$$

where v_1 is Beta's continuation payoff in period z when Alpha makes a sale and v_2 is Beta's continuation payoff in period z after either Beta makes the sale or no sale is made.[15] When Beta increases its nonrandom prices to $(\beta+1)d \leq p^*$, Beta earns

$$
\begin{aligned}
&\sum_{\{\alpha:\, \alpha < \beta+1\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F(\alpha d)][v_1] + F(\alpha d)[v_2]\} \\
&+ \Pr(p^{\alpha} = (\beta+1)d)\{[1- F((\beta+1)d)][(\beta+1)d + v_2)/2 + (v_1)/2] + F((\beta+1)d)(v_2)\} \qquad [3] \\
&+ \sum_{\{\alpha:\, \alpha > \beta+1\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F((\beta+1)d)][(\beta+1)d + v_2] + F((\beta+1) d)(v_2)\}
\end{aligned}
$$

---

[15] Under the Markov assumption, a vendor's continuation payoff in period z depends only on period z inventory levels up to z and z.

The difference between [3] and [2] is

$$\Pr(p^{\alpha} = \beta d)\{[1 - F(\beta d)][v_1] + [F(\beta d)][v_2]\}$$

$$- \Pr(p^{\alpha} = \beta d)\{[1 - F(\beta d)][(\beta d + v_2)/2 + v_1/2] + [F(\beta d)][v_2]\}$$

$$+ \Pr(p^{\alpha} = (\beta+1)d)\{[1 - F((\beta+1)d)][((\beta+1)d + v_2)/2 + (v_1)/2] + F((\beta+1)d)(v_2)\}$$

$$- \Pr(p^{\alpha} = (\beta+1)d)\{[1 - F((\beta d)][\beta d + v_2] + F((\beta d)][v_2]\}$$

$$+ \sum_{\{\alpha:\, \alpha > \beta+1\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F((\beta+1)d)][(\beta+1)d] \; - \; [1 - F((\beta d)][\beta d]\}$$

=

$$\Pr(p^{\alpha} = \beta d)\{[1 - F(\beta d)][(v_1/2 - v_2/2 - \beta d/2]\}$$

$$+ \Pr(p^{\alpha} = (\beta+1)d)\{[1 - F((\beta+1)d)][(\beta+1)d/2 - v_2/2 + v_1/2] + v_2\}$$

$$- \; \Pr(p^{\alpha} = (\beta+1)d)\{[1 - F(\beta d)][\beta d] + v_2\}$$

$$+ \sum_{\{\alpha:\, \alpha > \beta+1\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F((\beta+1)d)][(\beta+1)d] \; - \; [1 - F((\beta d)][\beta d]\}$$

=

$$\Pr(p^{\alpha} = \beta d)\{[1 - F(\beta d)][(v_1/2 - v_2/2 - \beta d/2]\}$$

$$+ \Pr(p^{\alpha} = (\beta+1)d)\{[1 - F((\beta+1)d)][(\beta+1)d/2 + (v_1 - v_2)/2 \,] - [1 - F(\beta d)][\beta d]\}$$

$$+ \sum_{\{\alpha:\, \alpha > \beta+1\}} \Pr(p^{\alpha} = \alpha d)\{[1 - F((\beta+1)d)][(\beta+1)d] \; - \; [1 - F((\beta d)][\beta d]\}$$

Each term in curly brackets is positive when d is sufficiently small. First note that from $S_1(z)$ and $S_2(z)$, $v_1 - v_2$ is close to Π^M when d is small. Using [1], $\beta d < p^* < \Pi^M/4 < p^M$. In addition, it was assumed that $(1 - F(x))x$ is increasing for $x < p^{M}$.[16] It follows that in equilibrium, either Beta final prices at p* with probability 1 or $\sum_{\{\alpha:\, \alpha < \beta\}} \Pr(p^{\alpha} = \alpha d) = 1$ if d is sufficiently small.

Using $S_3(z)$, if Beta is pricing at p* with probability 1, then Alpha will undercut when d is small enough. Of course, if $\sum_{\{\alpha:\, \alpha < \beta\}} \Pr(p^{\alpha} = \alpha d) = 1$, Alpha is undercutting with probability one. From $S_1(z)$ and [1], Beta gets at least $[1 - F(p^*)][\inf EP^{B}(x - 1, z , z , d)] \geq [1 - F(p^*)][(z + 1 - x)\Pi^{M} - \epsilon^*/2] \geq [1 - F(\gamma)][(z + 1 - x)\Pi^{M} - \epsilon^*/2] \geq (z + 1 - x)\Pi^{M} - \epsilon^*$, when d is small enough.

Step 2. $S_2(z + 1)$

In period z + 1, Alpha and Beta are choosing possibly random final prices which are realized at various αd and βd. Beta can earn no more than:

$$\begin{aligned}
&\sum_{\{(\alpha,\beta):\, \alpha < \beta\}} \Pr(p^{\alpha} = \alpha d \text{ and } p^{\beta} = \beta d) \{[1 - F(\alpha d)][\sup EP^{B}(x- 1, y, z, d)] + \\
&\qquad\qquad\qquad\qquad F(\alpha d)\sup EP^{B}(x, y, z, d)\} \\
&+ \sum_{\{(\alpha,\beta):\, \alpha > \beta\}} \Pr(p^{\alpha} = \alpha d \text{ and } p^{\beta} = \beta d) \{[1 - F(\beta d)][\beta d + \sup EP^{B}(x, y - 1, z, d)] + \\
&\qquad\qquad\qquad\qquad F(\beta d)\sup EP^{B}(x,y,z,d)\} \\
&+ \sum_{\{(\alpha,\beta):\, \alpha = \beta\}} \Pr(p^{\alpha} = \alpha d \text{ and } p^{\beta} = \beta d)\{[1 - F(\alpha d)][(1/2)(\alpha d + \sup EP^{B}(x, y - 1, z, d)) \\
&\qquad\qquad + (1/2)\sup EP^{B}(x - 1, y, z, d)] + F(\alpha d)\sup EP^{B}(x, y, z, d)\}.
\end{aligned}$$

---

[16] In the second line, note $(\beta+1)d/2 < \Pi^{M}/4 < (v_1 - v_2)/2$. Thus, the expression in the curly brackets is greater than $[1 - F((\beta+1)d)][(\beta+1)d] - [1 - F(\beta d)][\beta d]$, which is positive because the profit function is increasing.

Using $S_2(z)$, the expression inside each set of curly brackets is not greater than $(z + 1 - x)\Pi^M + \epsilon^*$ when d is sufficiently small.

Step 3. $S_3(z + 1)$

Suppose that for some x and $\epsilon^* < \Pi^M$, there does not exist $d_2(z + 1, \epsilon^*)$ such that $\sup EP^A(x, z + 1, z + 1, d) \leq \epsilon^*$ for all $d \leq d_2(z + 1, \epsilon^*)$. Then there exists a sequence $\{d^i\}$ converging to zero such that $\sup EP^A(x, z + 1, z + 1, d^i) > \epsilon^*$ for each d^i. For each d^i, select a subgame equilibrium $E(d^i)$ that yields more than ϵ^* to Alpha.

The argument is based on the calculation of an upper bound on Beta's payoff in $E(d^i)$, which converges to a number less than $(z + 1 - x)\Pi^M$ as d^i goes to zero. This contradicts $S_1(z + 1)$, and establishes $S_3(z + 1)$.

First note from that when d^i is small enough, Beta must be placing positive probability weight on at least one initial price $p'(d^i)$ that gives at least $3\epsilon^*/4$ to Alpha <u>in period z + 1</u>, given Alpha's random initial price. (From $S_3(z)$, Alpha's earnings after period z + 1 become trivial as d^i becomes smaller.) Beta earns its equilibrium profit in $E(d^i)$ when charging $p'(d^i)$, and assume in what follows that Beta is charging $p'(d^i)$.

Let s be the probability that Alpha chooses an initial price that gives her an expected payoff of at least $\epsilon^*/2$ in period z + 1, when Beta prices at $p'(d^i)$. Note that $s \geq \epsilon^*/(4\Pi^M - 2\epsilon^*)$, since $3\epsilon^*/4 \leq (1 - s)\epsilon^*/2 + s\Pi^M$.

Now if Beta prices at $p'(d^i)$, and Alpha chooses an initial price that gives Alpha an expected payoff of at least $\epsilon^*/2$ in period $z + 1$, Alpha's final price will always be less the highest price that Beta charges with positive probability when d^i is sufficiently small.[17] (Let $B(d^i)$ be the highest final price that Beta sets with positive probability. Note that since Alpha is earning at least $\epsilon^*/2$ in period $z + 1$, it must be that $B(d^i) > x$, where $(1 - F(x))x = \epsilon^*/2$. Alpha will not overcut $B(d^i)$, because then it cannot earn is earning at least $\epsilon^*/2$ in period $z + 1$. If Alpha matches $B(d^i)$, then Alpha earns

$$(1 - F(B(d^i))))((B(d^i) + w_1)/2 + w_2/2) + F(B(d^i))(w_2) =$$

$$(1 - F(B(d^i)))((B(d^i) + w_1)/2 - w_2/2) + w_2 \qquad [4]$$

where w_1 is Alpha's period z continuation payoff if Alpha makes the sale, and w_2 is Alpha's period z continuation if Beta makes the sale or no sale is made. If Alpha undercuts Beta by d^i, then Alpha gets $(1 - F(B(d^i) - d^i))((B(d^i) - d^i + w_1) + F(B(d^i) - d^i)(w_2) \geq$

$$(1 - F(B(d^i))((B(d^i) - d^i + w_1 - w_2) + w_2 \qquad [5]$$

---

[17] This is explained in the following parenthetical remark.

since $B(d^i) > x$, and by $S_3(z)$, $w_1 - w_2$ is close to zero when d^i is small. Thus, undercutting by Alpha is more profitable if [5] > [4], or $B(d^i)/2 + (w_1 - w_2)/2 > d^i$. Using $S_3(z)$ and the fact that $B(d^i) > x$, Alpha always undercuts $B(d^i)$, for small enough d^i.)

Finally, consider Beta's payoff when Beta prices at $p'(d^i)$. If Alpha chooses an initial price that gives Alpha an expected payoff of less than $\epsilon^*/2$, which happens with probability no more than $1 - s$, then the step 2 proof of $S_2(z + 1)$ shows that Beta's payoff is bounded above by a number that converges to $\Pi^M(z - x + 1)$ as d^i goes to zero.

On the other hand, if Alpha chooses an initial price that gives Alpha an expected payoff of at least $\epsilon^*/2$, which happens with probability at least s, Beta's payoff is what she gets when final pricing at $B(d^i)$. Alpha undercuts $B(d^i)$ with probability 1 -- given small enough d^i. Thus, the question is what Beta earns when it prices at $B(d^i)$. If Alpha makes the sale, $S_2(z)$ implies Beta's payoff is not more than a number that converges to $\Pi^M(z - x + 1)$ as d^i goes to zero. However, when Alpha does not make the sale, $S_2(z)$ implies Beta's payoff is not more than a number that converges to $\Pi^M(z - x)$ as d^i goes to zero. The probability that Alpha chooses a final price of at least $\epsilon^*/4$ is $r \geq \epsilon^*/(4\,\Pi^M - \epsilon^*)$, since $\epsilon^*/2 \leq (1 - r)\,\epsilon^*/4 + r\,\Pi^M$. When Alpha's final price is at least $\epsilon^*/4$, the probability that the buyer's value is less than Alpha's final price is at least $k\ \epsilon^*/4$. Thus, Alpha does not make the sale with probability at least $r\,k\,\epsilon^*/4$. ∎

Appendix B

Proof of proposition 2 If not, then one vendor, say Beta, is choosing some $y < n$ with positive probability. Beta earns the expectation of $\Pi^B(x, y, n, d) - c(n - y)$ over x, where $\Pi^B(x, y, n, d)$ is Beta's equilibrium profit when supplies of Alpha and Beta are x and y. Using $S_2(n)$ with $\epsilon = c/4$, this is not more than the expectation of $(z - x)\Pi^M + c/4 - c(n - y)$ over x, for d sufficiently small. Beta can instead earn the expectation of $\Pi^B(x, n, n, d)$ over x by not restricting supply. Using $S_1(n)$ with $\epsilon = c/4$, this is not less than the expectation of $(z - x)\Pi^M - c/4$ over x if d is sufficiently small. But $(z - x)\Pi^M - c/4 > (z - x)\Pi^M + c/4 - c(n - y)$. No supply restriction means sellers price near zero; see fn. 14. ▮

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