

Some fiscal calculus.

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What is the impact of fiscal policy on the economy? How large are the “multipliers” of government spending and tax cuts? This old question has recently received considerable attention, in particular in the context of the American Recovery and Reinvestment Act (ARRA) of 2009.

I contribute to answering that question by calculating fiscal multipliers in a baseline neoclassical growth model with endogenous labor supply and fiscal policy, allowing for government spending transfers, government debt and distortionary taxes on labor and capital income. The policy experiments are conducted holding transfers, consumption taxes and capital income taxes fixed, i.e. changes in taxation require changes in the distortionary labor tax. The model is simple and fairly standard. As an additional contribution, I provide the model in an elegant and tractable form, starting from a fairly general formulation and pointing out the key functional form properties for characterizing the dynamics. The formulation of the model here or versions thereof may be useful for a variety of purposes.

I shall argue, that short-run fiscal multipliers can be dramatically misleading. In the model here, the initial effect of a government spending as in the ARRA stimulates output, see figure 3, initially generating net present fiscal multipliers well in excess of unity, see figure 2. However, this turns into a prolonged below-trend performance of output, as the tax increases necessary to repay the increased debt impact on the economy, see figure 4 and 5. The net present fiscal multiplier for government spending turns negative too, as the horizon increases. Indeed, according to this

model and its parameterization, 5.8 dollars of output are lost eventually for every dollar of increased government spending, when discounting them to the current period. By contrast, for a tax cut and each discounted dollar given up in terms of taxing labor, one obtains an increase of 2.4 dollars in discounted output eventually.

Typical fiscal stimulus debates concentrate on output effects and not on the consequences for overall welfare. This paper follows that practice. It is therefore a positive, not a normative analysis. Whether these output responses are desirable or not needs to be evaluated by other means. For example, while the model here features a representative agent, it is not hard to derive this model from a heterogeneous agent model, in which agents possess different amounts of capital and care about government goods differently, resulting in heterogeneous welfare effects across the population. One therefore should not draw welfare conclusions from the consumption and leisure path for the representative agent.

Despite these caveats, the results here point to potentially drastic long-term consequences of fiscal stimuli. These long-term consequences ought to receive more and sufficient attention in the debates. Without their discussion, these debates appear to be severely incomplete.

The model is very stylized. Richer features may be desirable to provide a fully adequate discussion of fiscal policy effects. For example, much of the current debate appears to argue that Keynesian features such as sticky prices and rules-of-thumb consumers lead to a larger and more positive effect of a fiscal stimulus. The issue is taken up in Drautzburg-Uhlig (2010), who employ a New Keynesian model to study the issues raised here. It turns out that the initial impact on output is actually more muted in the New Keynesian model, as the wealth effect on labor supply is diluted in a Keynesian environment of sticky wages and demand-driven labor markets. Furthermore, the long-run feature of

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typical New Keynesian models are neoclassical in nature: therefore, the long-run issues raised here are surely not a matter of principle, but a matter of degree.

I. The model

Time is discrete, counting quarters. The representative agent enjoys consumption and leisure according to

$$U = E \left[\sum_{t=0}^{\infty} \beta^t \frac{(c_t \Phi(n_t))^{1-\eta} - 1}{1-\eta} \right]$$

plus possibly a separable term in government spending $(g_t)_{t=0}^{\infty}$, where c_t denotes consumption, n_t denotes labor, $1/\eta > 0$ is the intertemporal elasticity of substitution, and $\Phi(\cdot)$ is strictly positive, decreasing, concave and thrice differentiable. As is well known for time-separable preferences, this form of preferences is necessary in order to be consistent with long run growth, i.e. to be consistent with a balanced growth path of growing consumption and constant labor.

The agent maximizes his or her lifetime utility, subject to the constraints (1) and (2). The budget constraints are

$$(1) \quad (1 + \tau^c)c_t + x_t + b_t = R_t^b b_{t-1} + s_t + m_t \\ + (1 - \tau_t^n)w_t n_t + (r_t - \tau^k(r_t - \delta))k_t$$

where $c_t, x_t, b_t, w_t, n_t, r_t, k_t, R_t^b, s_t$ and m_t denote consumption, investment, real government bonds, wages, labor, capital rental rates, capital, government bond returns (fixed at $t-1$), lump sum transfers from the government and (unmodelled) lump sum transfers from the rest of the world due to a nonzero international asset position. Further, τ^c, τ_t^n, τ^k are taxes on consumption, wage income and capital income. Capital is accumulated according to

$$(2) \quad k_{t+1} = (1 - \delta)k_t + x_t$$

Production takes place in a competitive sector of firms with the production function

$$y_t = f \left(\frac{A_t n_t}{k_t} \right) k_t$$

where y_t denotes output and A_t is a (labor-

augmenting) technology parameter. I assume that f is positive, concave and differentiable thrice. The usual first-order condition deliver wages and capital rental rates,

$$w_t = A_t f' \left(\frac{A_t n_t}{k_t} \right) \\ r_t k_t = y_t - w_t n_t$$

I assume that $\zeta_t = \frac{A_t}{A_{t-1}}$ is a stationary exogenous stochastic process. In the non-stochastic case, $\zeta_t \equiv \bar{\zeta}$. With this, define $\tilde{\delta} = \bar{\zeta} - 1 + \delta$ and $\tilde{\beta} = \beta \bar{\zeta}^{1-\eta}$. I assume that $\beta, \bar{\zeta}$ and η are such that $0 < \tilde{\beta} < 1$. I assume that $m_t/A_t \equiv \bar{m}$.

Introducing the auxiliary variable d_t (for “deficit remaining”), I write the government budget constraints as

$$d_t = g_t + s_t + R_t^b b_{t-1} - \tau^c c_t - \tau^k (r_t - \delta) k_t \\ = b_t + \tau_t^n w_t n_t$$

For the simulations, I shall specify an exogenous stochastic process for g_t and s_t in section II. I assume that excess deficits are repaid at speed ψ per

$$\tau_t^n w_t n_t - A_t \bar{\tau}^n \bar{w} \bar{n} = \psi (d_t - A_t \bar{d})$$

for some $\psi \in (0, 1]$. Thus,

$$b_t - A_t \bar{b} = (1 - \psi)(d_t - A_t \bar{d})$$

Aggregate feasibility is

$$c_t + x_t + g_t = y_t + m_t$$

Equilibrium is defined in the usual way.

For the calibration, I follow Trabandt and Uhlig (2009), adapted to quarterly frequency. The basic parameters and their calibration are $\bar{\zeta} = 1.005, \delta = 0.02, \eta = 2$ and β per $\bar{R} = \zeta^\eta / \beta = 1.01$. The steady state levels of taxes, debt, spending and payments on foreign assets are $\bar{\tau}^n = 0.28, \tau^c = 0.05, \tau^k = 0.36, \bar{b}/\bar{y} = 4 * 0.63, \bar{g}/\bar{y} = 0.18$ and $\bar{m}/\bar{y} = 0.04$. This implies $\bar{c}/\bar{y} = 0.59, \bar{x}/\bar{y} = 0.27, \bar{s}/\bar{y} = 0.07$. A crucial parameter is the speed of tax adjustment. I set it at a rather slow pace, $\psi = 0.05$, still assuring stability. Key characteristics of the functions like the labor share θ or labor supply and demand elasticities ω_S, ω_D are given in table 1. I

assume a labor share of 62 percent and a Frisch elasticity of 1. I assume the production function to be Cobb-Douglas, $f(x) = x^\theta$. With this, $\kappa, \omega_S, \omega_D$ can be calculated.

abbrev.	expression	value
$\theta =$	$\frac{f'(\bar{n}/\bar{k})\bar{n}/\bar{k}}{f(\bar{n}/\bar{k})} = \bar{w}\bar{n}/\bar{y}$	0.62
$1/\omega_D =$	$\frac{-f''(\bar{n}/\bar{k})\bar{n}/\bar{k}}{f'(\bar{n}/\bar{k})}$	0.38
$\kappa =$	$\frac{-\Phi'(\bar{n})\bar{n}}{\Phi(\bar{n})} = \frac{1-\bar{\tau}^n}{1+\bar{\tau}^c} \theta \frac{\bar{y}}{\bar{c}}$	0.72
$1/\omega_S =$	$\left(\frac{\Phi'(\bar{n})\bar{n}}{\Phi(\bar{n})} + \kappa \right)$ (= 1/“Frisch elasticity” $-(1-1/\eta)\kappa$)	0.64

TABLE 1—KEY CHARACTERISTICS OF THE FUNCTIONS

I log-linearize the detrended equations around the steady state. Hats normally denote log-deviations, e.g. $\hat{c}_t = \log(\tilde{c}_t) - \log(\bar{c}) \approx (\tilde{c}_t - \bar{c})/\bar{c}$, where $\tilde{c}_t = c_t/A_t$ and \bar{c} is the steady state of \tilde{c}_t . I use (hundredth of) percentage points for the tax rate, $\hat{\tau}_t^n = \tau_t^n - \bar{\tau}^n$. Furthermore, $\hat{b}_t, \hat{d}_t, \hat{g}_t$ and \hat{s}_t are expressed relative to steady state output, e.g. $\hat{b}_t = (\tilde{b}_t - \bar{b})/\bar{y}$.

The labor market equilibrium is given by

$$\begin{aligned}\hat{w}_t &= \frac{1}{\omega_S} \hat{n}_t + \frac{1}{1-\bar{\tau}^n} \hat{\tau}_t^n + \hat{c}_t \\ \hat{w}_t &= \frac{1}{\omega_D} (\hat{k}_t - \hat{n}_t)\end{aligned}$$

Production and feasibility are

$$\begin{aligned}\hat{y}_t &= \theta \hat{n}_t + (1-\theta) \hat{k}_t \\ \hat{y}_t &= \frac{\bar{c}}{\bar{y}} \hat{c}_t + \frac{\bar{x}}{\bar{y}} \hat{x}_t + \hat{g}_t\end{aligned}$$

Government policy follows

$$\begin{aligned}\hat{d}_t &= \hat{g}_t + \hat{s}_t + \frac{1}{\beta} \hat{b}_{t-1} \\ &\quad + \frac{\bar{b}}{\beta \bar{y}} (\hat{R}_t^b - \hat{\zeta}_t) - \tau^c \frac{\bar{c}}{\bar{y}} \hat{c}_t \\ &\quad - \tau^k \left(1 - \theta - \frac{\delta \bar{x}}{\delta \bar{y}} \right) \hat{k}_t - \tau^k (1-\theta) \hat{r}_t \\ \psi \hat{d}_t &= \theta \hat{\tau}_t^n + \theta \bar{\tau}^n (\hat{w}_t + \hat{n}_t) \\ (1-\psi) \hat{d}_t &= \hat{b}_t\end{aligned}$$

Capital accumulation and returns satisfy

$$\begin{aligned}\hat{k}_t &= (1-\tilde{\delta}) \hat{k}_{t-1} + \tilde{\delta} \hat{x}_{t-1} - \hat{\zeta}_t \\ \hat{r}_t &= -\frac{\theta}{1-\theta} \hat{w}_t \\ \hat{R}_t^{(k)} &= \left(1 - (1 - (1 - \tau^k) \delta) \beta \bar{\zeta}^{-\eta} \right) \hat{r}_t \\ \hat{\lambda}_t &= -\eta \hat{c}_t - (1-\eta) \kappa \hat{n}_t \\ 0 &= E_t \left[\hat{\lambda}_{t+1} - \lambda_t + \hat{R}_{t+1} \right]\end{aligned}$$

where the last equation shall hold for $\hat{R}_{t+1} = \hat{R}_{t+1}^{(k)}$ as well as $R_{t+1} = \hat{R}_{t+1} = E_t[\hat{R}_{t+1}^{(b)}]$. The model can be solved with standard methods, as e.g. Uhlig (1999). Code for the latter is available at the AER website.

II. Results

To capture the fiscal stimulus of the American Recovery and Reinvestment act (ARRA), I assume that $t = 1$ and the date of the fiscal surprise correspond to the second quarter of 2009. I assume that \hat{g}_t follows an AR(2) with the two roots $\xi_1 = 0.933$ and $\xi_2 = 0.72$, or, equivalently, $\hat{g}_t = 1.653 \hat{g}_{t-1} - 0.672 \hat{g}_{t-2}$. As initial conditions, I pick $\hat{g}_0 = 0$ and $\hat{g}_1 = 0.32\%$ of \bar{y} . Initially, the resulting path for government spending follows closely the path documented by Cwik et al (2010), reaching a peak of 78% of \bar{y} in the sixth quarter, i.e. the second quarter of 2010, see figure 1. Spending then falls off more slowly here than in Cwik et al (2010), due the rather simple AR(2) for \hat{g}_t . Less technically, it reflects a skepticism that stimulus spending will truly return back to normal as quickly as envisioned by the ARRA. It may be appropriate to build this skepticism into a rational expectations model such as this one, and therefore I do. I also experiment using the process above for \hat{s}_t rather than \hat{g}_t : the results will be reported in an extended version of this paper. Furthermore, I calculate the effects of an initial tax cut.

Figure 2 shows net present value fiscal multipliers. To calculate the net present value fiscal multiplier at date t , I sum output up to that date, discounted at the steady state interest rate, and divide by government spending calcu-

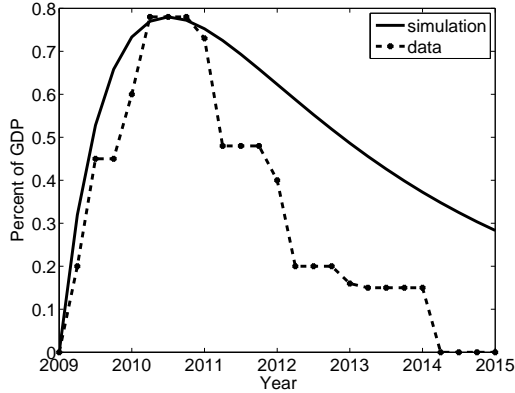


FIGURE 1. FISCAL STIMULUS: DATA PER CWIK ET AL (2010) VERSUS THE SIMULATION HERE.

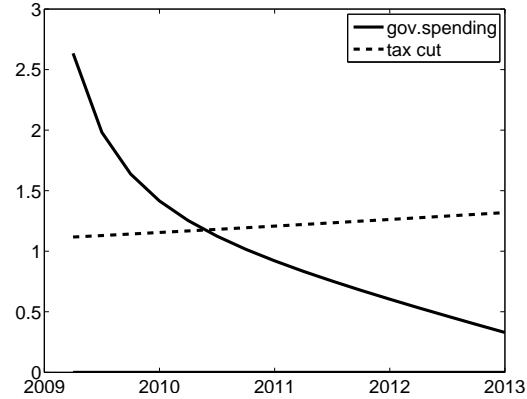


FIGURE 2. FISCAL MULTIPLIERS

lated the same way,

$$\varphi_t = \sum_{s=1}^t \bar{R}^{-s} \hat{y}_s / \sum_{s=1}^t \bar{R}^{-s} \hat{g}_s$$

noting that \hat{g}_t is expressed relative to \bar{y} . I proceed likewise for the fiscal multiplier for taxes, replacing \hat{g}_s by the tax revenue loss relative to \bar{y} , $-\theta\hat{\tau}_s^n - \theta\bar{\tau}^n(\hat{w}_t + \hat{n}_t)$. The figure shows the tax fiscal multiplier to start at about 1 and reach 1.3 at the beginning of 2013. By contrast, the net present government spending multiplier is initially substantially higher, starting at 2.6 and gradually declining to 0.3 at the beginning of 2013. Figure 3 shows the dynamics of output and government spending during that time. The model generates an initial boost in output due to the anticipation of the increase in government spending and taxes. The reduction in wealth leads the representative agent to consume less leisure, i.e. supply more labor, thereby boosting output initially. One may be tempted to conclude from this picture that government spending is successful in stimulating the economy and generating large fiscal multipliers. Indeed, these numbers are similar or even more optimistic than the multipliers for a permanent fiscal change provided in the appendix of Romer-Bernstein (2009), the official White House document providing an economic analysis for the ARRA.

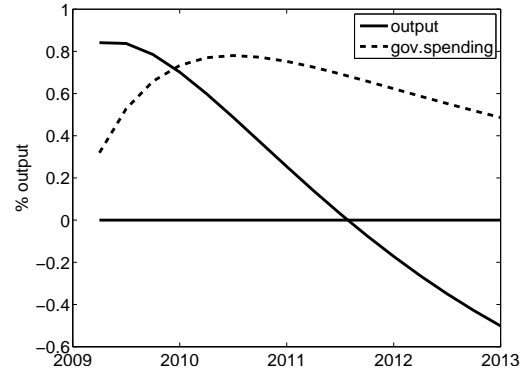


FIGURE 3. OUTPUT AND GOVERNMENT SPENDING: 4 YEARS.

This, however, is a misleading interpretation. The expansion in government spending needs to be financed. It is financed by debt initially and by higher taxes eventually. Figure 4 shows the resulting debt dynamics (in percent of \bar{y}) and labor tax rates (in percentage points) over the next 40 rather than the next 4 years. The increased tax burden in later years leads to disincentives on the labor market, creating a very persistent and prolonged below-trend performance for output. This can be seen in figure 5. That figure shows the output dynamics and government spending dynamics for the next 40 years, rather than 4 years as in figure 3. The net present value government spending multi-

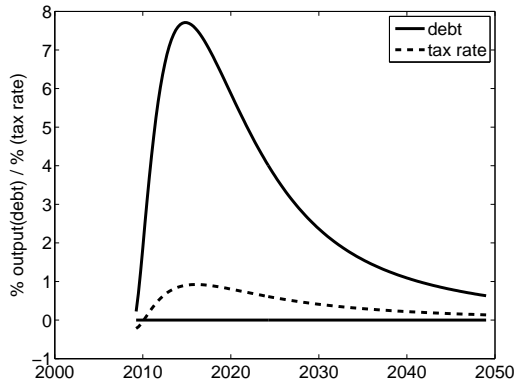


FIGURE 4. DEBT AND TAXES: 40 YEARS.

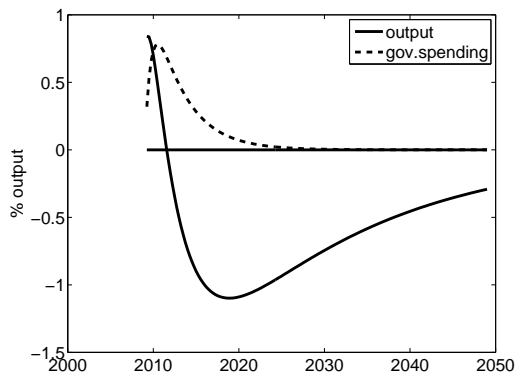


FIGURE 5. OUTPUT AND GOVERNMENT SPENDING: 40 YEARS.

plier for this experiment converges to a value of -5.8 eventually, i.e. for every dollar of extra government spending, 5.8 dollars of output are lost eventually, when discounting them to the beginning of 2009. For a similar exercise, but boosting transfers rather than government spending, the present value multiplier converges to -2.4. By contrast, the limit value for the net present tax multiplier is 2.4, i.e., each dollar given up in terms of taxing labor generates nearly 2.4 dollars in extra output, when discounted to the beginning of 2009.

III. Conclusions

I have presented an analysis of a government spending stimulus similar to the one employed in the American Recovery and Reinvestment Act and of a tax cut. I have shown that, initially, net present value fiscal multipliers for government spending may exceed unity for several years and possibly substantially so. But these fiscal multipliers may be highly misleading, as eventually 5.8 dollars of output are lost for each dollar spent on government stimulus, according to this model. The analysis here is a positive one, not a normative one: the “price” of the output loss later on may well be worth “suffering” in order to “gain” the initial boost in output. But a discussion of fiscal stimulus without pointing out this price, i.e. the eventual and prolonged growth slowdown in output, appears incomplete. I argue that more attention and debate should be devoted to this important aspect of fiscal stimulus.

REFERENCES

- Cogan, John F., Tobias Cwik, John B. Taylor and Volker Wieland. Forthcoming. “New Keynesian versus Old Keynesian Government Spending Multipliers,” *Journal of Economic Dynamics and Control*.
- Drautzberg, Thorsten and Harald Uhlig. 2010. “Fiscal Stimulus and Distortionary Taxation,” draft, University of Chicago, http://www.frbatlanta.org/documents/news/conferences/10fiscal_policy_uhlig.pdf
- Romer, Christina and Bernstein, Jared. 2009. “The Job Impact of the American Recovery and Reinvestment Plan,” memo, The White House, http://www.ampo.org/assets/library/184_obama.pdf
- Trabandt, Mathias and Harald Uhlig. 2009. “How far are we from the slippery slope? The Laffer curve revisited.” NBER Working Paper 15343, NBER, Cambridge, MA.
- Uhlig, Harald. 1999. “A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily,” in R. Marimon and A. Scott: *Computational Methods for the Study of Dynamic Economies*. Oxford and New York: Oxford University Press, pp. 30-61.