Bargaining with Arrival of New Traders or New Information

William Fuchs* Andrzej Skrzypacz†

PRELIMINARY DRAFT

September 14, 2006

Abstract

A seller meets a potential buyer who has private information about her valuation of the asset. They bargain dynamically over the transaction price. The bargaining is affected by the possibility that some event occurs that would end the bargaining with some division of the surplus. The arrivals of events are driven by exogenous Poisson processes. We characterize the unique stationary equilibrium of this game and in particular the dynamics of trade and prices in the limit as the time between offers goes to zero. As a general result we provide conditions for when the Coase conjecture does not hold. The main relevant factor is that the seller's expected payoff conditional on arrival of an event is sensitive to the buyer's value. We show that the expected time to trade is a non-monotonic function of the arrival rate. Applying the model to arrival of a second trader (buyer or seller) with common value, we show how market tightness and thickness affect the division of surplus and the time to trade. When buyer valuations fall, average transaction prices drop and the time on the market gets longer.

1 Introduction

You have put your house on the market. So far only one buyer has expressed interest. He has just informed you that your original price is too high and asks you if you could bring the price down. What do you consider before responding? Out of many factors that you may take into account two of the most important considerations are: 1) How likely it is that other serious buyers will show up in the short run. 2) How likely it is that if you wait to reduce the price, the current buyer will find another house and "disappear".

Many markets are characterized by a combination of search and negotiated prices, without a centralized clearing house. The housing market is one of them but these "thin markets" are ubiquitous. One of the key characteristics of such markets is that trade/bargaining over price takes time and the

*University of Chicago
†Stanford GSB

bargaining dynamics are heavily influenced by the market conditions. For example, the asking price of a house takes time to drop, and how long it takes may depend on whether it is a "sellers' market" or a "buyers' market." This paper is a step towards connecting bargaining theory and demand-supply theory.

We present a model of a thin market in which a seller bargains over a price of a non-divisible asset with a privately informed buyer. As bargaining continues, an external event can happen that influences the competitive positions of the players. For example, a second buyer can arrive, a second seller can arrive or new public information may get released.

One of our main findings is that delay is a very natural consequence of bargaining in the context of a thin market: the possibility of arrival of new buyers (and subsequent competition between them) leads to delay in reaching compromise. In equilibrium the seller decreases prices over time, but does it slowly. This creates a non-trivial amount of delay even as bargaining frictions disappear and even though it is common knowledge that immediate trade is efficient. In contrast, the seminal papers by Gul, Sonnenschein and Wilson (1986) (GSW) and Fudenberg, Levine and Tirole (1986) (FLT) have shown that if the bargaining is insulated from external events then the equilibrium outcome exhibits Coasian dynamics (Coase 1972). That is, as bargaining frictions disappear, so that the seller is allowed to make offers very frequently, trade takes place with no delay.[1]

We characterize the equilibrium and analyze the determinants of delay and division of surplus. Interestingly, although the delay part of the Coase conjecture does not hold, the prediction about seller payoffs does hold: as bargaining frictions disappear the seller payoffs converge just to his outside option (of waiting for the arrival of the event). Moreover, we show that delay and surplus division are tied to how the seller's outside option changes with the buyer's valuation. The more sensitive is the seller (post arrival) payoff to the current buyer type, the longer, and hence more inefficient, the negotiations will be. For example, the less diverse are tastes of potential buyers, the longer the bargaining takes (see Section 4.1).

Delay seems to be cyclical in many markets. For example, in the housing market it is perceived that when prices are lower (a down market) houses stay longer on the market. In the labor market, a down market is usually correlated with longer unemployment spells. Standard demand-supply analysis does not have predictions how prices should correlate with time on the market. Yet, such patterns seem to be consistent with our model. In particular, as we show in Section 4 a weaker distribution of the buyer values leads to lower prices and longer bargaining (even keeping the relative arrival rates of buyers and sellers constant).

[1] That is not to say that there are no bargaining models that exhibit delay. For example, delay occurs in a model with two sided private information about fundamentals and overlap in values (e.g. Cramton 1984 or Cho 1990), with irrational players (Abreu and Gul 2000) , with higher order beliefs (Feinberg and Skrzypacz 2005) with disagreement about continuation play (Yildiz 2004) or with a different bargaining protocol (Admati and Perry 1987). The novelty of our approach is that we take the FLT/GSW framework as is and show that embedding it in a thin market leads to delay. As we discuss below, the intuition behind delay is closely related to Evans (1989), Vincent (1989) and Deneckere and Liang (2001).

Finally, we show how relative market conditions influence the division of surplus. In the full information bargaining models, such as Rubinstein (1982), the division of surplus is highly dependent on the relative impatience of players. In our model the potential arrival of outside events acts similar to discounting, introducing 'impatience' that depends on the market conditions (you may be impatient because you are afraid the buyer will find another house, the buyer may be impatient because he is afraid that somebody else will make you a high offer). The relative market conditions intuitively affect division of surplus, creating a natural connection between bargaining and demand-supply analysis: prices are lower in buyer's markets (with relatively more sellers) and lower in seller's markets (with relatively more buyers).

The main intuition why there is delay in equilibrium is closely related to the bargaining models with interdependent values, as presented by Evans (1989), Vincent (1989) and Deneckere and Liang (2001). In these models the seller does not know the cost of supplying the asset and the buyer has private information that determines both the value and the cost. If the lowest possible value is below the average cost, delay must occur. The reasoning is that otherwise the individual rationality constraint of at least one of the agents would be violated: to satisfy buyer's IR no delay implies prices not higher than the lowest buyer's value, but then the seller would lose money on average. In our model the seller knows his physical cost of delivering the asset, but the buyer has information about the (endogenous) opportunity cost: by trading today the seller forgoes the option to trade after an event arrives. As long as the post-arrival seller profits depend on the buyer type (which is the case whenever a competing buyer arrives), the delay is necessary. The main difference between our model and the previous work on bargaining with interdependent values is that the interdependence is created by market conditions and hence we can obtain interesting insights about thin markets. On the methodological side, we present most of the analysis in the continuous time limit, which greatly simplifies the analysis. We believe that this approach is promising for many other applications.

There are previous bargaining papers that allow for of arrival of new traders (in particular buyers) without obtaining equilibrium delay. This difference in results is caused by different assumptions about post-arrival competition, mainly that the post-arrival profits do not depend on the current buyer type. For example, Inderst (2003) only allows the seller to choose whether to keep the original buyer or switch to the new one but if he does switch, then the value of the original value is irrelevant for his continuation value.[2] As a result, in his model the Coase conjecture continues to hold.

Our work is also related to other models of dynamic trade with asymmetric information and with no frictions, for example Noldeke and Van Damme (1990), Swinkels (1999) and Kremer and Skrzypacz (2006). The main difference between these papers and the setup we propose to analyze is that they have monopoly only on one side of the market and competition on the other side, while we propose to study a temporary bilateral monopoly. Similarly to Kremer and Skrzypacz (2006) the outside events

[2] The same happens in Alberto Trejos and Randall Wright (1995) where the newly arrived traders simply displace the old ones.

lead to delay in trade, but the equilibrium dynamics are very different (for example, in the current model we obtain perfect separation of types through type, while with competition on one side of the market such separation is not possible).

The paper is organized as follows: Section 2 presents the general model and discusses some possible applications. Section 3 contains general preliminary analysis, Section 4 briefly describes several extensions and Section 5 concludes. Most proofs are in the Appendix.

2 The Model

To describe the model, we start with a reduced-form description of a general bargaining game with arrival of new events. Then, we provide examples of applications that fit this general structure and that range from arrival of new buyers and sellers with common or private values to arrival of new information about the asset's value.

2.1 General Bargaining

There is a seller and a buyer. The seller has an indivisible good (or asset) to sell. The buyer has a privately known type $v \in [0,1]$ that represents his value of the asset and that is distributed according to a *c.d.f.* $F(v)$ that has atomless distribution and full support.

The time is discrete and periods have length Δ. The timing within periods is as follows. In the beginning of the period an event $\omega \in \Omega$ arrives with probability $\Delta\lambda(\omega)$ that ends the game. There is a finite set of possible events, Ω, and their (Poisson) arrival rates are exogenous.[3] We treat the events as a reduced form of some continuation play for now, but in the next section we present possible applications that fit the description here. If no event arrives, the seller makes a price offer p. Then the buyer decides whether to accept this price or to reject it. If he accepts, the game ends. If he rejects, the game moves to the next period. A strategy of the seller is a mapping from the histories of rejected prices to current period price offers. A strategy of the buyer is a mapping from the history of rejected prices to an acceptance strategy (which specifies the set of prices that the buyer accepts in the current period).

The payoffs are as follows. If the game ends with buyer accepting price p at time t, then the seller's payoff is $e^{-rt}p$ and the buyer's payoff is $e^{-rt}(v-p)$, where r is a common discount rate.[4] If

[3] In particular, we assume that the arrival rates $\lambda(\omega)$ are independent of v and of the history of the game. Only one event can arrive in any given period.

[4] We will focus on the case $\Delta \to 0$, i.e. no bargaining frictions, so it is more convenient to count time in absolute terms rather than in periods. Period n corresponds to real time $t = n\Delta$.

the game ends with event ω arriving at time t, then the payoffs are:

$$e^{-rt}W_{\omega}(v) \text{ for the buyer,}$$
$$e^{-rt}\Pi_{\omega}(v) \text{ for the seller.}$$

For notational convenience let $\Lambda = \sum \lambda(\omega)$ denote the total arrival rate of events. Also, let $W(v) = \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} W_{\omega}(v)$ and $\Pi(v) = \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} \Pi_{\omega}(v)$ denote the expected payoffs conditional on any arrival and the buyer's value being v.

We assume:

Assumption 1

i) $\Pi(v) + W(v) \leq v$, so that immediate trade is efficient.

ii) $W(v)$ is continuous and weakly increasing, with $v - W(v)$ strictly increasing.

iii) $\Pi(v)$ is continuous and strictly increasing.

iv) $\Pi(0) = W(0) = 0$, so that no player expects negative payoff regardless of the realized v.

These assumptions are not too restrictive and are satisfied in many environments. We now describe a few applications in which they are satisfied.[5]

2.2 Applications

The above reduced game can be completed by a lot of models of the arrival of events. As the title of the paper suggests, we are focusing on two types of events: new traders and new information. Some of the following examples we consider in a greater detail below.

1. Arrival of a second buyer with a pure common value. Suppose that there is only one event that can arrive, that corresponds to a second buyer arriving to the market. Furthermore, suppose that the second buyer has the same value as the first buyer, and upon arrival the seller runs an English auction to allocate the asset. Then the payoffs are:

$$W(v) = 0, \quad \Pi(v) = v$$

2. Arrival of a second buyer with independent private value and English auction. Suppose that the only event that can arrive is a second buyer with a value independent of the first buyer. The second buyer has value v_2 distributed according to a distribution $G(v)$ (that satisfies the same assumption as $F(v)$ does). Upon arrival the seller runs an English auction: starts with a low price and keeps increasing it until one of the buyers drops out. He allocates the asset to the other buyer then and

[5] For comparison, as we discussed in the Introduction, Inderst's (2003) model violates (iii) because the outside option of the seller is not increasing in the current buyer's valuation in his environment.

charges the price at which the auction stopped. As in the English auction it is a dominant strategy to bid until reaching one's value (we focus attention on the equilibrium in weakly undominated strategies) the payoffs are:

$$\begin{aligned} W(v) &= G(v)\left(v - E\left[V_2 | V_2 \leq v\right]\right), \\ \Pi(v) &= G(v)\, E\left[V_2 | V_2 \leq v\right] + (1 - G(v))\, v. \end{aligned}$$

Where $E\left[V_2 | V_2 \leq v\right]$ is the expected value of v_2 conditional on it being below v. Direct calculation shows that Assumption 1 is satisfied by these reduced-form payoffs (for example, $W(v) + \Pi(v) = v$).[6]

3. Arrival of a second buyer with correlated private value and English auction. The previous example can be expanded to allow for correlation of the private values. Suppose that the distribution of the second buyer value is $G(v_2 | V_1)$. In this case the functions are

$$\begin{aligned} W(v) &= G(v | V_1 = v)\left(v - E\left[V_2 | V_2 \leq v\right]\right), \\ \Pi(v) &= G(v | V_1 = v)\, E\left[V_2 | V_2 \leq v\right] + (1 - G(v | V_1 = v))\, v \end{aligned}$$

and again they satisfy Assumption 1.

4. Arrival of buyers and sellers Suppose that now there are two events that can happen: either a second seller with an identical good arrives or a second buyer with identical valuation arrives. The arrival rates are $\lambda(\omega_s) = \lambda_s$ and $\lambda(\omega_b) = \lambda_b$. If the buyer arrives, the game ends like in point 1 above. If the second buyer arrives, the sellers compete away all surplus, selling the asset at 0. The payoffs in this case are:

$$W(v) = \frac{\lambda_s}{\Lambda} v, \quad \Pi(v) = \frac{\lambda_b}{\Lambda} v.$$

5. Arrival of Information Finally, suppose that a public information can arrive that reveals the value of the buyer. Additionally, suppose that the bargaining situation (power) can change depending on the state of nature $\omega \in \Omega$. That is, the players split the surplus according to shares $\alpha_\omega, (1 - \alpha_\omega)$. Therefore the payoffs are:

$$\begin{aligned} W(v) &= v \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} (1 - \alpha_\omega), \\ \Pi(v) &= v \sum_{\omega} \frac{\lambda(\omega)}{\Lambda} \alpha_\omega . \end{aligned}$$

[6] Note that with different values for the two buyers different auction formats may yield different revenue (even if F and G are identical). Modeling the auction as an English auction is most convenient as the equilibrium strategies do not depend on the information the second buyer has about the history of the bargaining. We come back to the discussion of the impact of the auction format on the division of surplus and delay in Section 5.

3 Characterization of Equilibria

Bargaining games are usually studied in discrete time.[7] This choice is dictated by the problem of defining the game in continuous time.[8] However, the description of the equilibrium in our game seems to be particularly tractable in continuous time, while the strategies in discrete-time are quite complicated. For clarity of exposition we characterize the continuous-time limits of the equilibrium conditions by taking $\Delta \to 0$. Details about the equilibrium in continuous time are provided in the Appendix

A stationary strategy for the buyer is a reservation price function $p_b(v)$ that does not depend on the history of the game. A (pure) stationary strategy for the seller is a pricing function $p_s(t)$ that specifies the equilibrium path of prices over time. Off the path seller prices $p_{OFF}(t)$ are induced by $p_s(t)$ by stationarity (if the seller happens to be off the equilibrium path, he looks at the lowest price so far and then continues the path from that level, as if no time has passed since).

In the Appendix we show that there exists a stationary equilibrium in pure strategies. Given existence, it is relatively easy to show that $p_b(v)$ is continuous and strictly increasing, $p_s(t)$ is strictly decreasing and as $\Delta \to 0$, $p_s(t)$ converges to a continuous function (the intuition for the last observation is that a jump in prices would make the buyers trading right before the jump strictly better off waiting for 1 second and getting a strictly lower price, but then the seller would be better off jumping with the price sooner).

We now construct the limit of the equilibrium strategies as $\Delta \to 0$. The Equilibrium strategies $p_s(t)$ and $p_b(v)$ induce unique functions: $\{t^*(v), p^*(v), k^*(t)\}$ that specify, respectively, the time at which buyer with type v trades, the price he pays and the cutoff type of the buyer that remains in the game after time t (note that $k^*(t)$ is an inverse of $t^*(v)$). In the limit the induced $t^*(v)$ is strictly monotone. For any given continuous and strictly monotone $\{t^*(v), p^*(v)\}$ we can also go the other way to pin down the unique strategies $p_s(t)$ and $p_b(v)$ that induce them by letting $p_b(v) = p^*(v)$ and $p_s(t) = p^*(k^*(t))$. To characterize the equilibrium it is easier to work with the induced functions $\{t^*(v), p^*(v), k^*(t)\}$ instead of $\{p_s(t), p_b(v)\}$.

Seller's problem. Define $\overline{\Pi}(k) = \int_0^k \Pi(v)\frac{f(v)}{F(k)}dv$ which is the seller's expected payoff conditional on some event arriving immediately and current cutoff being k. Note that since $p_b(v)$ is strictly increasing (and stationary), from the point of view of the seller the state of the game is summarized by one number: the highest-type cutoff, k, that is the highest type still remaining in the game. Denote by $V(k)$ the expected payoff of the seller given current cutoff k. Because the seller cannot commit to future prices, he chooses every period a new price maximizing total sum of expected discounted payoffs, expecting that in the future he would re-optimize over prices given his beliefs. Given a k, the best response

[7] See for example FLT or GSW and all papers surveyed in Ausubel, Cramton, and Deneckere (2001).

[8] See Simon and Stinchcombe (1989) for some of the issues with modeling games with perfect monitoring in continuous time.

problem of the seller is:

$$V(k) = \max_{p} \Delta\Lambda\overline{\Pi}(k) + (1-\Delta\Lambda)\left[p\frac{F(k)-F(k_{+\Delta})}{F(k)} + \frac{F(k_{+\Delta})}{F(k)}e^{-\Delta r}V(k_{+\Delta})\right]$$

where $k_{+\Delta}$ satisfies $p_b(k_{+\Delta}) = p$, so that it is the cutoff type that accepts price p, and $\frac{F(k)-F(k_{+\Delta})}{F(k)}$ is the probability of buyer accepting p. Given the buyer strategy, $p_b(v)$, we can write equivalently the best response problem by letting the seller choose the next-period cutoff, $k_{+\Delta}$, and require $p_b(k_{+\Delta}) = p$. Or, we can write that the seller chooses the speed at which he "runs through types", $\frac{k_{+\Delta}-k}{\Delta}$. The best response problem then becomes:

$$V(k) = \max_{\frac{k_{+\Delta}-k}{\Delta}} \Delta\Lambda\overline{\Pi}(k) + (1-\Delta\Lambda)\left[p(k_{+\Delta})\frac{F(k)-F(k_{+\Delta})}{F(k)} + \frac{F(k_{+\Delta})}{F(k)}e^{-\Delta r}V(k_{+\Delta})\right].$$

Subtracting $e^{-\Delta r}V(k)$ from both sides of the problem, collecting terms and dividing by Δ we get:

$$\begin{aligned} V(k)\frac{\left(1-e^{-\Delta r}\right)}{\Delta} &= \max_{\frac{k_{+\Delta}-k}{\Delta}} \Lambda\left(\overline{\Pi}(k) - \frac{F(k_{+\Delta})}{F(k)}e^{-\Delta r}V(k_{+\Delta})\right) \\ &\quad + \left((1-\Delta\Lambda)p(k_{+\Delta}) - e^{-\Delta r}V(k_{+\Delta})\right)\frac{F(k)-F(k_{+\Delta})}{\Delta F(k)} \\ &\quad + e^{-\Delta r}\frac{V(k_{+\Delta})-V(k)}{\Delta} \end{aligned}$$

It is easy to argue that given a strictly downward-sloping 'demand' $p_b(v)$, in the limit $k_{+\Delta}-k \to 0$. The intuition is that jumping to a strictly lower type $v' < k$ the seller obtains revenue $p_b(v')$ conditional on trade, while by running continuously yet sufficiently fast through types yields almost the whole area under the demand curve $p_b(v)$ between k and v'. In the limit, as $\Delta \to 0$, this second strategy is strictly better.[9] If the limit $\dot{k} \equiv \frac{dk^*(t)}{dt} = \lim_{\Delta\to 0}\left(\frac{k_{+\Delta}-k}{\Delta}\right)$ exists, then it must satisfy:[10]

$$rV(k) = \max_{\dot{k}\in(-\infty,0]} \Lambda\left(\overline{\Pi}(k) - V(k)\right) + \left(p(k) - V(k)\right)\frac{f(k)}{F(k)}\left(-\dot{k}\right) + V'(k)\dot{k} \tag{1}$$

which is the limit of the above seller optimality condition as $\Delta \to 0$. This condition has a clear interpretation directly in continuous time. The left-hand side is the expected equilibrium payoff expressed in flow terms. The right hand side represents the possible sources of the flow: upon arrival (which happens with a probability flow Λ) the game ends with the seller earning in expectation $\overline{\Pi}(k)$ (and as the game ends he forgoes $V(k)$). With a flow probability $\frac{f(k)}{F(k)}\left(-\dot{k}\right)$ the buyer accepts current offer,

[9] This reasoning requires that $p_b(v)$ is strictly decreasing in the limit. As we show below, it is indeed the case.

[10] The limit of $k^*(t)$ is monotone and continuous, hence it is differentiable almost everywhere. If it is not differentiable at t, then the right derivative has to satisfy this condition.

$p(k)$, which also ends the game. Finally, if the game does not end immediately, the continuation payoff drops, as the seller becomes more pessimistic about v, as captured by $V'(k)\dot{k}$.

Note that (1) is linear in $\dot{k}$, so that a continuous and strictly decreasing $t(v)$ can be consistent with equilibrium only if the seller is indifferent over all possible $\dot{k}$.[11] This requires that the coefficients on $\dot{k}$ add up to 0:

$$\begin{aligned}(p(k)-V(k))\frac{f(k)}{F(k)} &= V'(k) \\ &\Updownarrow \\ f(k)\,p(k) &= \frac{\partial}{\partial k}\left[V(k)\,F(k)\right]\end{aligned}$$

Furthermore, if indeed the seller is indifferent over all $\dot{k}$ it is easy to calculate $V(k)$ by evaluating a strategy of $\dot{k}=0$, that is of asking high enough prices to avoid trade until the arrival:

$$V(k)=\frac{\Lambda}{\Lambda+r}\overline{\Pi}(k) \tag{2}$$

That implies that the equilibrium prices have to satisfy:

$$p(v)=\frac{\Lambda}{\Lambda+r}\Pi(v) \tag{3}$$

That pins down the unique candidate for the (limit of) equilibrium $p^*(v)$. Since $p_b(v)=p^*(v)$, and we assumed that $\Pi(v)$ is strictly increasing, the obtained demand is strictly downward sloping (as we assumed arguing that the seller never wants to jump with prices down discontinuously).

Buyer's problem. We now turn to the buyer's best response problem. Denote by $B(v)$ the expected payoff of buyer with value v (at the beginning of the game). Looking at the direct-revelation representation of the buyer's strategy, he plays a best response if and only if :

$$B(v)=\max_{v'} e^{-(r+\Lambda)t(v')}\left(v-p\left(v'\right)\right)+\int_0^{t(v')}\Lambda W(v)\,e^{-(\Lambda+r)t}dt \tag{4}$$

and $v'=v$ is a solution to this problem. In words, the buyer knowing the equilibrium $p_s(t)$ can mimic another type v' to trade at a different price, $p(v')$ at the corresponding time $t(v')$. The first part on the RHS reflects the surplus from trading before the arrival of an event and the second part stands for the possibility that the arrival happens before $t(v')$. This expression looks at the problem at time 0, but if it is satisfied, due to the stationarity of the equilibrium (and the environment), the off-equilibrium

[11] This linearity is the source of Coasian dynamics when $\Lambda=0$: in that case for any strictly decreasing $p_b(v)$ the seller wants to run down the demand function as fast as possible. Therefore the equilibrium $p_b(v)$ in the limit becomes flat at 0. The outside option in our model provides a counterbalance for the seller's temptation to run down the demand curve, leading to a strictly downward-sloping $p_b(v)$.

incentives are satisfied as well.[12]

Since we have pinned down $p(v)$ from the seller's problem, we use the buyer's problem to characterize the (limit of) $t(v)$. First, we can argue that the equilibrium has to satisfy a boundary condition $t(1) = 0$, because otherwise the trade would never happen until arrival.[13] Furthermore, for $v < 1$ $t(v) > 0$, otherwise we would have an atom of trade at time zero which is never optimal for the seller.[14] Second, consider the local incentives for the cutoff type $v = k_{+\Delta}$.It must be indifferent between trading at price $p(k)$ and waiting for Δ and then either trading at a bit lower price, $p(k_{+\Delta})$ arrival and earning $W(k_{+\Delta})$:

$$\underbrace{k_{+\Delta} - p(k)}_{\text{trade now}} = e^{-\Delta r}(\Delta\Lambda \underbrace{W(k_{+\Delta})}_{\text{arrival}} + (1-\Delta\Lambda)\underbrace{(k_{+\Delta} - p(k_{+\Delta}))}_{\text{trade tomorrow}})$$

Subtracting $e^{-\Delta r}(1-\Lambda\Delta)(k_{+\Delta} - p(k))$ from both sides, dividing by Δ and taking $\Delta \to 0$ we get the following limit of the indifference condition:

$$(r+\Lambda)(k - p(k)) = \Lambda W(k) - p'(k)\dot{k}$$

It again has a direct interpretation: the LHS is the cost of delaying trade (due to discounting and possibility of arrival) and the RHS is the benefit of waiting consisting of the reduction of price and the payoffs from possible arrival.[15] Substituting the candidate $p^*(v)$ that we found in the seller's problem, (3), yields:

$$-\dot{k} = (\Lambda + r)\frac{(r+\Lambda)k - \Lambda(\Pi(k) + W(k))}{\Lambda\Pi'(k)} \tag{5}$$

which together with a boundary condition $t(1) = 0$ pins down uniquely a candidate $t^*(v)$. Since by Assumption 1 $\Pi(k) + W(k) \leq k$ and $\Pi(k)$ is strictly increasing, this candidate $t^*(v)$ is strictly decreasing. Finally, the buyer local incentive compatibility conditions are sufficient because the solution $t(v)$ is strictly decreasing and so the buyer's objective function (4) is supermodular in v' and v.

Equilibrium. Summarizing, the limit of the equilibrium as $\Delta \to 0$ is:

[12] If any price $p < p(k)$ is offered we assume the buyer believes that from then on the sequence of prices will continue falling following those of the original equilibrium after p. With these beliefs all the buyer types that would remain after the price p was offered correspond to those in the original equilibrium and hence if it was optimal for the seller and buyer to follow the original equilibrium after p it will be optimal to follow it now. Now suppose the seller offers a higher price than expected. No buyer would accept this offer since they believe that the seller would offer the price he was supposed just a second later. Similar arguments can be constructed no matter how many times the seller has deviated from the equilibrium in the past.

[13] If so, then $p_b(v) < W(v)$ and the seller would have a profitable deviation to trade immediately with some types close to 1 at prices below $W(v)$, a contradiction.

[14] The formal proof of this statement can be found in the Appendix.

[15] This expression is equivalent to the first order condition of problem (4).

Proposition 1 *For any generic* Δ *there exists a unique stationary equilibrium. In the limit, as* $\Delta \to 0$*, the equilibrium strategies converge to* $\{p_s(t), p_b(v)\}$ *characterized (uniquely) by* (3) *and* (5) *with the boundary condition* $t(1) = 0$.

The proof of uniqueness for any Δ is in the Appendix. Once existence and uniqueness are determined, the limits of the equilibrium strategies follow from the above discussion, since we have only used necessary conditions of the best response problems.

Additionally, the seller expected equilibrium payoffs converge to (2). The buyer's payoff (at time 0) can be found using either directly $\{t^*(v), p^*(v)\}$, or applying the envelope theorem to problem (4):

$$B'(v) = e^{-(r+\Lambda)t(v)} + \frac{\Lambda}{\Lambda + r}\left(1 - e^{-(r+\Lambda)t(v)}\right) W'(v)$$

(and using the boundary condition $B(0) = 0$).

We can prove the following general properties of the equilibrium:

Proposition 2 *1) If the seller has a positive expected value upon arrival,* $\Pi(v) > 0$*, then there will be delay in the bargaining for any* $0 < \Lambda < \infty$*. Furthermore, the time to trade is increasing in* $\Pi(v)$.
2) (Coase conjecture): as $\Lambda \to 0$*, the time to trade and the transaction price converges to converges to* 0 *for all types.*
3) (Competition Effect): the effect of Λ *on the expected time to trade is non-monotonic: as* $\Lambda \to 0$ *or* $\Lambda \to \infty$ *the expected time to trade converges to zero while it is strictly bounded away from zero for intermediate values of* Λ.

Part (1) of the proposition is one of our main results. It shows that when the bargaining is subject to external influences, delay is to be expected. Part (2) shows that there is continuity with the results of GSW and FLT since as we take the probability of arrivals to zero trade takes place immediately and the buyer captures all the surplus. Finally part 3 shows that time on the market is non-monotonic in the search frictions. As the frictions to find new traders disappear, as in a perfectly competitive market, the expected time to trade also goes to zero.

Arrival of new traders or outside options is necessary for delay but another important ingredient is that the seller's outside value depends on the buyer's type. In particular, we can establish the following general comparative statics:

Proposition 3 *Consider two environments, one with* $\Pi_1(v)$ *and the other with* $\Pi_2(v)$*. Assuming either* $W_1(v) = W_2(v)$ *or* $\Pi_1(v) + W_1(v) = \Pi_2(v) + W_2(v)$ *:*
1) If $\Pi_1'(v) \geq \Pi_2'(v)$ *for* $\forall v$*, then trade is on average more efficient in the environment with* $\Pi_2(v)$.
2) In the limit as $\Pi'(v) \to 0$ $\forall v$*, trade happens immediately with the buyer capturing all the surplus.*

The second part of this Proposition shows that the Coase conjecture holds in the limit as $\Pi'(v) \to 0$ $\forall v$. Given our assumption $\Pi(0) = 0$, $\Pi'(v) \to 0$ $\forall v$ implies that $\Pi(v) \to 0$ $\forall v$. To separate slope versus level effects consider the case where $\Pi'(v) = 0$ *but* $\Pi(v) = c > 0$ $\forall v$. In this case, in equilibrium the seller offers price $p = \frac{\Lambda}{\Lambda+r}c$ and either trade happens immediately or there is no trade until arrival. For there to be both delay and trade before arrival we need $\Pi'(v) > 0$. Intuitively what happens is that the seller makes a first offer $p = \frac{\Lambda}{\Lambda+r}\Pi(1)$. Since this offer is accepted by the highest types of the buyer, the seller's outside option decreases a bit and next period he is willing to make lower offers. In this way he slowly skims through all buyer types.

But why does it happen slowly? Why don't we get almost immediately to $p = 0$, like in the Coase conjecture? The reason is that if the seller ran 'the clock' too fast then some buyer types would have an incentive to wait for a lower price - their reservation prices would decrease. But then the seller would prefer to stop trading, since he would get a higher expected payoff from just waiting for an arrival than from trading at these low prices. On the other hand, the seller cannot run too slowly through the demand either, since then the reservation prices would be so high, that the seller would prefer to collect the whole area below the demand *before* the arrival. Therefore the buyer price decreases has to be such that the reservation prices of the buyer keep the balance between the incentive to speed up and slow down the trade.

Following this logic, if $\Pi_1'(v) \geq \Pi_2'(v)$ $\forall v$, then under Π_1 the seller's outside option drops faster as his belief of the current buyer cutoff type falls. This makes him offer lower prices at $k' = v - \varepsilon$, that is prices as a function of k decrease at a faster rate for the steeper $\Pi(v)$. Hence, if the seller ran the clock (with respect to $k(t)$) at the same speed, prices would drop faster in time under Π_1. But then the buyers would have an incentive to wait for lower prices, leading to a contradiction that the k changes through time. To keep the current cutoff types willing to trade at the current prices the seller has to go through the types slower under Π_1.

This result allows us to compare our dynamics to existing literature. For example, in Inderst (2003) (and other papers that have the new buyers replace the existing buyer), $\Pi'(v) = 0$.[16] Taking the limit $\Pi'(v) \to 0$ in our model leads to the same limiting outcome. It is essential for there to be delay that the outside value of the seller depends on the buyer's type. The more sensitive the outside value of the seller to the buyer's private information, the greater the inefficiency. As we explained in the Introduction, the correlation of the seller's outside option with the buyer value endogenously creates a bargaining environment with interdependent values, as studied by Evans (1989), Vincent (1989) and Deneckere and Liang (2001), and hence some of dynamics are similar (yet, by explicitly studying the arrivals of new traders we can provide additional insights about thin markets).

[16] Inderst (2003) assumes that upon the second buyer arriving, the seller can only choose to keep on bargaining with the current buyer or switch to bilateral bargaining with the new one, which implies $\Pi'(v) = 0$ in his model.

4 Arrival of New Traders

In this section we return to the original motivation of studying thin markets. In particular, we apply the general model to analyze bargaining dynamics when we new traders can arrive to the market. We are mostly interested in determinants of delay/time on the market and division of surplus.

Suppose that there are two events that can happen: either a second seller with an identical good arrives or a second buyer with identical valuation arrives. The arrival rates are $\lambda(\omega_s) = \lambda_s$ and $\lambda(\omega_b) = \lambda_b$. If the second buyer arrives, the game ends with the seller capturing all the surplus. If the second seller arrives, the sellers compete away all surplus, selling the asset at 0. In this context we can refer to $\Lambda = \lambda_b + \lambda_s$ as the market thickness: how frequently new traders are found. The larger Λ, the thicker the market. We can also use $\frac{\lambda_b}{\lambda_s}$ as a measure of relative market tightness. It is a buyers' market if $\frac{\lambda_b}{\lambda_s}$ is low, so that sellers are abundant relative to buyers.

The expect payoffs conditional on arrival in this case are:

$$W(v) = \frac{\lambda_s}{\Lambda} v \ , \ \Pi(v) = \frac{\lambda_b}{\Lambda} v \ .$$

and clearly they assumptions of the general model.

Using the equilibrium conditions (3) and (5) we can calculate the limiting strategies (as $\Delta \to 0$) in a closed form:

$$p^*(v) = \frac{\lambda_b}{\Lambda + r} v \ , \qquad t^*(v) = -\frac{\lambda_b}{r(\Lambda + r)} \ln v \ . \tag{6}$$

and the (limits of the) value functions are:

$$V(k) = \frac{\lambda_b}{\Lambda + r} \int_0^k v \frac{f(v)}{F(k)} dv \ ,$$

$$B(v) = \frac{\lambda_s v + r v^{\frac{\lambda_b + r}{r}}}{\Lambda + r}.$$

We start with simple observations about the (continuous-time limit of) equilibrium:

Proposition 4 *In (limit of) equilibrium in the model with entry on both sides of the market:*
1. (Market Tightness) Keeping $\Lambda = \lambda_b + \lambda_s$ *constant (the sum of arrival rates of the second buyer and seller), an increase in the ratio* $\frac{\lambda_b}{\lambda_s}$*, implies longer equilibrium time on the market, a higher seller's expected payoff and a lower buyer's payoff. In the limit, as* $\frac{\lambda_b}{\lambda_s} \to 0$ *we get immediate trade with the buyer capturing all the surplus.*
2. (Market Thickness) Keeping the ratio $\frac{\lambda_b}{\lambda_s}$ *fixed, delay is non-monotonic in the sum* $\lambda_b + \lambda_s$*. It converges to zero as* $\lambda_b + \lambda_s \to \infty$ *and also as* $\lambda_b + \lambda_s \to 0$*, while it is greater than zero for intermediate values.*

The first result shows that trade is more efficient when it is a buyers' market. This is because the higher the likelihood of arrival of the second seller, the more impatient the current seller gets, which makes him offer lower prices. In the limit, if only new sellers can arrive then trade takes place immediately and the buyers capture all the surplus as in FLT or GSW.

The second result shows that the efficiency is not monotonic in the liquidity of the market. In the limit as we approach perfect competition ($\lambda_b + \lambda_s \to \infty$) trade takes place immediately. Trade is also immediate when there is a bilateral monopoly with no possibility of arrival. But when we have a thin market there is some delay in trade.

These results can potentially be useful in understanding the impact of market makers on the market equilibrium and how their bid ask-spreads are related to their higher contact frequency.

From the seller's perspective, one could expect that not only market tightness but that also market strength, as captured by the distribution of buyer valuations, are important in determining the price offers. Surprisingly, note that (6) shows that the price path $p_s(t)$ of seller's offers and the reservation prices of the buyers, $p_b(v)$, are invariant to the distribution $F(v)$! Does it mean that the distribution of values has no impact on the expected trade dynamics? No. In fact, this characterization allows us to establish two results about the impact distribution of values has on time on the market and efficiency of trade:

Proposition 5 *In the common value case:*
1) (Weak markets and time on the market) Consider two distributions of buyer's values F and H such that F first order stochastically dominates H. Then expected time to trade is longer if the distribution of values is H (and average prices are lower).
2) (Information dispersion and efficiency of trade) Consider two distributions of values F and H such that F second order stochastically dominates H. Then the equilibrium is more efficient under distribution of values H.

This proposition points to an interesting finding that trade takes longer in markets with weaker distributions of valuations. This could help explain some of the cyclical patterns in real estate markets and in labor markets. In a downturn prices in both these markets respond sluggishly but both time to sale and exit out of unemployment tend to respond much more significantly, as indicated by the following citations:

> "One important point to keep in mind, however, is that while declines in nominal house prices are relatively rare, the volume of housing market transactions tends to be more responsive to a slowing economy. A flattening out of an observed price series may in fact mask a buildup of inventory of unsold houses." FRBSF Economic Letter 2002-13; May 3, 2002.
>
> "In the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the

search model predicts that the two variables should have nearly the same volatility." Shimer (2005)

Clearly more research, especially on further endogenizing the post-auction payoffs and arrival rates (as well as empirical research) is needed to determine if our model in fact an important explanation of these cyclical patterns.[17]

4.1 Taste Diversity and Time on the Market.

So far we have assumed that all buyers have the same valuation. In many markets, however, it is natural to think that there are different groups of potential buyers of the asset, and that even though valuations within a group can be very similar, they would differ across groups quite a bit. For example, families with school age children could be one group with similar valuations for a given house. The group of retirees, on the other hand, could value the same house differently. The first group would put more weight on the quality of the school district while the latter care more about the quality of the walking paths. Similarly, if a firm is being sold, there are different groups of potential buyers such as competing firms and private equity funds that have different motives for purchasing the target.

To illustrate the effects of diverse taste groups of potential buyers on the bargaining dynamics, we parameterize the problem as follows. Assume there are n different groups of buyers. All members of a given group share the same valuation but valuations across groups are *i.i.d.* according to $F(v)$. Now, when the second buyer arrives, with probability $\gamma = \frac{1}{n}$ he belongs to the same group (and has the same valuation) as the current buyer (and this is common knowledge). Otherwise, with probability $(1-\gamma)$, he belongs to a different group and his value is independent of the first buyer value. Therefore, a larger γ stands for a less diverse market place. In either case an English auction is used to allocate the good. For simplicity assume $\lambda_s = 0$.

In this case the expected payoffs conditional on arrival are:

$$\begin{aligned} W(v_1) &= (1-\gamma) F(v_1)(v_1 - E[v_2|v_2 \leq v_1]) \\ \Pi(v_1) &= \gamma v_1 + (1-\gamma)(F(v_1) E[v_2|v_2 \leq v_1] + (1-F(v_1)) v_1) \end{aligned}$$

Applying the general analysis above, we can establish the following comparative statics with respect to the taste diversity:

Proposition 6 *The (limit of) the stationary equilibrium as* $\Delta \to 0$, *has the following comparative statics with respect to an increase in the number of groups,* n $(\downarrow \gamma)$:

[17] For example, by changing which side has private information in the model, one can reverse the result and get higher prices with longer delay. We believe that putting the private information about values on the buyer side is realistic for the housing market, but one cannot expect this to be true for all markets. Therefore, the result should be interpreted that the model delivers cyclical behavior, rather than it predicts a particular sign of the correlation.

i) The expected time to trade decreases.
ii) The surplus for the seller falls.
iii) For any t the price offered is lower.

Part (i) of the Proposition follows from noting that $\frac{\partial \Pi'(v_1)}{\partial \gamma} = F(v_1) > 0$ and using the result from Proposition 3. (ii) and (iii) follow from noting that $\Pi(v_1)$ is decreasing in n since the second term of $\Pi(v_1)$ is smaller than v_1 and using equations (2) and (3) which respectively characterize the seller's value and prices.

This result suggests that sellers would benefit more from specializing in a narrow market. Intensively targeting a given group of potential buyers rather than casting a very wide net.

5 Extensions of the Model

Before we conclude, we discuss two additional applications of the model: the impact of auction format and arrival of information.

5.1 Arrival of Information

So far we have looked at different events which involved the arrival of new traders. Instead we could consider events which are characterized by the arrival of information.[18] As stated in Example (5) a possibility is to assume that public information can arrive that reveals the value of the buyer. Additionally, the bargaining situation (power) can change depending on the state of nature $\omega \in \Omega$. That is, the players split the surplus according to shares $\alpha_\omega, (1-\alpha_\omega)$. In this case the payoffs are:

$$\begin{aligned} W(v) &= v(1-\bar{\alpha}), \\ \Pi(v) &= v\bar{\alpha}. \end{aligned}$$

where $\bar{\alpha} = \sum_{\omega} \frac{\lambda(\omega)}{\Lambda}\alpha_\omega$ is the expected bargaining power of the seller.

The offered prices and the time to trade are given by:

$$p^*(v) = \frac{\Lambda}{\Lambda + r}\bar{\alpha}v, \qquad t^*(v) = -\frac{\bar{\alpha}\Lambda}{r(\Lambda + r)}\ln v$$

Using Proposition (3), or directly from the equations above, we can show that as the expected bargaining power of the seller decreases:

i) The expected time to trade decreases.

[18] The model can also allow for the possibility of arrival of both, new information and new traders.

ii) The surplus for the seller falls.

iii) For any t the price offered is lower.

Finally, a case we believe to be interesting but which we leave for future work is when the buyer's valuation is not fully revealed by a unique signal. Instead, there is a gradual revelation of the buyer's type as time goes by and more and more signals arrive.

5.2 The Auction Format

In the applications we have analyzed so far we have assumed that the seller would run an English auction with no reserve price. McAfee and Vincent (1997) have shown that without commitment and symmetric bidders there is revenue equivalence between first price and second price auctions. Furthermore, when the time between auctions goes to zero the seller's expected revenues converge to those of a second price auction with no reserve. In our setup since first and second buyers are not symmetric at the beginning of the auction[19], different auction formats will yield different revenues.

Optimal auctions usually treat weaker bidders more favorably. With i.i.d. ex-ante distributions of the two buyers, the first buyer is going to have a weaker (truncated) distribution. However, treating the first bidder more favorably in the auction can make him more stubborn during the bargaining phase. Can that lead to time-inconsistency of the optimal auction choice of the seller (i.e. that he would like to choose one format ex-ante and another one ex-post)?

Using the analysis above we can show that in fact no such time-inconsistency would arise. The intuition is that since the lack of commitment to prices drives the seller's payoff down to his outside option, maximizing ex-post revenues, maximizes ex-ante payoffs as well. [20]

6 Conclusions

To be added

7 Appendix

To be added.

[19] The asymmetry arises endogenously even with i.i.d. valuations in our setup because of the updating that takes place during the previous bargaining with the first buyer.

[20] Modeling the impact of different auction formats is somewhat delicate because in general optimal bids depend on the beliefs the new buyer has about the value of the first buyer, and these in turn depend on how much of prior bargaining he can observe. The analysis is tractable when we assume that he observes the whole history.

References

[1] Abreu D, Gul F. (2000)."Bargaining and Reputation," Econometrica 68 (1): 85-117

[2] Admati, Anat and Motty Perry (1987), "Strategic Delay in Bargaining," Review of Economic Studies, 54, 345-364.

[3] Ausubel, Lawrence M., Peter Cramton, and Raymond J. Deneckere (2001). Bargaining with Incomplete Information. Robert J. Aumann and Sergiu Hart, eds., Handbook of Game Theory, Vol. 3, Amsterdam: Elsevier Science B.V.

[4] Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky, "The Nash Bargaining Solution in Economic Modeling," RAND Journal of Economics, Summer 1986, 17 (2), pp. 176–188.

[5] Cho, In-Koo (1990), "Uncertainty and Delay in Bargaining," Review of Economic Studies, 57, 575-596.

[6] Coase, Ronald H. (1972), "Durability and Monopoly," Journal of Law and Economics, 15, 143-149.

[7] Cramton, Peter (1984), "Bargaining with Incomplete Information: An Infinite-Horizon Model with Continuous Uncertainty," Review of Economic Studies, 51, 579-593.

[8] Deneckere, Raymond J. and Meng-Yu Liang (2001), "Bargaining with Interdependent Values," mimeo, University of Wisconsin-Madison.

[9] Evans, Robert (1989), "Sequential Bargaining with Correlated Values," Review of Economic Studies, 56(4),499-510.

[10] Feinberg, Yossi and Andrzej Skrzypacz (2005) "Uncertainty about Uncertainty and Delay in Bargaining." Econometrica 73 (1) pp. 69-91.

[11] Fudenberg, D., Levine, D., and Tirole, J. (1986). Infinite horizon models of bargaining with incomplete information, in Game Theoretic Models of Bargaining A. Roth, Ed., pp. 73-98.London New York: Cambridge Univ. Press.

[12] Gul, F., Sonnenschein, H., and Wilson, R. (1986). Foundations of dynamic monopoly and the Coase Conjecture, J. Econ. Theory 39, 155-190.

[13] Hall, R. and Paul R. Milgrom (2006) "The Limited Influence of Unemployment on the Wage Bargain." Mimeo Stanford University.

[14] Inderst, R. (2003). The Coase Conjecture in a Bargaining Model with Infinite Buyers. Working Paper LSE.

[15] Kremer, Ilan and Andrzej Skrzypacz (2006) "Dynamic Signaling and Market Breakdown." Forthcoming Journal of Economic Theory

[16] McAfee, P. and Vincent D. (1997). “Sequentially Optimal Auctions. Games and Economic Behavior" Vol. 18, 246-276.

[17] Noldeke G. and E. Van Damme (1990) " Signalling in a Dynamic Labour Market," Review of Economic Studies" Vol 57 (1990) pp. 1-23

[18] Rubinstein, Ariel (1982), “Perfect Equilibrium in a Bargaining Model,” Econometrica, 50, 97-109.

[19] Simon, L. and Stinchcombe, M. (1989). Extensive Form Games in Continuous Time: Pure Strategies." Econometrica Vol. 57, pp. 1171-1214

[20] Shimer, R. The Cyclical Behavior of Equilibrium Unemployment and Vacancies (2005) American Economic Review, 95(1): 25-49.

[21] Swinkels, Jeroen M, (1999). "Education Signalling with Preemptive Offers," Review of Economic Studies, Blackwell Publishing, vol. 66(4), pages 949-70

[22] Trejos, A. and Wright, R. (1995) "Search, Bargaining, Money and Prices," The Journal of Political Economy, Vol. 103, No. 1. (Feb., 1995), pp. 118-141.

[23] Vincent, Daniel R. (1989), “Bargaining with Common Values,” Journal of Economic Theory, 48, 47-62.

[24] Yildiz, Muhamet (2004) "Waiting to Persuade" Quarterly Journal of Economics Vol:119, Issue: 1 (February 2004), 223-248