Self–Control and Random Strotz Representations*

Eddie Dekel[†] Barton L. Lipman[‡]

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Abstract

We show that any preference with what Gul and Pesendorfer [2001] call a self–control representation also has a random Strotz representation, where the agent expects future choices to be made by a future self whose preference is not currently known with certainty. Thus one cannot use preferences over menus to determine whether or not an agent expects to exhibit self control.

1 Introduction

Gul and Pesendorfer [2001] (henceforth GP) introduce a novel representation of temptation. Classical models such as Strotz's consistent planning [1955] view temptation in terms of multiple selves, where an agent expects to choose tomorrow according to a different preference from the one he has today and evaluates this choice according to his current preference. GP's model allows what they term self control, where the agent's future self chooses in a manner at least partially consistent with his current preferences albeit at a cost.

To be more precise, they consider two representations of a preference over menus. Given a compact set B, let $\Delta(B)$ denote the set of lotteries over B and let X denote the set of *menus* — that is, the set of compact, nonempty subsets of $\Delta(B)$. The preference

*We thank Jawwad Noor for helpful conversations.

†Economics Dept., Northwestern University, and School of Economics, Tel Aviv University E–mail: dekel@nwu.edu.

‡Boston University. E–mail: blipman@bu.edu.

over X is denoted $\succ$. A function $w : \Delta(X) \to \mathbf{R}$ is *linear* if $w(\lambda\alpha + (1-\lambda)\beta) = \lambda w(\alpha) + (1-\lambda)w(\beta)$ for all $\lambda \in [0,1]$ and all $\alpha, \beta \in \Delta(B)$. We say that $w : \Delta(B) \to \mathbf{R}$ is an *expected utility function* if it is continuous and linear.

GP's first representation, which they call the *overwhelming temptation representation*, is based on Strotz's approach. Here we have two expected utility functions $u, v : \Delta(B) \to \mathbf{R}$ and a menu x is evaluated by

$$V_{OT}(x) = \max_{\beta \in B_v(x)} u(\beta)$$

where $B_v(x)$ is the set of best elements of x according to v. That is,

$$B_v(x) = \{\beta \in x \mid v(\beta) \geq v(\alpha), \quad \forall \alpha \in x\}.$$

Intuitively, v represents the preference of the future self who will choose from the menu as he wishes, breaking ties in favor of the current self who has utility function u.

A natural generalization of this, not considered by GP, is what we will call a *random Strotz representation.*

Definition 1 *A* random Strotz representation *is an expected utility function u and a measure μ over the set of expected utility functions such that the preference $\succ$ is represented by*

$$V_{RS}(x) = \int \max_{\beta \in B_w(x)} u(\beta)\mu(dw).$$

This is the overwhelming temptation representation of GP but where the agent is not sure what his future self's preference will be.

The other representation considered by GP is what they call a *self–control representation.*

Definition 2 *A* self–control representation *is a pair of expected utility functions (u, v) such that the preference $\succ$ is represented by*

$$V_{SC}(x) = \max_{\beta \in x}[u(\beta) + v(\beta)] - \max_{\beta \in x} v(\beta).$$

GP emphasize the idea that in the self–control representation, the agent chooses from the menu the item which maximizes $u + v$, not v. In this sense, he shows partial self control by compromising between u and v instead of simply maximizing v. One intriguing interpretation offered by GP which highlights this idea can be seen by writing the representation as

$$V_{SC}(x) = \max_{\beta \in x}[u(\beta) - c(\beta, x)]$$

where $c(\beta, x) = [\max_{\alpha \in x} v(\alpha)] - v(\beta)$. This representation is written as if the agent chooses the β which maximizes $u(\beta) - c(\beta, x)$ which is the β which maximizes $u + v$. Under this interpretation, $c(\beta, x)$ is the cost of resisting temptation by choosing β instead of maximizing v.

It is important to emphasize that GP's representations are based on a preference over menus, not over choices from menus. That is, the only object the modeler "sees" is the preference over menus, not choices from menus. Thus the interpretations sketched above regarding the nature of choices from menus are only interpretations.

What we will show in this note is that one cannot use preferences over menus to distinguish self control from random temptation. Specifically, every preference with a self–control representation also has a random Strotz representation. In this sense, there are multiple consistent interpretations of the same preference *over* menus which have different implications for choices *from* menus.

2 The Result

Theorem 1 *Fix any self–control representation (u, v) and the corresponding V_{SC}. Then there exists a random Strotz representation V_{RS} such that for every menu x, $V_{SC}(x) = V_{RS}(x)$.*

Proof. Let W denote the set of expected utility preferences such that $w \in W$ iff there exists $A \in [0, 1]$ with $w = v + Au$. Define a measure μ over W by taking the uniform distribution over A. That is, for a set $E \subseteq W$, we have

$$\mu(E) = \Pr[\{A \in [0, 1] \mid v + Au \in E\}],$$

where $\Pr(\cdot)$ is the uniform distribution. Finally, let V_{RS} denote the random Strotz representation generated by this measure.

Fix any menu x. Let $\beta^*(A)$ denote any element of x which maximizes u over the set $B_{v+Au}(x)$. Let $\hat{u}(A) = u(\beta^*(A))$ and let $\hat{v}(A) = v(\beta^*(A))$. So

$$V_{RS}(x) = \int_0^1 u(\beta^*(A))\, dA = \int_0^1 \hat{u}(A)\, dA.$$

Define

$$\mathcal{U}(A) = \hat{v}(A) + A\hat{u}(A) = \max_{\bar{A}\in[0,1]} \hat{v}(\bar{A}) + A\hat{u}(\bar{A}).$$

From the usual argument characterizing incentive compatibility with transferrable utility (see, *e.g.*, Mas–Colell, Whinston, and Green [1995], Proposition 23.D.2, page 888, or Milgrom and Segal [2002], Theorem 2), we have

$$\mathcal{U}(1) - \mathcal{U}(0) = \int_0^1 \mathcal{U}'(A)\,dA = \int_0^1 \hat{u}(A)\,dA = V_{RS}(x).$$

But $\mathcal{U}(1) = \max_{\beta\in x}[v(\beta) + u(\beta)]$, while $\mathcal{U}(0) = \max_{\beta\in x} v(\beta)$. Hence the left–hand side is $V_{SC}(x)$. ∎

3 Discussion

The result of the previous section raises numerous questions, regarding both the interpretations of earlier results in the literature and the possibilities of further connections. We briefly discuss some of these issues here.

First, GP interpret their self–control representation as identifying second period choice behavior where that behavior reveals that the agent can control his behavior. In particular, suppose the preference has the property that $\{\alpha\} \succ \{\alpha, \beta\} \succ \{\beta\}$. GP interpret $\{\alpha\} \succ \{\beta\}$ as saying that the agent prefers to commit herself to α than to β. They interpret $\{\alpha\} \succ \{\alpha, \beta\}$ as saying that β tempts the agent to deviate from α when both are options. Finally, they interpret $\{\alpha, \beta\} \succ \{\beta\}$ as saying that the agent is able to resist this temptation and avoid choosing β. While this interpretation is not unreasonable and corresponds in a natural way to what we see in the self–control representation, we get a very different view from the random Strotz representation. In it, $\{\alpha\} \succ \{\beta\}$ is again interpreted as saying the agent would rather commit to α than to β. However, $\{\alpha\} \succ \{\alpha, \beta\}$ is interpreted as saying that there is a positive probability the agent succumbs to temptation and chooses β when it is available, while $\{\alpha, \beta\} \succ \{\beta\}$ says that the probability the agent succumbs is less than one. In short, from the preference over menus alone, we cannot deduce second period choice behavior. In particular, we cannot say whether the agent will resist or expects to resist temptation. At best, we can construct various internally consistent stories about what the agent expects to happen in the future. Only observations of second period choice can confirm or refute such hypotheses.

Second, we turn to a more specific version of the previous observation. GP show that their overwhelming temptation representation can be thought of as a limiting case of a self–control representation. More specifically, they observe that for any menu x,

$$\lim_{\lambda \to \infty} \left[\max_{\beta \in x} [u(\beta) + \lambda v(\beta)] - \max_{\beta \in x} \lambda v(\beta) \right] = \max_{\beta \in B_v(x)} u(\beta).$$

Our result shows that the self–control representation with functions u and λv corresponds to a random Strotz representation with a uniform distribution over the set of preferences of the form $\lambda v + Au$ for $A \in [0, 1]$. As $\lambda \to \infty$, this distribution collapses to point mass on v. Thus the random Strotz representation collapses to GP's overwhelming temptation representation, giving another view of the same convergence result. This shows that we cannot use preferences over menus to distinguish the effects of increasing temptation from the effects of a particular decrease in uncertainty about temptation.

Third, consider a *random self–control representation* — that is, one where the v is random. More specifically, suppose we have a representation of preferences of the form

$$V_{RSC}(x) = \int \left\{ \max_{\beta \in x} [u(\beta) + v(\beta)] - \max_{\beta \in x} v(\beta) \right\} \eta(dv)$$

for some measure η over the set of expected utility preferences. It is immediate from Theorem 1 that we can rewrite this as a random Strotz representation. Simply apply the result "v by v." Since a self–control representation is continuous, this construction must generate a continuous random Strotz representation. An open question is the converse: Can any continuous random Strotz representation be written in the form of a random self–control representation for some measure η? Note that the restriction to continuous representations is needed; since the overwhelming temptation representation is discontinuous in general, random Strotz representations can be discontinuous, while a random self–control representation cannot.

We can prove a partial version of this converse. Fix a continuous random Strotz representation satisfying the finiteness axiom of Dekel, Lipman, and Rustichini [2007] (henceforth DLR). It is not hard to show that the preference it represents satisfies weak order (of course), continuity, independence, and a property termed *weak set betweenness* by DLR, namely:

Axiom 1 (Weak Set Betweenness) *If $\{\alpha\} \succeq \{\beta\}$ for all $\alpha \in x$ and $\beta \in y$, then $x \succeq x \cup y \succeq y$.*

DLR show that any preference satisfying weak order, continuity, independence, and finiteness has what they call a finite additive EU representation. They conjecture that adding weak set betweenness implies that the preference has what they term a simple representation, which is just a random self–control representation with a finite support. Recently, in a private communication, John Stovall has shown us a proof of this result.

The implication is that a continuous random Strotz representation which satisfies DLR's finiteness axiom must also have a random self–control representation. We are currently working on proving that the finiteness axiom is not needed for this result, yielding a full converse.

Fourth, a particular instance of this converse concerns the Steiner point. While the Steiner point has several equivalent definitions, the definition most useful for our purposes is that the Steiner point of a set x is the "expected maximizer" over the set of a uniformly drawn v. That is, let $\beta^*(x, v)$ denote a maximizer of v over the set x (which is unique for almost all v for any given x). The Steiner point of x, say $\beta^*(x)$, is the expectation over v of $\beta^*(x, v)$ where the distribution over v is uniform. It is easy to see, then, that the representation $u(\beta^*(x))$ is a random Strotz representation. Hence if our conjecture is correct, the Steiner point representation is also a random self–control representation. In particular, it seems likely to equal the random self–control representation emerging from a uniform distribution over v.

Finally, we remark that the basic point that the self–control representation can be rewritten in terms of a random determination of which self has control has been made before, though in a very different way. In particular, Benabou and Pycia [2002] note that the self–control representation can be written as the equilibrium payoff of a game between the current and future self engaging in a costly battle for control. Also, Chatterjee and Krishna [2006] show that a preference with a self–control representation also has a representation of the form

$$(1 - \rho_x) \max_{\beta \in x} u(\beta) + \rho_x \max_{\beta \in B_v(x)} u(\beta)$$

where ρ_x is interpreted as the probability of succumbing to temptation when facing menu x. Unfortunately, the properties of the function ρ_x make it difficult to interpret in general. The appealing aspect of our result, we believe, is that the random Strotz representation is such a natural alternative formulation.

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