Small Numbers, Large Meaning:

A Sensitivity Analysis of the Stern Review on Climate Change

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Abstract

The "Stern Review" (The Economics of Climate Change) assesses the costs of climate change and the benefits of avoiding it. It has attracted much attention and some criticism. The feature criticized most strongly by fellow economists is the Review's assumption of a rate of pure time preference close to zero. The central conclusion of the Review, namely that the costs of mitigation today are lower than those of adaptation tomorrow, is linked to this assumption. The debate has quickly focussed on the ethical validity of alternative judgements on the rate of time preference. Comparatively little effort has been made to assess the quantitative impact of alternative parameter values on the conclusions of the Review.

We try to fill this gap to facilitate further discussion. We carry out a sensitivity analysis of alternative parameter values for the rate of time preference and the intertemporal elasticity of substitution. The results show that the conclusions of the Review are sensitive to alternative parameter values. Using values which are standard in applied economics will lower the cost of global warming by a factor of 8 or more, invalidating the Review's policy conclusions. The debate on the ethical validity of the underlying assumption therefore is relevant. However, the results raise a bigger question. If climate change is happening – and we have no reason to doubt this – how wise it is to base crucial policy choices on a methodology so dependent on one judgement call?

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I. Introduction

The "Stern Review" on the economics of climate change (Stern 2007) is based on the comprehensive analysis of a huge amount of data. Its conclusions have commanded global attention. The most important of these conclusions is that the costs of mitigation, if mitigation is started early enough, will be lower than the costs of climate change, if nothing is done. Waiting therefore is not an option. Urgent action is imperative – and sufficient.

The Review expresses the welfare loss of climate change in terms of lost consumption per capita. It estimates these costs at 5% and up to 20% of global consumption in perpetuity.[2] Mitigation costs are expressed as a fraction of global output. To reach the desired level of 550ppm CO2e, which the Review regards as suitable and attainable, green house gas emissions will need to be cut, which is costly. Mitigation cost will rise as a share of GDP and by 2050 will have reached approximately 1% of GDP. The Review is not clear about how much mitigation costs will rise after that point. With the costs of climate change at 5% of consumption, and consumption assumed at 80% of GDP, however, the costs of climate change are 4% of GDP, compared to mitigation costs of 1% of GDP. From these headline numbers, the Review concludes the obvious: Mitigation is preferred because it is cost effective.

A number of criticisms have been levelled against the report, most of them concerned with a lack of rigour in analysis and data treatment. They include the following (we have addressed those most relevant for the argument that follows in greater detail in appendices).

- The Review utilizes two different growth rates, and hence discount rates, for calculating the costs of mitigation and the costs of climate change (see appendix D).

[2] Not, as stated in the Summary of Conclusion "at least 5% of global GDP" (Stern 2007, p. xv) - an endless source of errors in news reports about the Review.

- The Review uses different measurement units for mitigation costs (expressed as a fraction of global output) and the costs of climate change (expressed as a fraction of consumption) (see appendix D).
- The time horizon in the report extents to 2050 for the discussion of mitigation costs, and is infinite in the case of climate change (climate change is costly up to the year 2200).[3]
- The Review assumes that the population grows at an annual rate of 0.6% until the year 2200 and then stops growing. No reason is given for this assumption. However, it ensures analytical tractability of the Review (see appendix B).
- Some of the costs estimated for climate change are inconsistent (cf. Stern 2007 p. 186 and p. 333).

However, the point that has attracted most attention among economists is the assumption of a rate of time preference of almost zero which underlies the conclusions of the Review.[4] It has been noted that a higher rate of time preference would presumably lead to different conclusions – in particular, to lower estimates of the cost of climate change.[5] In a similar vain, it has been argued that changes in the intertemporal elasticity of substitution of consumption, which the Review fixes at unity, has the potential of altering the estimated costs of climate change in the same direction.[6]

This relates to an important ethical judgement. A rate of pure time preference of zero means that the utility of future generation has the same weight as the utility of present generations: In general terms, today' consumption is of equal utility than tomorrow's consumption. Most professional economists would say that future utility ought to be weighted lower than present utility – within the life of one person, and also when comparing across generations. Others, a minority in today's profession, would argue that

[3] The mitigation cost calculation is incomplete. It shows how to cut emissions by 25% from 2005 to 2050. It does not show how to cut emissions by 50% by 2100, as required by the emission trajectory targeted in the Review.
[4] The Review uses a discount rate of 0.1, allowing for the minute probability of a catastrophic event leading to extinction of the human race.
[5] Henderson et al (2006), Nordhaus (2006)
[6] Dasgupta (2006)

from an ethical point of view the utility of any future generation (including those not yet born) ought to be as valuable as the utility of those living today.

The value of time preference, of course, is not an observable parameter. Its value lies in the eyes of the beholder.

We have no desire to enter this debate here, and we certainly will not try to resolve it. However, despite a focus of the discussion of the Stern Review on this ethical dimension, comparatively little effort has been made to assess the actual, quantitative impact of alternative parameter values for the rate of time preference. We try to fill this gap, to facilitate further discussion, by carrying out a sensitivity analysis of alternative parameter values for the rate of time preference and the intertemporal elasticity of substitution.

The results of this analysis confirm that the conclusions of the Review are highly dependent on the assumed parameter value. The strength of the Stern Review lies in its ability to condense a complex problem – the question how to act in the face of global warming – into juxtaposing two numbers, the cost of mitigation and the costs of climate change. This strength may hide a weakness. The simulation shows the extent to which one of these values -- the cost of climate change -- and with it all the policy implications of the Review, depend on a single judgement call.[7]

II. Theoretical Framework

A simple model helps to illustrate. To capture the dimension of time, each generation in this model is assumed to live for one period. C denotes consumption and N the number of people. Consumption streams over time are ranked according to the following social welfare function:

[7] The costs of mitigation are calculated on a year by year basis and therefore are not affected by changes in the discount rate. As discussed in footnote 3, the Review does not provide sufficient information to sum these values over time

$$\sum_{t=0}^{\infty} e^{-\delta t} U(C_t) N_t$$

In this function δ indicates the rate of pure time preference or the social discount factor, (δ>0 and hence $e^{-\delta}$ is between 0 and 1). It is this parameter that has been criticised by economists as too low in the Stern Review: A value of zero for δ (and thus $e^{-\delta} = 1$) implies indifference between (equal utility of) consumption today and consumption tomorrow. The higher δ, the more present consumption is preferred over future consumption. The Stern Review sets $\delta = 0.1\%$. We will explore different values further below

$U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ is an instantaneous utility function, describing the utility derived from consumption at a given point in time. Following the Review, we assume that the intertemporal elasticity of substitution of consumption is one, i.e. $\sigma = 1$: $U(C) = \log(C)$. As indicated earlier, this value for σ has also been questioned as too low by economists critical of the Review. We will report sensitivity analysis with respect to this parameter as well.

The Stern Review assumes population growth at 0.6% per year, i.e. $N_t = N_0 \exp(0.006\, t)$ from 2000 through 2200, from which point onward the population remains constant. No reason is given for this sudden stop in population growth. We suspect it is merely to keep calculations tractable - but it is an important step: Without this assumption, the Review's social welfare function is not defined (see appendix B). However, in the interest of comparable simulations, we keep this same assumption.

Without climate change, people in the global economy would enjoy a consumption stream described by a law of motion e_t. The Stern Review assumes that per capita consumption without climate change will grow by 1.3% annually, i.e. it assumes $e_t = e_0 \exp(0.013\, t)$.

However, "business as usual" involves an increasing stock of GHG emissions leading to global warming, and global warming will reduce goods available for consumption. Changes in agriculture, migration, catastrophic events etc. will diminish the stream of future consumption by a fraction $1-\alpha_t$. This fraction will increase over time because the damages from climate change are assumed to increase due to the increased stock of GHG in the atmosphere. Climate change therefore will change the consumption stream to $\alpha_t e_t$, where α_t decreases, and at best remains constant, over time. The Review assumes that damages from climate change cease in the year 2200. After 2200, consumption grows again at 1.3%, starting from $\alpha_T e_T$, without further damages.[8]

Using the above assumptions about population and consumption growth, the social welfare function can be written as (Stern, 2007, p. 184. See appendix A below):

$$W = \sum_{t=0}^{T-1} N(t)\ln C(t)e^{-\delta t} + \left(\frac{N_T \ln C_T}{\delta} + \frac{N_T g}{\delta^2}\right)e^{-\delta T}$$

The Review measures the cost of climate change as a fraction of the consumption stream that would have resulted without climate change. This requires computing the decline in consumption associated with climate change, i.e. to find a fixed fraction τ of the original consumption stream which will mirror the utility of the consumption stream with climate change (see figure 1).

[8] The decision not to estimate economic damages 200 years from now does in itself reveal preferences with regard to the appropriate rate of time preference. At a rate of time preference as low as 0.1%, consumption of generations 200 years from now would still provide substantial utility today - with a weight of approximately 80%!

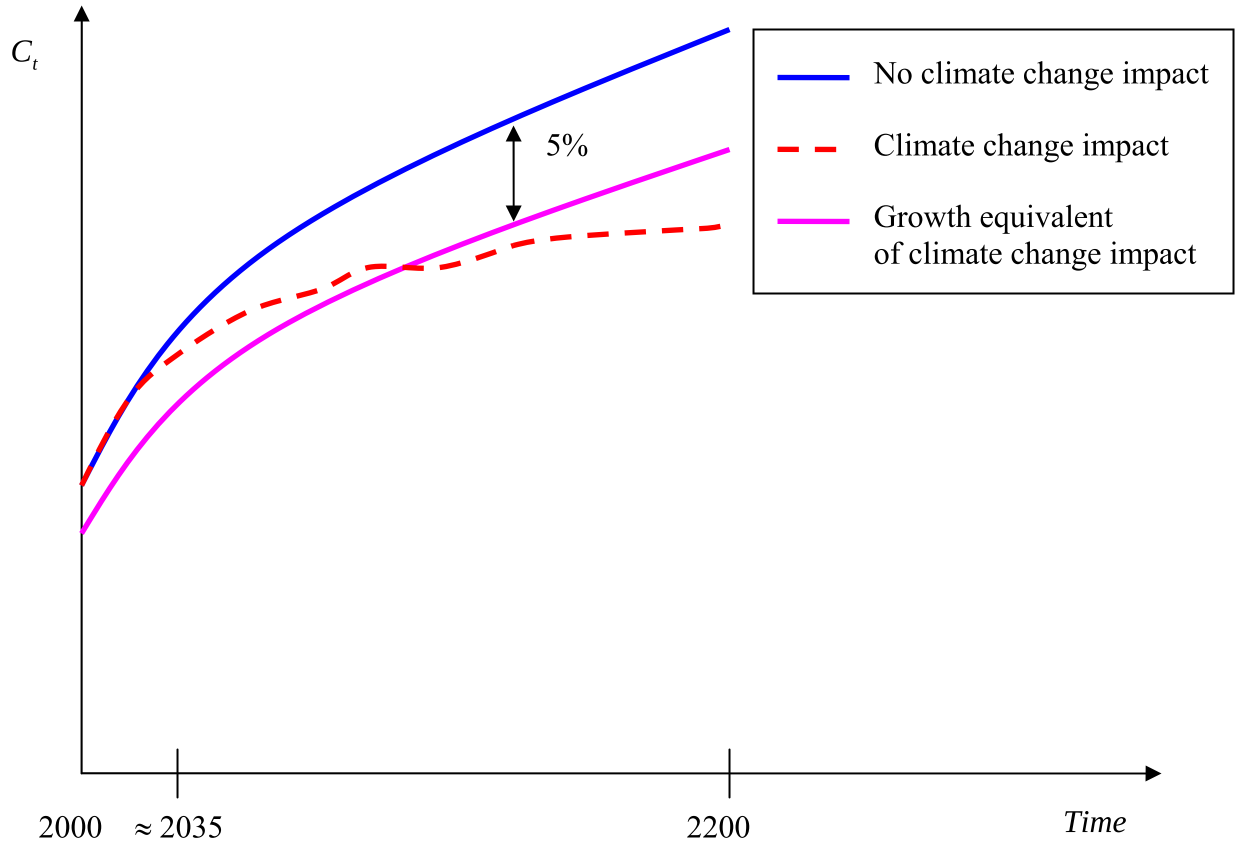

Figure 1: Cost of climate change as a fraction of potential consumption

The utility of consumption with damages from climate change is:

$$W_{CC} = \sum_{t=0}^{T-1} e^{-\delta t} N(t) \ln(\alpha_t e_t) + \left(\frac{N_T \ln e_T \alpha_T}{\delta} + \frac{N_T g}{\delta^2} \right) e^{-\delta T}$$

The equivalent utility of consumption without climate change, but with a uniform decline in consumption is:

$$W_{NCC} = \sum_{t=0}^{T-1} e^{-\delta t} N(t) \ln(\tau\, e_t) + \left(\frac{N_T \ln(\tau\, e_T)}{\delta} + \frac{N_T g}{\delta^2} \right) e^{-\delta T}$$

Thus, τ is defined as $W_{CC} = W_{NCC}$, or equivalently:

$$\log \tau = \frac{\sum_{t=0}^{T-1} e^{-\delta t} N(t) \ln(\alpha_t) + \frac{N_T \ln \alpha_T}{\delta} e^{-\delta T}}{\sum_{t=0}^{T-1} e^{-\delta t} N(t) + \frac{N_T}{\delta} e^{-\delta T}}.$$

It can be shown that $\frac{d\tau}{d\delta} > 0$ (see appendix C). This means that the estimated damages $1-\tau$ will decline as higher values for δ are considered. The question we want to answer is how large this derivative is. How do the costs of global warming in the framework of the Stern Review change as higher values for the pure rate of time preferences are substituted?

The following section provides the results of the numerical analysis.

III. Numerical Results

The ideal way to investigate the sensitivity of the results of the Review to the underlying assumption about time preference would have been to use the same program. However, the model runs were not available upon request. We therefore developed a separate simulation model.

The main difficulty of course is the appropriate choice of damages from climate change represented by the coefficients α_t, which diminish the available stream of consumption. All estimates of the costs of global warming will crucially depend on the choice of the path of α_t over time – both, the point in time when damages first occur, and the speed with which they are assumed to propagate. Given that the model runs leading to the results in the Stern Review are not made available, these are also potentially the largest results of error in replicating the Review's results.

Luckily, the Review itself undertakes partial sensitivity analysis for low values of δ. We therefore are able to calibrate the time path for damages from climate change to match the data presented in the Review. In addition, we can ensure that the damage estimates in our calculation react less sensitive to changes in the rate of time preference δ than those in the Review over the relevant range (i.e. costs in our runs decline less rapidly as δ increases than costs published in the Stern Review).

The results reported below should therefore, if anything, understate the magnitude with which the costs of global warming react to changes in the discount rate. Figure 2 illustrates how our estimates provide a lower bound for the impact of changes in the rate of time preference on the costs of climate change.

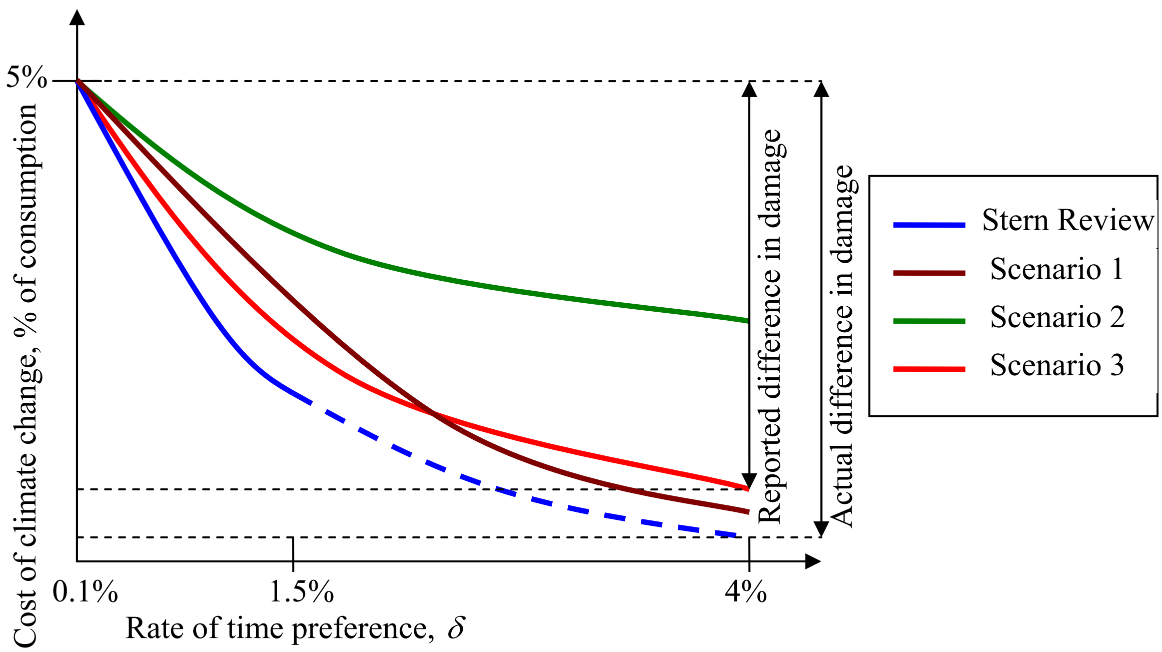

Figure 2: the costs of climate change as a function of the rate of time preference

We simulate three scenarios to capture the path of damages from climate change on consumption.

- In the first scenario, all damages happen in the middle of the period considered in the Review, i.e. in 2100. This depicts a time path for consumption, where consumption before 2100 follows a "business as usual" path, not affected by climate change and then drops permanently to the new, lower path, driven by the one-off cost of climate change:

  $e_t = C_0 e^{gt}, t < 2100$

  $e_t = C_T e^{gt}, t \geq 2100$

The consequences of this drop are calibrated so as to match the stipulations of the Review. Assuming a rate of time preference of 0.1%, the cumulative damage comes to 5% of consumption over time, the same as in the Review.

- Scenario 2 is the same as scenario 1 but this time all damages are incurred in the year 2060 instead of 2100. Again, the scenario is calibrated so as to generate 5% of foregone consumption at a rate of time preference of 0.1% as the cumulative costs of climate change.

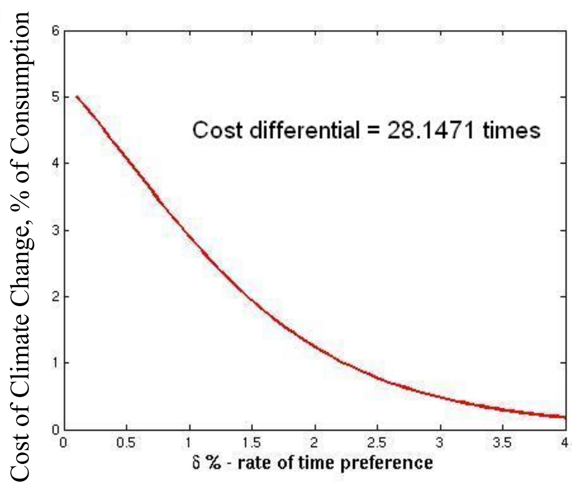

Figure 3 a

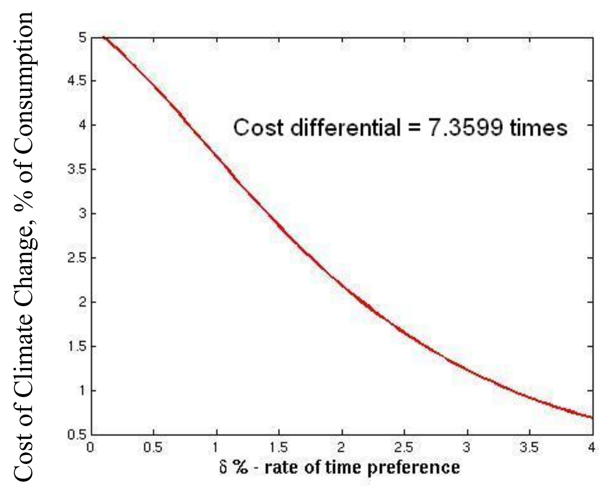

Figure 3 b

Cost of climate change as a function of the rate of time preference

Both simulations show a considerable "overstatement" of the cost of climate change in the Stern Review, compared to a situation where higher rates of time preference are deemed desirable (figure 3a and b). However, these scenarios also depart from the sensitivity analysis performed in the Review (see table 1). Their costs are less sensitive to the rate of time preference than the Review suggests.

Both of our scenarios assume that the losses from climate change occur as a one-off cost. The Review leaves open the possibility of costs accumulating over time, after they have started to occur. Our third scenario allows for this possibility. Allowing for costs to accumulate over time will increase the sensitivity of cost estimates to changes in the rate of time preference. The rational is easy to see: An accumulation of costs means that a higher proportion of them is shifted into the future. As costs are shifted into the future, changes in the discount rate will have a higher impact.

- In the third scenario, the assumption that the negative impact occurs at a single point in time is replaced by a decline in consumption growth after an initial period during which nothing happens.

The Review states that threshold concentrations of 550ppm are likely to be reached between 2035 and 2050 (e.g., Stern 2007, p. 193). In line with this statement we assume that the path of consumption growth will start to decline in response to climate change in 2035:

$$C_t = e_{2000} e^{gt}, t \leq 2035$$

$$C_t = \frac{e_{2000}}{e^{D(t-2035)}} e^{gt}, t \geq 2035$$

D denotes the percentage by which consumption drops below the balanced growth path after the damages from climate change reveal themselves from 2035. Figure 4 shows that the costs on the Stern Review would fall by a factor of more than 20 if the rate of time preference were 4%.

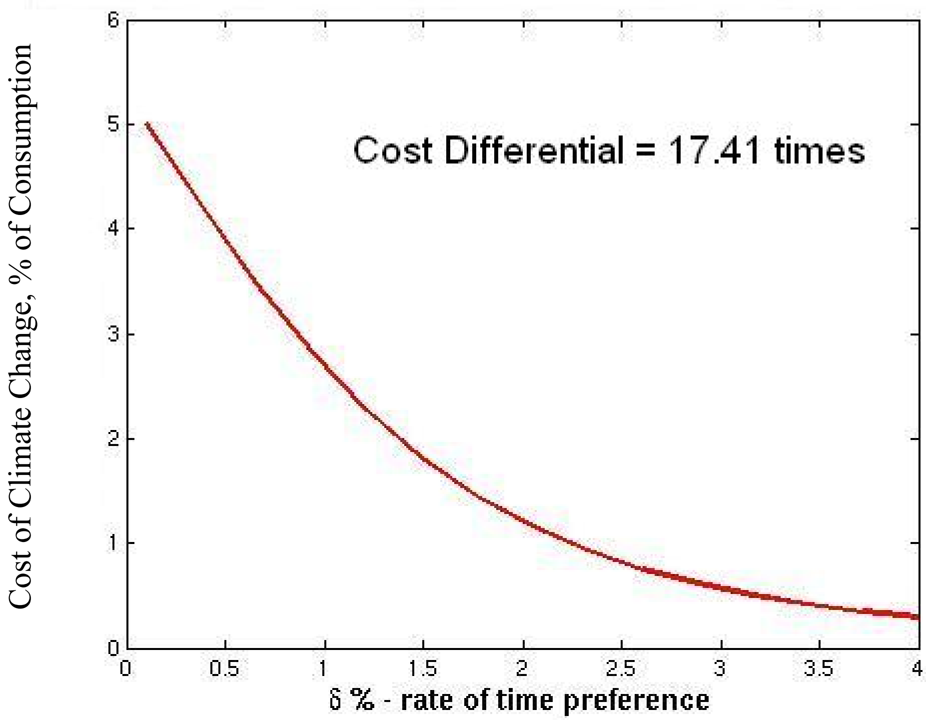

Figure 4: Cost of climate change as a function of the rate of time preference

The third scenario is closest to the sensitivity results of the Stern Review. Table 1 depicts the sensitivity analysis for the most prominent case, in which the damage from global warming amounts to 5% of consumption expenditures over time and contrasts it with the values generated by the three scenarios discussed above. Clearly, the third one comes closest. It also is in the spirit of providing a low estimate of how the costs from climate change would decline with an increase in the rate of time preference.

Damage function exponent	Pure time discount rate (percent)	Stern review estimates	First scenario	Second scenario	Third scenario
Low range	0.1	5.0	5.00	5.00	5.00
	0.5	3.6	4.07	4.45	3.88
	1.0	2.3	2.90	3.64	2.68
	1.5	1.4	1.93	2.86	1.80

Table 1: Sensitivity of costs of climate change to the rate of time preference

It therefore appears appropriate to use the methodology of the third scenario to replicate the sensitivity analysis to match all the specifications of damages discussed in the Review. Table 2 below shows the parameter values and the sensitivity analysis considered in the Stern Review.

Damage function exponent	Pure time discount rate (percent)	Baseline climate; market impacts + risk of catastrophe (mean)	Baseline climate; market impacts + risk of catastrophe + non-market impacts (mean)	High climate; market impacts + risk of catastrophe + non-market impacts (mean)
Low range	0.1	5.0	10.9	14.4
	0.5	3.6	8.1	10.6
	1.0	2.3	5.2	6.7
	1.5	1.4	3.3	4.2
High range	0.1	6.0	14.2	21.9
	0.5	4.3	10.2	15.8
	1.0	2.7	6.4	9.8
	1.5	1.7	4.0	5.9

Table 2: Sensitivity analysis in the Stern Review (Stern 2007, p. 668)

Table 3 below reports the damage estimates resulting from our model. It extends the range of parameter values for the rate of time preference to take account of values of 3% and 4% as encountered in the economic literature,[9] and cited by critics of the Review.[10]

In terms of methodology, we calibrate our model to match the available results for the six specifications introduced in the Review at a value of time preference of 0.1%. We then run it for the parameter values considered in the Review, and extend it for values up to 4%.

Damage function exponent	Pure time discount rate (percent)	Baseline climate; market impacts + risk of catastrophe	Baseline climate; market impacts + risk of catastrophe + non-market impacts	High climate; market impacts + risk of catastrophe + non-market impacts
Low range	0.1	5.00	10.9	14.4
	0.5	3.88	8.52	11.3
	1.0	2.68	5.93	7.91
	1.5	1.80	4.00	5.35
	2.0	**1.20**	**2.69**	**3.60**
	3.0	**0.56**	**1.27**	**1.70**
	4.0	**0.29**	**0.65**	**0.87**
High range	0.1	6.0	14.20	21.9
	0.5	4.66	11.14	17.36
	1.0	3.22	7.79	12.27
	1.5	2.16	5.27	8.37
	2.0	**1.45**	**3.55**	**5.67**
	3.0	**0.68**	**1.67**	**2.69**
	4.0	**0.35**	**0.85**	**1.38**

Table 3: Simulation results

[9] E.g., Lucas (1990).
[10] Nordhaus (2006).

The results have the "lower bound" feature desirable when the original data is unavailable: For all cases and parameter values covered in the Review, our reported cost estimates are higher and therefore the implied impact of changes in the rate of time preference is lower than in the Review. This tendency is likely to continue also for parameter values larger than 1.5% (not covered in the Review) which gives confidence that our results will *under*estimate the impact of changes in the rate of time preference on the costs of climate change reported in the Stern Review.

Depending on the preferred rate of time preference, the results indicate that the Stern Review could substantially overstate the potential costs of climate change. If a rate of time preference of 3% were to be considered accurate, the Review would overestimate the costs of climate change by a factor of 8; if a discount of 4% were to be deemed appropriate, it would overestimate costs by a factor of 16 (see tables 3 and 4).

Damage function exponent	Baseline climate; market impacts + risk of catastrophe	Baseline climate; market impacts + risk of catastrophe + non-market impacts	High climate; market impacts + risk of catastrophe + non-market impacts
Low range	**8.93**	**8.58**	**8.47**
High range	**8.82**	**8.50**	**8.14**

Table 4: Simulation results with rate of time preference of 3%

Damage function exponent	Baseline climate; market impacts + risk of catastrophe	Baseline climate; market impacts + risk of catastrophe + non-market impacts	High climate; market impacts + risk of catastrophe + non-market impacts
Low range	**17.41**	**16.77**	**16.55**
High range	**17.14**	**16.71**	**15.87**

Table 5: Simulation results with rate of time preference of 4%

Using the parameters in the Review, mitigation costs of 1% of GDP (1.25% of consumption) imply that a factor of 4 would be sufficient to derail the Review's conclusion that mitigation is the preferred option. The cost differentials reported in tables 4 and 5 are high enough to reverse the Review's central conclusion.

The elasticity of substitution

A second variable which has been criticized is the Stern Review's choice of fixing the intertemporal elasticity of substitution of consumption. In the Review, the value of σ is unity, whereas in parts of the economic literature, σ assumes higher values.[11]

Higher values of σ also decrease the estimated costs of climate change.

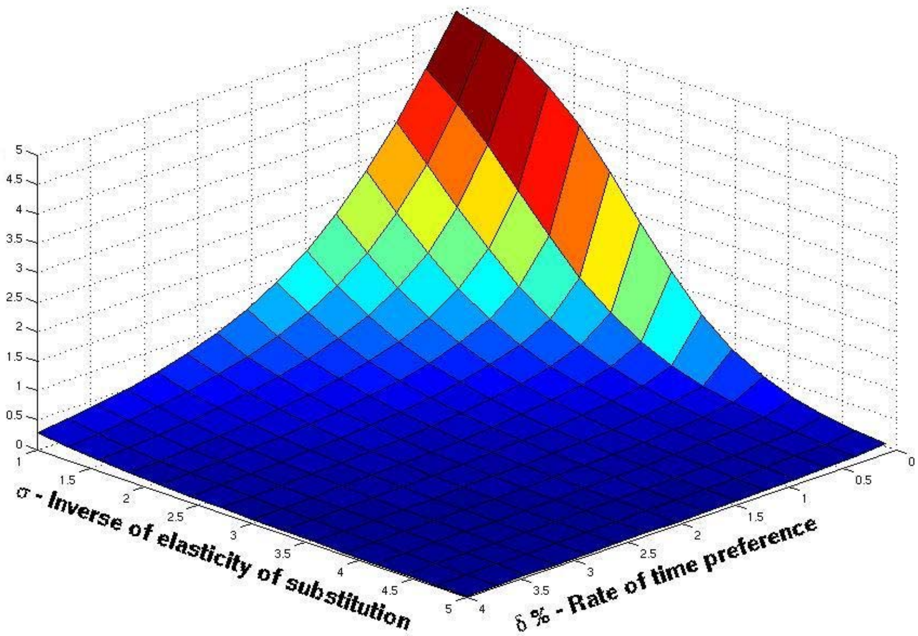

Figure 4

Figure 4 shows how the estimated costs of climate change depend on the choice of the rate of time preference and the elasticity of substitution. For example, choosing $\sigma = 2$ and $\delta = 4\%$ – values considered entirely appropriate by many in the profession – would decrease the estimated costs of climate change by a factor of 33 in our scenario, compared to the familiar assumption of $\delta = 0.1\%$ and $\sigma = 1$.

[11] Whereas values of 1 or 2 for σ are common in the literature on economic growth (e.g., Lucas 1988, 1990), the academic literature on asset pricing (where σ controls for risk aversion) suggests much higher values of up to 4-5 and even 10 (cf. Cochrane 2001).

IV. Conclusion

The policy implications of the Stern Review rest to a large extent on the conclusion that the costs of climate change justify action now, because they are considerably higher than the costs of mitigation. The simulation results above show that this conclusion is indeed based on the assumption of a rate of time preference, which is very low. Cost differentials by a factor of 4 are sufficient to reverse the central policy conclusion of the Review. Calibrating the model at values of 3-4% for the rate of time preference (the social discount factor), which are common in applied economics, decreases the costs of climate change by at least a factor of 8.

The rate of time preference is the single most important factor driving the results of the Stern Report. It is an unobservable parameter, the value of which reflects personal and ethical judgement. The conclusions of the Review are not robust to variations in this parameter.

This certainly means that the debate on the ethical foundation of this parameter is of great importance. However, it also raises the question how wise it is to base crucial policy decisions on a methodology so dependent on one judgement call.

V Literature

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APPENDIX

(A) Derivation of the social welfare function in the Stern Review

The Stern Review (Stern 2007, p. 184, equation (6)) states that the social welfare function can be written as:

$$W = \sum_{t=0}^{T-1} N(t)\ln C(c)e^{-\delta t} + \left(\frac{N_T \ln C_T}{\delta} + \frac{N_T g}{\delta^2}\right)e^{-\delta T}$$

The derivation starts from the following general objective function:

$$W = \sum_{t=0}^{\infty} N(t)\ln C(t)e^{-\delta t} = \sum_{t=0}^{T-1} N(t)\ln C(t)e^{-\delta t} + \sum_{t=T}^{\infty} N(t)\ln C(t)e^{-\delta t}, T = 200$$

The Review assumes that $N(t) = N_0 e^{nt}$ for $t < T$, $n = 0.6\%$ and $N(t) = N(T), t > T$. It also assumes that the damages from global warming cease in 2200, with no more damages after 2200, i.e. $C(t) = C_T e^{g(t-T)}, t > T$.

With these assumptions we get:

$$\sum_{t=T}^{\infty} N(t)\ln C(t)e^{-\delta t} = N_T e^{-\delta T}\sum_{t=T}^{\infty}\left(\ln C_T + g(t-T)\right)e^{-\delta(t-T)} \approx$$
$$\approx N_T e^{-\delta T}\int_T^{\infty}\left(\ln C_T + g(t-T)\right)e^{-\delta(t-T)}dt$$

$$\int_T^{\infty}\left(\ln C_T + g(t-T)\right)e^{-\delta(t-T)}dt = -\left.\frac{\left(\ln C_T + g(t-T)\right)e^{-\delta(t-T)}}{\delta}\right|_T^{\infty} + \int_T^{\infty}\frac{g}{\delta}e^{-\delta(t-T)}dt = \frac{\ln C_T}{\delta} + \frac{g}{\delta^2}$$

Q.E.D.

(B) Population growth assumptions

The Review (Stern 2007, p. 183, footnote 36) assumes that the global population stops growing after 2200. No rational is provided as to why this may be the case, but it seems to have been motivated by the desire to keep the social welfare function defined for low rates of time preference. For, if the population would grow forever at a constant rate larger than the rate of time preference (as is the case for population growth of 0.6% and a rate of time preference of 0.1% assumed in the Review), the social welfare function does not exist: The infinite sum $\sum_{t=0}^{\infty} N(t) \ln C(t) e^{-\delta t}$ can not converge to a finite number, because $n > \delta$ (0.6%>0.1%) and because consumption increases over time. In this case, the results would only reflect the authors prior: The damages from climate change could be depicted as arbitrarily high or low, simply by fixing the time horizon T to achieve the desired outcome.

(C) Proof that estimated damages decline with higher values of the rate of time preference

We prove this result for the general case. The only assumption needed is that α_t is non-increasing over time and that there is some t_0 such that $\alpha_{t_0} > \alpha_{t_0+1}$. Intuitively this means that damages do actually occur and that they are irreversible: As the economy continues to grow, its production potential will always reflect those damages (i.e., output will always be lower than it "would have been" without climate change).

In the general case, the growth equivalent fraction of consumption τ is defined as:

$$\sum_{t=0}^{\infty} \lambda^t N_t \ln(e_t \tau) = \sum_{t=0}^{\infty} \lambda^t N_t \ln(e_t \alpha_t)$$

To be concise, we denote $\lambda = e^{-\delta}$. This general specification allows for stabilization after 2200, when further damages from climate change cease to exist: i.e. N_t, α_t are constant after 2200. Calculating $\ln \tau$ from the expression above gives:

$$\ln \tau = \frac{\sum_{t=0}^{\infty} \lambda^t N_t \ln(\alpha_t)}{\sum_{t=0}^{\infty} \lambda^t N_t}$$

Denote $Y_t = N_t \ln \alpha_t$. Since $0 < \alpha_t < 1$ are non-increasing, Y_t / N_t are negative and non-increasing as well. The derivative of $\ln \tau$ with respect to λ is

$$\frac{d\ln\tau}{d\lambda} = \frac{\sum_{t=0}^{\infty} t\lambda^{t-1}Y_t \sum_{t=0}^{\infty}\lambda^t N_t - \sum_{t=0}^{\infty} t\lambda^{t-1}N_t \sum_{t=0}^{\infty}\lambda^t Y_t}{\left(\sum_{t=0}^{\infty}\lambda^t N_t\right)^2}$$

The sign of this fraction coincides with the sign of its numerator:

$$\sum_{t=0}^{\infty} t\lambda^{t-1}Y_t \sum_{t=0}^{\infty}\lambda^t N_t - \sum_{t=0}^{\infty} t\lambda^{t-1}N_t \sum_{t=0}^{\infty}\lambda^t Y_t = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} n\lambda^{n+m-1}\left(Y_n N_m - Y_m N_n\right)$$

If $n = m$, the corresponding term in the sum above is zero. With $n \neq m$, the terms $(n,m),(m,n)$ can be considered together:

$$n\lambda^{n+m-1}\left(Y_n N_m - Y_m N_n\right) + m\lambda^{n+m-1}\left(Y_m N_n - Y_n N_m\right) = (n-m)N_n N_m \lambda^{n+m-1}\left(\frac{Y_n}{N_n} - \frac{Y_m}{N_m}\right)$$

If $n > m$, since $\frac{Y_n}{N_n} \leq \frac{Y_m}{N_m}$, this term is non-positive. Similarly, if $n < m$, $\frac{Y_n}{N_n} \geq \frac{Y_m}{N_m}$ and again, the term is non-positive.

Finally, since $t_0 : \frac{Y_{t_0}}{N_{t_0}} > \frac{Y_{t_0+1}}{N_{t_0+1}}$ exists, the corresponding term in the sum is negative.

Thus, the sum is strictly negative. This means that, as the value of the rate of time preference δ increases and λ decreases, $\ln\tau$ and thus τ increase and the estimated damage $1-\tau$ therefore decreases.

Q.E.D.

(D) Different discount rates for the cost of global warming and the cost of mitigation

The Stern Review estimates that the costs of mitigation would be around 1% of world product in 2050, assuming that world product will grow at 2.5% per year from 2005 onward.[12] At the same time, the Review assumes a growth rate of 1.3% for per capita consumption when computing the costs of global warming.[13] The assumption of constant

[12] Stern (2007) p. 262 Table 9.3 footnote (a)
[13] Stern (2007) p.184

population growth of 0.6%[14] and a constant (80%) share of consumption in global production[15] imply a global rate of growth of 1.9%. The costs of mitigation are therefore underestimated.

We recalculate the costs of mitigation under the assumption that global output grows at 1.9%. Since world product is assumed to grow at 2.5% for 45 years starting from 2005, the costs of mitigation are equal to:

$$M = 1\% * WP_{2005} \exp(45 * 2.5\%)$$

If global output grows at 1.9% instead, it will be equal to just $WP_{2005} \exp(45 * 2.5\%)$ in 2050. The costs of mitigation M will constitute $x\%$ of world product where x is defined by the equation:

$$M = 1\% * WP_{2005} \exp(45 * 2.5\%) = x\% * WP_{2005} \exp(45 * 1.9\%)$$

$$x = \exp(45 * [2.5\% - 1.9\%])\% \approx 1.3\%$$

The corrected mitigation cost estimate thus is 30% higher than the one discussed in the Review.

Moreover, the costs of global warming and the costs of mitigation are measured in different units, i.e. fractions of consumption and world output respectively. If one were to use the corrected estimate of 1.3% of global output, adjusted with the assumption that consumption constitutes 80% of global output, mitigation costs would be $\frac{1.3\%}{0.8} = 1.64\%$ instead of the 1% argued in the Review.

(E) Text of the program

```
% This program is used to evaluate sensitivity of findings of Stern review
% to the assumed social discount factor value
clear
g=0.013; % consumption growth
n=0.006;
delta=[0.001 0.005 0.01 0.015 0.02 0.03 0.04];

%delta=linspace(0.001,0.04,15);
```

[14] Stern (2007) p. 183 footnote 36
[15] Stern (2007) p. 183 Box 6.3

```
sigma1=linspace(1,5,15);%[1 2 3 4 5]

lambda=exp(n*ones(size(delta))-delta);

t0=2000;
T=2200;

C0=1;
%CT=0.9471; % third scenario 5%
%CT=0.8849; %3rd 10.9%

CT=0.8482; % 3rd 14.4%
%CT=0.9366; % 3rd 6.0%
%CT=0.8503; %3rd 14.2
%CT=0.7697 %3rd 21.9

%CT=0.9471; % third scenario 5%
%   CT=0.9489; % second 5%
%   CT=0.9478 % first scenario 5%

%   WP=22*exp(-5*0.025); % world product 22 trillion pounds in 2005 (does not matter
anyway)
%   C0=C0*WP;
%   CT=CT*WP;

TimeLeft=35;

declineExp=(log(C0/CT)/(T-t0-TimeLeft));

W0=zeros(length(sigma1),length(lambda));
W1=W0;
W11=W0;
W01=W11;
denominator=W0;

for i=1:length(sigma1)
for t=t0:T-1
        W0(i,:)=W0(i,:)+(U1(C0*exp(g*(t-t0)),sigma1(i)))*lambda.^(t-t0);
        W1(i,:)=W1(i,:)+(U1((exp(log(C0)-declineExp*max(t-t0-TimeLeft,0)))*exp(g*(t-
t0)),sigma1(i)))*lambda.^(t-t0);

        denominator(i,:)=denominator(i,:)+lambda.^(t-t0);

%    if t<t0+100
%        W1(i,:)=W1(i,:)+(U1(C0*exp(g*(t-t0)),sigma1(i)))*lambda.^(t-t0);
%    else
%        W1(i,:)=W1(i,:)+(U1(CT*exp(g*(t-t0)),sigma1(i)))*lambda.^(t-t0);
%    end
end
%W1(i,:)=W1(i,:)+(U1(CT*exp(g*(T-t0)),sigma1(i)))*((lambda.^(T-t0))./delta);
W1(i,:)=W1(i,:)+(U1((exp(log(C0)-declineExp*max(T-t0-TimeLeft,0)))*exp(g*(T-
t0)),sigma1(i)))*((lambda.^(T-t0))./delta);

W0(i,:)=W0(i,:)+(U1(C0*exp(g*(T-t0)),sigma1(i)))*((lambda.^(T-t0))./delta);
denominator(i,:)=denominator(i,:)+((lambda.^(T-t0))./delta);

if i==1
tau(i,:)=exp((W1(i,:)-W0(i,:))./denominator(i,:));
else
    tau(i,:)=exp(log(W1(i,:)./(W0(i,:)))/(1-sigma1(i)));
end
end


%plot(delta*100, ratio(1,:), delta*100, zeros(size(delta)))

plot((delta)*100,(1-tau(1,:))*100, 'LineWidth', 2.0 , 'Color' , 'r')
title('Consumption decrease (%) as a function of the rate of time preference','FontSize',
14.0, 'FontWeight', 'Bold')
xlabel('\delta % - rate of time preference', 'FontSize', 14.0, 'FontWeight', 'Bold')
```

```
ylabel('Decrease in consumption %','FontSize', 14.0, 'FontWeight', 'Bold')
text(1.5,4.5,['Overstatement = ',num2str((1-tau(1,1))/(1-tau(1,end))), '
times'],'FontSize', 14.0, 'FontWeight', 'Bold' )

surf((delta)*100, sigma1, (1-tau)*100)
view([1,1,1])
xlabel('\delta % - Rate of time preference', 'FontSize', 20.0, 'FontWeight','Bold',
'Rotation',19, 'Position', [20 21.4 20])
ylabel('\sigma - Inverse of elasticity of substitution', 'FontSize', 20.0,
'FontWeight','Bold', 'Rotation',-19,'Position', [2,2.5,-3])
zlabel('Decrease in Consumtion (%)', 'FontSize', 20.0, 'FontWeight','Bold', 'Rotation',90)
title('Dependence of cost of climate change on rate of time preference and elasticity of
substitution','FontSize', 20.0, 'FontWeight','Bold')
```