The Role of Firm Heterogeneity in Determining Labor Market Outcomes

Mathis Wagner*
University of Chicago
PRELIMINARY AND INCOMPLETE

May 10, 2006

Abstract

I construct and estimate an equilibrium search model with two-sided heterogeneity and on-the-job-search. Both firms and workers are heterogenous in a productivity parameter, and solve their respective optimization problems in an environment with undirected search. Market equilibrium is achieved by the entry and exit of firms run by profit maximizing entrepreneurs. The structure of the model allows for a wide-range of wage-setting mechanisms; three of which, an exogenous output-sharing rule, a surplus-splitting rule and a Bertrand competition mechanism, the paper deals with explictly. The paper delivers a theory of equilibrium wage dispersion and labor turnover, and relates these to observable and unobservable worker and firm characteristics. The structural model is estimated using matched a uniquely comprehensive employer and employee Austrian panel data. The full distribution of time-varying worker and firm productivities and transition rates are estimated. Preliminary results suggest that firm heterogeneity explains one-quarter of residual (after accounting for observable worker characteristics) wage dispersion, and person-specific unobserved heterogeneity for the remaing three-quarters. Firm effects explain perhaps 45% of inter-industry wage differentials, and up to 30% of wage variation for low-skilled workers. Unobserved worker heterogeneity explain nearly all of gender wage differentials and wage dispersion amongst high-skilled workers. Firm effects account for three-quarters of the explained variation in wage growth rates. Over the lifecycle workers switch to firms with higher productivity. Firm heterogeneity explains the firmsize-wage effect; turnover and the fraction of the workforce recruited from the pool of unemployed are decreasing in firm productivity.

*I would like to thank Gary Becker, Steven Davis, Derek Neal, Robert Shimer, Robert Topel, Michael Wagner-Pinter and numerous colleagues and seminar participants at the University of Chicago for advice and helpful discussions. I am most grateful to Synthesis Forschung and Günter Kernbeiss for providing the data used in this paper. All errors are my own. Email address: mwagner@uchicago.edu.

1 Introduction

The central proposition of this paper is that the huge and well-documented dispersion in productivity amongst firms[1] may have important implications for understanding two central facts of labor markets; the large amounts of wage dispersion that can not be attributed to observable worker characteristics, and the high degree of labor turnover, even at low frequencies, found in labor markets. Beginning with these observations I construct and estimate an equilibrium model of a labor market with heterogenous workers, firms and entrepreneurs who interact in an environment with search frictions, where there is on-the-job search. The basic setup borrows most closely from the matching with labor market frictions literature.[2] However, my model removes the requirement of a matching function, by modelling an explicit entrepreneur and firm optimization problem.[3] The number of firms is determined by the occupational decision of a class of entrepreneurs, who are modelled as having an exogenously determined managerial ability distribution, and can choose to start a firm or become workers. The structure of the model allows for a wide range of wage-setting mechanisms. The paper deals with three of these mechanisms explictly, an exogenous output-sharing rule, a variant on the Nash-bargaining assumption widely used in the literature, and the Bertrand competition mechanism of Postel-Vinay and Robin (1999).

The modelling strategy allows for structural identification of time-varying person and firm productivity parameters, to my knowledge the first paper to do so, as well as job destruction, job offer and unemployment rates. I use a new Austrian dataset which is uniquely comprehensive, it covers all employees in Austria from 1972 to 2003, to estimate the model. The findings speak to a number of issues much debated in the literature: the contribution of firm and worker heterogeneity to the cross-sectional wage distribution, the role of search versus human capital accumulation in wage growth over the lifecycle, inter-industry wage differentials, the firmsize wage effect, mobility decisions of workers over the lifecycle, turnover and the composition of hiring at firms, productivity shocks versus structural change as a source of job creation and destruction, and firm entry and exit. The model also addresses key questions of the macroeconomic search literature, the Beveridge curve and the relative volatility of wages, vacancies and unemployment.

There are two main theoretical innovations. The matching function is replaced with an entrepreneur and firm decision problem, allowing for a more explicit treatment of firm-worker interactions, as well as making the model more amenable to estimation using microeconomic data. Second, the structure is flexible enough to accomodate a diversity of wage-setting theories, providing a unified framework within which to compare them. Finally, the model results in a log-linear wage determination function, which make its results directly comparable to

[1] See Bartelsman and Doms (2000) for a survey, Syverson (2004) and Dunne, Foster, Haltiwanger and Troske (2004).

[2] See Pissarides (2000) for an overview.

[3] The matching function has, I would argue, no clear empirical microeconomic counterpart. For this reason most of the empirical literature using matching models has focused on macroeconomic issues, see for example [] and Shimer (2005). Moreover, in most of this literature there is no real sense in which there is a firm. Rather job vacancies are treated in isolation of each other.

the large reduced-form literature in the areas on which it touches.

Thus far, my preliminary results suggest that the firm effect accounts for around one-quarter of the residual variation in wages (after having accounted for observable worker characteristics), with the person effect explaining the rest. Firm effect and the unobserved part of the person effect are slightly positively correlated, a correlation coefficient of 0.08. The variation in the firm effect explains perhaps 45% of inter 2-digit industry wage differentials, and as much as 30% of wage dispersion amongst low-skilled workers. It explains little of gender wage differentials, which are accounted for entirely by the unobserved component of the person effect, and only around 14% of the residual wage variation for workers with a university degree. For understanding wage growth the firm effect's explanatory power is twice that of observables (experience and tenure), albeit still only 8%.

The data provides broad support for the predictions of the model. Over the lifecycle workers do switch to firms with higher estimated productivity, and there is a positive relationship between the firm effect and average experience and tenure levels at the firm. The firmsize-wage effect can nearly entirely be explained by the fact that higher productivity firms pay more and are larger. Workers' probability of leaving a firm is decreasing in the firm effect. Turnover and the fraction of a firm's workforce hired form the pool of unemployed are lower at more productive firms.

After a brief review of the related literature, Section 3 describes the model and Section 4 outlines identification and estimation method. In Section 5 I discuss the data source and some main features of the Austrian labor market. In Section 6 I discuss the results and Section 7 concludes.

2 Related Literature

When interested in how firm heterogeneity induces wage dispersion and job mobility, a natural starting point is the search literature, of which there are two major strands. Both strands of the literature assume the existence of labor market frictions that create rents, which are then divided between workers and firms. The part of the literature that has become synthesized in what has become known as the Burdett-Mortensen model, Mortensen (2003) provides a comprehensive overview, assumes that all rents go to firms.[4] Firms post wages both to compensate and induce workers to join the firm;[5] wage dispersion exists in equilibrium both if heterogenous or homogenous firms are assumed. In closely related models, typically known as matching models, workers and firms are matched with each other via a matching function. Search is truly undirected in that firms do not strategically set wages to attract workers, rather wage-setting is determined by Nash-bargaining. This literature begins with Diamond (1982) and Pissarides (1985), it is summarized in Pissarides (2000).

[4]Early papers are by Phelps (1970), Mortensen (1970) and Rothschild (1973), Butters (1977), Burdett and Judd (1983), Mortensen (1990) and Burdett and Mortensen (1998).

[5]In this the literature is related to the efficiency wage literature, in that wages play a dual role. In this case to both compensate and attract workers, a similar idea to the turnover theory of efficiency wages, see for example Salop (1979).

Common to all this literature, with the notable exception of Acemoglu and Shimer (2000) is that productivity differentials are exogenous.

Whilst this paper is based on search in markets with labor market frictions, it is nevertheless worth bearing in mind as a point of reference the literature on search in competitive markets. The defining paper of this literature is Jovanovic (1979), and the Roy (1951) model, who suggests that there is learning about the productive value of a worker-firm match. Mobility arises on account of imperfect information about match-quality, as workers leave low productivty matches to, in expectation, higher productivity matches. Jovanovic and Moffitt (1990) use this idea to explain why gross inter-sectoral mobility is much higher than net flows, Topel (1991) to estimate the amount of firm-specific human capital, and Topel and Ward (1992) and Neal (1999) to explain the high mobility rates amongst young workers. These models have in common that wage dispersion is induced by differences in match, not firm, productivity. The mechanisms described by Jovanovic have only recently been introduced into a setting with labor market frictions by Moscarini (2005). The model in this paper could be fruitfully expanded to incorporate these ideas, but in this more parsimonious version such mechanisms have been shutdown.

Most empirical work on search models with labor market frictions has focussed on the Burdett-Mortensen model.[6] Koning, Ridder and van den Berg (1995) estimate the simplest Burdett-Mortensen model, Bowler, Kiefer and Neumann (1995) estimate a version of the model with heterogeneous productivity. Recently, with the increased availability of matched emploer-employee panel datasets, have richer models been estimated. Bontemps, Robin and van der Berg (1995) use French data to estimate a model with heterogeneous firm productivity, Christensen et al. (2005) use Danish data to estimate a Burdett-Mortensen model with on-the-job search. Postel-Vinay and Robin (2002) introduce Bertrand competition in a model of on-the-job search. They allow for two-sided heterogeneity and structurally recover firm productivity parameters and the variance of person productivity. It is this latest structural empirical literature that this paper is most closely related to. The basic setup, however, draws upon the simpler Pissarides (2000) setup. As a consequence the estimation method is more akin to Abowd, Kramarz and Margolis (1999). This makes the results less easily comparable with previous attempts at structural estimation of search models, but more easily comparable to the far larger reduced-form literature in these areas.

The estimation results speak to a large and diverse literature, which I will discuss when appropriate in the results section. There are, however, a few related papers worth mentioning at the outset. Most obviously the estimating equations and computational methodology are similar to that of Abowd, Kramarz and co-authors.[7] A difference is that the estimating equations in this paper are derived from an explicit model within which the results can be interpreted and given explicit meaning. Furthermore, the structure of the model allows for estimation of a richer set of parameters, such as time-varying firm effects and transition rates for workers, which in turn can be related to a wider set of questions than is possible in a

[6] The earliest papers are Wolpin (1987) and Eckstein and Wolpin (1990) that estimate Albrecht and Axell (1984) search model of workers with heterogenous reservation wages.

[7] Abowd, Kramarz and Margolis (1999), Abowd, Finer and Kramarz (1999) and Abowd, Creecy and Kramarz (2002).

reduced-form framework. The papers results also speak to research on worker mobility and job creation and destruction, which has been summarized and inspired by Davis, Haltiwanger and Schuh (1996). The structure provided by the model allows for a causal interpretation of many of the findings, for example are job losses caused by idiosyncratic shocks to firms or aggregate changes in the exogenous job loss rate, and the data allow for a comparison of the Austrian with the US labor market.[8] Further reference points are the the debate on whether inter-industry wage differentials are explained by firm or person-specific heterogeneity, see Gibbons and Katz (1992) and Murphy and Topel (1990) for example; and the literature on the firmsize wage effect summarized by Oi and Idson (1999).

3 Model

In this section I describe a labor market with search frictions, where a continuum of heterogeneous workers face a continuum of heterogenous firms and produce a unique good. Firms are created by entrepreneurs who have the choice between being workers or starting a firm. They make job offers to workers using an undirected search technology, so as to attain their optimal size. Workers switch firms as they receive better job offers. I consider three mechanisms about the way in which firms and workers share output, an exogenously determined output sharing parameter, a variation on Nash-bargaining and Bertrand competition between firms. In the subsequent sections I take the model to the data.

3.1 Setup

3.1.1 Workers:

There is a continuum of workers of mass N_w. For now I treat workers as homogenous, with productivity p, and extend the model to the heterogenous worker case in Section []. Workers maximize the present discounted value of wages and discount the future at an exogenous and constant rate $\rho > 0$. When unemployed workers receive a flow utility bp. Job search is assumed equally effective on-the-job and when unemployed, workers are contacted by firms at a rate λ and can choose to accept or reject wage offers.

3.1.2 Entrepreneurs:

There is a stock of potential entrepreneurs characterized by a productivity parameter f. They have the choice of starting a firm, endowing that firm with their productivity parameter and receiving the value of that firm (present discounted value of profits). Alternatively, they can become a worker and join the pool of unemployed. f has a cdf Φ on the support $[0, f_{\max}]$. For simplicity I normalize the mass of potential firms to one, so that the cdf of firms also gives the distribution of the mass of firms.

[8] See Huber et al (2004) for work on some of these issues using the same data source used in this paper.

3.1.3 Firms:

Firms differ in the parameter f (which is derived from the entrepreneuer who runs the firm), where f should be thought of raising labor productivity. Firms maximize the present discounted value of profits, they use a discount rate $\rho > 0$. The only input into production is labor and the production function is constant returns to scale.

$$q_j = n_j f_j p \qquad \text{for all } j \tag{1}$$

Search is undirected and firms contact workers at an endogenously given (Poisson) vacancy rate v_j resulting in a flow of new hires $\eta_j = H(v_j, \zeta_j)$, where where ζ_j is a vector of state variables to be determined. Worker-firm matches are subject to an exogenous job destruction rate δ. Workers may also choose to leave the firm on receiving a job offer from another firm, such that the total exit rate of workers from the firm is $\mu_j = M(\delta_j, \xi_j)$, where where ξ_j is a vector of state variables to be determined. The law of motion for the size of a firm is:

$$\dot{n}_j = \eta_j - \mu_j n_j \tag{2}$$

Firms have a cost of hiring workers which is convex in the number of job offers they send out. I assume that creating vacancies is costly and convex in the number of vacancies:[9]

$$c_j(v) = \psi_j \frac{1}{2} v_j^2, \quad \frac{\partial c}{\partial v} > 0, \frac{\partial^2 c}{\partial v^2} > 0 \tag{3}$$

3.1.4 Wage setting:

Wage setting in this economy follows an output sharing rule:

$$w_{ij} = s f_j p_i \tag{4}$$

where w_{ij} is the wage of individual i at firm j, and s is an output sharing parameter. In this paper I consider three types of sharing rules. The simplest is one where the sharing parameter is exogenously determined and common to the entire labor market. I will make this assumption for the exposition of the model and as a baseline case for estimation. In section [] I allow the sharing-rule to follow the common surplus-sharing assumption. Finally, in section [] I consider the sharing-rule under the assumption of Bertrand competition between firms.

3.1.5 Bellman equations:

Firm j's value function solves the Bellman equation given by:

$$\rho V(f_j, n_j) = \max_{v_j} \{(1-s) n_j f_j p - c(v_j) + \dot{n} V_n\} \tag{5}$$

[9]This functional form is for exposition reasons, in the empirical section I will use the more flexible functional form $\psi_j v_j^\phi$ where $\phi > 1$.

For a worker i the value of being unemployed solves the Bellman equation:

$$\rho U(p,b) = bp + \lambda[\int_{W_{\min}}^{W_{\max}} W d\Omega(W) - U] \tag{6}$$

For a worker the value of being employed at firm j solves the Bellman equation:

$$\rho W(p, f_j, U) = sf_j p + \delta[U - W] + \lambda \int_{W}^{W'_{\max}} (W' - W) d\Omega(W') \tag{7}$$

where $\Omega()$ is the cumulative distribution of the value of job offers in the labor market, over the interval $[W_{\min}, W_{\max}]$. For clarity of exposition I will drop subscripts hereafter.

3.2 Steady State

A steady state in this economy is defined as one where firmsizes and unemployment rates are constant.

$$\dot{n} = \dot{u} = 0 \tag{8}$$

3.2.1 Workers:

Proposition 1 *A worker's value function is increasing in f. Hence, workers switch to firms with higher productivity.*

Take the derivative of (7) with respect to f and rearrange to obtain:

$$\frac{\partial W}{\partial f} = \frac{sp}{\rho + \delta + \lambda(1 - \Omega(W))} > 0 \tag{9}$$

Workers choose jobs to maximize the value of employment, hence $\frac{\partial W}{\partial f} > 0$ implies that they will choose the firm with the higher productivity parameter f.

Proposition 2 *Workers' wage profiles are in expectation increasing and concave in labor market tenure. Wage growth is in expectation decreasing and convex.*

The essence of this proof is that workers switch to more productive firms, so over their labor market tenure their wage is increasing. However, the probability of receiving a higher wage offer is decreasing in the productivity of the worker's current firm, hence the wage profile is concave. See Appendix A.1 for a formal proof.

Proposition 3 *Workers job switch rate is in expectation decreasing and convex in labor market tenure.*

The probability of changing jobs conditional on having received a job offer is simply the probability that the wage offered is higher than the worker's existing wage. Since in expectation workers' wages are increasing at a decreasing rate, the probability they receive a wage offer higher than their current wage is decreasing. For a formal proof since Appendix A.1.

Note that the model has the implication that if a worker loses her job exogenously she becomes equivalent to a worker with no labor market experience.

Proposition 4 *More productive firms employ, in expectation, a more experienced workforce, i.e. a workforce with higher labor market tenure.*

The intuition for this result follows from proposition []. If firm productivity is in expectation increasing in labor market tenure, then labor market tenure is in expectation increasing in firm productivity. For a formal proof see Appendix A.

Proposition 5 *The minimum wage an unemployed worker is willing to accept is bp and therefore the minimum productivity of a firm that wishes to attract any workers is given by* $f_u = \frac{b}{s}$.

The minimum wage acceptable to an unemployed worker is given by $U = W_{\min}(f_u, p, U)$, which when substitute equations (7) and (6) into this expression yields:

$$w_{\min} = sf_u p = bp \tag{10}$$

3.2.2 Firms:

Outflows of workers from firms: Workers' decision rules imply that workers leave due to exogenous job destruction, or because they receive an offer from a firm with a higher productivity. Hence, the outflow rate of workers from a firm is

$$\mu = \delta + \lambda(1 - \Gamma(f)) \tag{11}$$

where $\Gamma(f)$ is the cdf of the firm productivity levels associated with job offers

$$\Gamma(f) = \frac{\int_{f_{\min}}^{f} v(f')d\Phi(f')}{\int_{f_{\min}}^{f_{\max}} v(f')d\Phi(f')} \tag{12}$$

Proposition 6 *Worker outflow rates are decreasing in firm productivity.*

$\frac{\partial \mu}{\partial f} = -\lambda\Gamma'(f) < 0$, since $\Gamma'(f)$ is a pdf and hence always positive.

Corollary 7 *Average tenure at a firm is increasing in firm productivity.*

The expected tenure at a firm T is simply the reciprocal of the flow probability of leaving the firm, $T = \frac{1}{\mu}$, (see Appendix A.2 for a proof). Hence,

$$\frac{\partial T}{\partial f} = -\frac{1}{\mu^2}\frac{\partial \mu}{\partial f} \tag{13}$$

$$= \frac{\lambda}{\mu^2}\Gamma'(f) > 0 \tag{14}$$

Inflows of workers to firms: The inflow of workers to a firm with productivity f is a function of the vacancy rate, such that

$$\eta = v[u + (1-u)\frac{1}{L}\int_{f_{\min}}^{f} m(f', \varpi)d\Phi(f')] \tag{15}$$

where $n_j = m(f_j, \varpi)$, ϖ is a vector of state variables to be determined in equilibrium, and $L = \int_{f_{\min}}^{f_{\max}} m(f', \varpi)d\Phi(f')$, the total number of workers actually employed. Workers who receive a job offer from a firm accept that offer if they are either unemployed or at a firm with lower productivity.

Proposition 8 *The probability of hiring a contacted worker is increasing in firm productivity.*

Denote the probability of hiring a contacted worker as

$$\pi(f, u) = u + (1-u)\frac{1}{L}\int_{f_{\min}}^{f} m(f', \varpi)d\Phi(f') \tag{16}$$

, then

$$\frac{\partial \pi}{\partial f} = (1-u)\frac{n}{L}\Phi'(f) > 0 \tag{17}$$

Proposition 9 *Higher productivity firms recruit a smaller fraction of their labor from the pool of unemployed.*

The fraction of workers a firm hires from the unemployed pool is, from (15), $u\left(u + (1-u)\frac{1}{E}\int_{f_{\min}}^{f} m(f', \varpi)d\Phi(f')\right)^{-1}$, which is decreasing in f.

Job offer rate: In steady state $\dot{u} = 0$. This implies a relationship between the unemployment rate, the exogenous job destruction rate and the rate at which workers receive job offers:

$$\lambda = \frac{\delta(1-u)}{u} \tag{18}$$

Firm vacancy-employment ratios: In steady state $\dot{n} = 0$, which when substituting for η (15) and μ (11) into (2) implies

$$\theta(f, \lambda, u) \equiv \frac{v}{n} = \frac{\mu}{\pi} = \frac{\delta + \lambda(1 - \Gamma(f))}{u + (1-u)\frac{1}{L}\int_{f_{\min}}^{f} m(f', \varpi)d\Phi(f')} \tag{19}$$

where θ is the vacancy-employment ratio for the firm, or one over the marginal efficiency of hiring workers, i.e. the ratio of the probability of a worker leaving the firm over the probability of a contacted worker joining the firm.

Proposition 10 *A firm's vacancy-employment ratio is decreasing in its productivity level.*

The probability of a worker leaving μ is decreasing in f, and the probability of hiring a contacted working π is increasing in f. Consequently more productive firms have lower vacancy to employment ratio.

Substituting for λ using (18) yields an expression for a firm's vacancy-employment ratio as a function of the firm's productivity and the unemployment rate.

$$\theta(f, u) \equiv \frac{v}{n} = \frac{\delta}{u} \frac{u + (1-u)(1-\Gamma(f))}{u + (1-u)\frac{1}{L}\int_{f_{\min}}^{f} m(f', \varpi) d\Phi(f')} \tag{20}$$

Notice that for values of $f \, \epsilon \, [f_{\min}, \, f_{\max}]$ θ will be in the interval $[\frac{\delta}{u^2}, \delta]$.

Optimal firm size, output and profits: The value function that solves the firm's Bellman equation, see Appendix A.2, is:

$$V = \frac{(1-s)fp}{\rho+\mu} n + \frac{1}{\rho}\left[\eta\frac{(1-s)fp}{\rho+\mu} - c(v)\right] \tag{21}$$

The firm maximizes this value function with respect to v. Taking first order conditions and imposing the steady state condition that $\dot{n} = 0$, which implies $n = \frac{\eta}{\mu}$ and from (20) $\frac{1}{\theta} = \frac{n}{v} = \frac{\eta}{\mu}\frac{1}{v}$ results in:

$$\frac{(1-s)fp}{\rho+\mu}\frac{1}{\theta} = \frac{1}{\rho}c'(v) \tag{22}$$

which given the functional form for $c(v)$ (3) and recalling that $v = n\theta$ yields the optimal size of a firm with productivity parameter f in steady state:

$$n^*(f, u) = \frac{\kappa}{\psi\theta^2}(1-s)fp \tag{23}$$

where $\kappa = \frac{\rho}{\rho+\mu}$.

Proposition 11 *Firm size is increasing in firm productivity.*

$$\frac{\partial n}{\partial f} = \frac{\kappa}{\psi\theta^2}(1-s)p + \left(\frac{1}{\psi\theta^2}\frac{\partial\kappa}{\partial f} - \frac{2\kappa}{\theta^3}\frac{\partial\theta}{\partial f}\right)(1-s)fp > 0 \tag{24}$$

since $\frac{\partial\kappa}{\partial f} > 0$ (recall that $\frac{\partial\mu}{\partial f} < 0$) and $\frac{\partial\theta}{\partial f} < 0$.

Proposition 12 *More productivity firms pay higher wages and are larger, i.e. there is a positive firmsize wage effect.*

From (4) $\frac{\partial w}{\partial f} = sp > 0$ and the previous proposition showed $\frac{\partial n}{\partial f} > 0$. As a result firmsize and wage will be positively correlated.

Proposition 13 *The value of a firm is strictly increasing in firm productivity.*

Given n^* (23) and $v^* = n^* \frac{\mu}{\pi}$ the first derivative of the steady-state value function of the firm (21) with respect to f is:

$$\frac{\partial V}{\partial f} = (n + \frac{1}{\rho}) \frac{\partial y}{\partial f} + y \left(\frac{1}{\pi} \frac{\partial \pi}{\partial f} - \frac{n}{\mu} \frac{\partial \mu}{\partial f} \right) > 0 \tag{25}$$

since $\frac{\partial \pi}{\partial f} > 0$ and $\frac{\partial \mu}{\partial f} < 0$, where $y = \frac{(1-s)fp}{\rho+\mu}$, see Appendix A.2. There are two sources of additional profits for more productive firms, the present discounted value of higher output given the labor force, $(n+\frac{1}{\rho})\frac{\partial y}{\partial f}$, and the value of the increase in size due to a decrease in the exit rate of workers and an increase in the probability of hiring a worker, $y\left(\frac{1}{\pi}\frac{\partial \pi}{\partial f} - \frac{n}{\mu}\frac{\partial \mu}{\partial f}\right)$.

3.2.3 Entrepreneurs:

Entrepreneurs have the choice between starting a firm, with productivity level f_j, or becoming an unemployed worker. The value of unemployment (6) is independent of an entrepreneur's f, $\frac{\partial U}{\partial f} = 0$, whilst $\frac{\partial V}{\partial f} > 0$, from the previous proposition. Therefore there will for each level of unemployment be some minimum productivity level $f = \zeta(u)$ such that:

$$V(\zeta(u), u) = U(u) \tag{26}$$

where $\frac{\partial \zeta(u)}{\partial u} < 0$.

The minimum productivity firm in steady state will be given by the constraint imposed by either the participation decision of the entrepreneur, or the participation decision of the worker. Whichever of these two is larger will be binding, so that:

$$f_{\min} = \max\{\frac{b}{s}, \zeta(u)\} \tag{27}$$

where $\frac{\partial f_{\min}}{\partial u} = 0$ for $\zeta(u) < \frac{b}{s}$ and $\frac{\partial f_{\min}}{\partial u} < 0$ for $\zeta(u) \geq \frac{b}{s}$.

3.2.4 Economy aggregates:

Mass of firms: Given $f_{\min}$ (27) the expression for the mass of firms in the economy, as a function of the unemployment rate is:

$$N_f(u) \equiv 1 - \Phi(f_{\min}) \tag{28}$$

where $\frac{\partial N_f}{\partial u} = 0$ for $\zeta(u) < \frac{b}{s}$ and $\frac{\partial N_f}{\partial u} > 0$ for $\zeta(u) \geq \frac{b}{s}$.

Unemployment rate: The number of workers in the economy times the employment rate has to equal the sum of all workers employed by firms. This relationship defines the steady state unemployment rate in the economy:

$$(N_w + \Phi(f_{\min}))(1-u) = N_f(u)\int_{f_{\min}}^{f_{\max}} n(f,u)dG(f) \tag{29}$$

Since $\frac{\partial f_{\min}}{\partial u} \leq 0$, $\frac{\partial N_f}{\partial u} \geq 0$ and $\frac{\partial n}{\partial u} > 0$ there will be some steady state unemployment rate u^* which solves (29).

Job offer rate: Using (18) we obtain the steady state job offer rate in this economy:

$$\lambda_w^* = \frac{\delta(1-u^*)}{u^*} \tag{30}$$

Note that I could have solved for the unemployment rate instead. Then used the fact that the number of job offers sent out by firms has to equal the number of job offers received by workers to complete the system of equations.

Comparative statics: I will consider three types of shocks that may affect this labor market, i. an aggregate shock to firm productivity f, ii. a change in the exogenous job destruction rate δ, and iii. a change in the level of unemployment benefits b.

Aggregate shock to f:

Proposition 14 *An aggregate shock to firm productivity induces an inverse relationship between unemployment and the aggregate vacancy rate, i.e. a Beveridge curve type relationship.*

An increase in all firms' f will increase the optimal size of existing firms (23), and induce some firm entry (as some entrepreneurs' f increase above the threshold). As a consequence there is a decrease in the unemployment rate (29) and increase in the job offer rate (18) and the aggregate vacancy rate ($\int_{f_{\min}}^{f_{\max}} v(f)d\Phi(f)$). The increase in the job offer rate will raise $f_{\min}$, but (by contradiction) not enough to offset the first-order effects. A decrease in all firms' f will have the opposite effect. Hence, aggregate shocks to firm productivity result in an inverse relationship between unemployment and the vacancy rate, i.e. there exists a Beveridge curve type relationship.

Changes in δ: An increase in the exogenous job destruction rate will increase the vacancy-employment ratio θ (20), as a consequence decrease the optimal firm size (23) and increase the unemployment rate. The effect on the job offer rate is ambiguous, from (18) $\frac{\partial \lambda_w}{\partial \delta} = \frac{u(1-u)-\delta\frac{\partial u}{\partial \delta}}{u^2}$, since $\frac{\partial u}{\partial \delta} > 0$, the net effect [may] be ambiguous. Profits fall which induces an exit of firms. [to be completed].

Changes in b: An increase in b only has an impact on aggregate outcomes if the minimal acceptable wage to an unemployed worker is the binding margin on which entrepreneurs make their decision, i.e. $f_u(b) \geq \underline{f}$. If so then an increase in b will induce some entrepreneurs to become workers, hence increasing the unemployment rate. The analysis is symmetric for a decrease in b.

3.3 Heterogenous Workers

I now extend the model to allow for heterogenous workers. Workers are assumed heterogenous in their productivity p, which is continuously distributed over the support $[p_{\min}, p_{\max}]$ with a cumulative distribution function Π. Denote by $\bar{p}$ the expected mean productivity in the population, $\bar{p} = \int_{p_{\min}}^{p_{\max}} p d\Pi(p)$. All workers are contacted at the same rate λ and receive a flow utility bp when unemployed. Entrepreneurs p is drawn independently from their f.

Notice that in this model all sorting mechanisms have been shutdown so that the distribution of worker productivities are in expectation the same in all firms and equal to $\bar{p}$. To see this note that:

- Search is undirected, so that firms have an equal chance of meeting any worker in the labor market.
- Unemployed workers accept job offers from all firms, since $f_{\min} \geq \frac{b}{s}$.
- All employed workers accept job offers only from more productive firms, to see this observe that (9) holds true for all p.
- Firms are willing to hire all workers, once contacted, irrespective of their p. Production is constant returns to scale, from (21) $\frac{\partial V}{\partial n} > 0$, the only constraint on firm hiring is the cost of sending out job offers.

So despite f and p being complements in production there is no expected relationship between the average quality of the workforce and the productivity of the firm. Whilst under perfect competition there would be positive assortative matching, see Becker (1973) and Sattinger (1979), the information imperfection that results in undirected search in this model prevents any kind of sorting.

Consequently, I assume that the average productivity of workers at a firm is the equal to the average productivity in the population, $\bar{p}_j = \bar{p}$ for all j firms. Then all the implications of the model follow as before, except that in the firm's problem p is replaced with $\bar{p}$, and the key equations become:

$$V = \frac{(1-s)f\bar{p}}{\rho+\mu}n + \frac{1}{\rho}\left[\eta\frac{(1-s)f\bar{p}}{\rho+\mu} - c(v)\right] \quad (31)$$

$$n^*(f,u) = \frac{\kappa}{\psi\theta^2}(1-s)f\bar{p} \quad (32)$$

The random search assumption is, of course, more contentious in the context of heterogenous workers. What it implies is that within a labor market, however defined, firms can not direct their search to workers of a certain productivity. But rather firms contact workers by some other mechanism, personal contacts and networks, chance, idiosyncratic preferences, which from the perspective of the modeller or econometrician can be depicted as being random. Extending this framework to include directed search would be a promising way of exploring sorting in markets with search frictions.

3.4 Surplus Sharing

One alternative wage-bargaining mechanism frequently used in the literature is Nash-bargaining, which has been extended by Moscarini (2005) to on-the-job search. Moscarini makes two simplifying, but not innocuous, assumptions, which Shimer (2005) discusses in detail and proposes an alternative to. I will proceed with the Moscarini assumptions. The first is that once a worker has decided to join a firm her outside option becomes unemployment. Whether realistic or not this assumption simplifies the analysis vastly since only the current firm (and the value of unemployment) is important to wage determination and not the identity of firms who may have made a job offer to the worker. The second assumption is that the firm does not take into account the impact of its wage policy on turnover.

Given Moscarini's assumptions a worker always chooses between the value of the job offered, W_{ij} as given by (7) and unemployment, U_i as given by (6). If a firm does not fill a vacancy the vacancy is destroyed, i.e. a job offer only ever reaches one worker. The value of a filled vacancy to the firm j is given by:

$$\rho J_j(p, f) = (1 - s_{ij}) f_j p_i - \lambda(1 - \Omega(W'))J - \delta J \tag{33}$$

Nash-bargaining implies that:

$$w_{ij} = \arg\max (W_{ij} - U_i)^{\alpha} J_{ij}^{1-\alpha} \tag{34}$$

Given Moscarini's second assumption the first-order condition to this problem satisfies:

$$\alpha J_{ij} = (1 - \alpha)(W_{ij} - U_i) \tag{35}$$

In Appendix A.4 I rearrange the first-order condition to derive an expression for the wage

$$w = \alpha f p + (1 - \alpha) b p + \alpha \lambda \int_{f_{\min}}^{f} J' d\Phi(f') \tag{36}$$

When dividing both sides by fp and substituting for J' from (33) yields

$$s = \alpha + (1 - \alpha)\frac{b}{f} + \alpha\lambda \int_{f_{\min}}^{f} \frac{(1 - s) f'}{\rho + \delta + \lambda(1 - \Phi(f'))} \frac{1}{f} d\Phi(f') \tag{37}$$

It is clear that the sharing-rule is independent of the worker's productivity. Intuitively this is true since J, W and U are all linear functions of p.

3.5 Bertrand Competition:

In the literature the main alternative to Nash-bargaining has been the wage-posting assumption of the Burdett-Mortensen model. Postel-Vinay and Robin (1999, 2002) propose a more complicated wage-setting mechanism in an environment with on-the-job search, where employers initially pay the reservation wage of the worker and then match outside offers, i.e. Bertrand competition for workers. The assumption is that all the match surplus goes to the firm. In this section I derive the sharing parameter under the Postel-Vinay and Robin assumptions.

Postel-Vinay and Robin (2002) derive the wage under these assumptions, see Appendix A.5 for an outline of the deriviation. The wage of a worker now depends on both the firm the worker is currently at, f, and the outside option the worker had when deciding to stay at that firm, which I shall denote by x.

$$w(p,x,f) = px - \frac{\lambda_w}{\rho+\delta} p \int_x^f (1 - V(x))dx \tag{38}$$

and

$$s(p,x,f) = \frac{1}{f}[x - \frac{\lambda_w}{\rho+\delta} \int_x^{f'} (1 - V(x))dx] \tag{39}$$

such that the share is decreasing in the productivity of the firm the worker is at and increasing in the highest productivity firm that this worker has received an outside offer from. Firms typically pay below marginal productivity, and workers may take wage cuts on switching employers on account of the option value of being at a high productivity firm.

4 Data

This section outlines the main features of the dataset *Synthesis Erwerb* and explains the construction of key variables. Next it describes the main features of the Austrian labor market. It compares labour market outcomes to those in the US and finds them quite similar. Additional information is provided in Appendix C.

4.1 Description of Synthesis Erwerb data

The data used in this paper has been obtained from *Synthesis Forschung*. The data provides continuous information on employees engaged in the labor market from 1972 to 2003. It is based on social security records for employees who are employed, unemployed, or on maternity leave. A total of [128,782,463] observations correspond to worker-firm matches generating positive income. Observations are employer and year specific, i.e. an observation starts either on the January 1 of a year or when a worker joins that employer, an observation ends on December 31 of a year or when a worker leaves that employer.

Only employees for whom social security contributions are made are covered by the dataset, this includes nearly all employees in Austria, with exceptions as follows.[10] The self-

[10] No records are kept for individuals who are not in the labor market, or work outside Austria.

employed, who currently account for around 10% of the laborforce in Austria, are covered by a different social security institution. Coverage for certain public employees increased in the mid-1990s. Moreover, certain workers with very short-term contracts, who account for around 7% of the laborforce, were also excluded from social security contributions and are not covered by the dataset until the 1990s. I will be using the expressions 'firm' and 'establishment' interchangeably throughout this paper, since I have no way of distinguishing between the two.

4.1.1 Observed Variables:

The data contains information on income earned and contracted hours worked during each observation. The wage measure I use is the hourly wage, the income earned during each observation divided by the number of hours worked. The income measure is reliable since social security contributions depend on it. The hours measure is likely to have a lot of measurement error, contracted hours being only a very rough measure of actual hours worked, 83% of full-time employees have contracted hours of either 161.7, 163.8 or 168 hours per month.

Prelimimary analysis suggests that in firm measures, industry classification and region, are well measured. Worker variables associated with a job much less so. Education and occupation are poorly measured, since both of these vary too much than is plausible over the lifecycle, with as much as 30% of workers switching education and 40% switching occuption between years. [need to sort this out] Gender is well measured with no one switching gender thoughout the entire sample.

4.1.2 Tenure and actual experience variables:

Tenure is measured as the cumulative number of hours worked at a given employer, whilst actual experience measures the cumulative number of hours an individual has been employed.[11] Tenure measures, for those where the employer-employee relationship begins after the beginning of the sample, are likely to be accurate. Actual labor market experience measures can not be constructed for those entering the labor market before January 1, 1972. Hence, I obtain experience and tenure variables for 69% of the data, only tenure for another 19% and neither for 12% of the data. I proxy experience and tenure for the initial observation of some of the people unobserved at the beginning of their career by matching on sex, education, full-time or part-time employment, age (in years), region, two-digit industrial classification, and number of children (only for women). The R-squared is 0.51 for the experience regression and 0.26 for the tenure regression. Thus I construct both variables for 92% of the data.

[11] I divide these measures by 5.5 times the days in the year (the average hours recorded for a full-time employee), so that experience and tenure are expressed in years of full-time employment.

4.1.3 Observation selection:

Since workers switch jobs mid-year and retain several jobs at the same time there are frequently multiple entries for an individual in a single year. A single entry per worker and year, a 'dominant' job, was selected by first selecting the worker-firm match from which the worker derived the most income that year (reducing the number of observations by 27,989,711), then, in case of ties, selecting the worker-firm match with the longest duration in that year (reducing the number of observations by 4,801). The remaining number of multiple observations involve repeated identical matches between the same firm and worker within a single year, amongst those the first observation was chosen (reducing the number of observations by 77,780).

For the purpose of most of the empirics in this paper workers in agriculture, mining, tobacco, utilities, transportation, non-profit organisation and the public sector were dropped from the analysis. This reduces the number of observations by 27% to 73,284,765, an average of around 2.3 million per year. For the actual estimation I also exclude all observations corresponding to firms with less than an average of ten employees and those where the firm effect is identified by less than thirty observations (a firm effect is identified by every observation employees of that firm at other firms).

4.2 The Austrian labor market

This section outlines several features of the Austrian labor market, additional tables are provided in Appendix B.

4.2.1 Laborforce composition:

During the sample period, 1972 to 2003, the Austrian labor market was characterized by a steady growth in employment. Male labor market participation rates declined in the 1970s from 85% and have since stabilized at around 80%. Meanwhile, female labor market participation steadily increased, from under 50% in the early 1970s to over 65% now. Unemployment rates have been stable and low, whereby we can distinguish between the 1970s, where unemployment rates were 1-2%, and the period after the second oil price shock where they fluctuated between 3% and 4%. GDP. Wage growth rates decline throughout the sample period.[12]

Austria's schooling system encourages a large number of individuals, especially men, to start an apprenticeship (46% of observations). Many, 28% of observations, only have compulsory schooling (up to age 15) and comparatively few have high school completion or a university degree, 10% and 6% of observations respectively. There is a trend away from apprenticeships towards more high school and university completion.

[12] Numbers are based on Eurostat data.

4.2.2 Wages and wage-bargaining:

Log wages are close to normally distributed in Austria and levels and trends for mean and median near identical, with the median slightly above the mean. Wage growth has been decelerating rapidly throughout this period. The standard deviation of log wages fell steadily from 0.62 to 0.60 in 1993 after which it increased to 0.65 in 2002. Wage level regression produce similar result as those for other countries. There is the usual concave experience and tenure profiles. The return to experience is around 7% per annum (Abowd, Karmarz and Margolis (1999) find experience returns of a similar magnitude for France,around 0.06, and somewhat higher for the US, around 0.1) and the return to tenure below 2% per annum. More education is associated with higher wages, the return on university education over compulsory education is around 0.35 log wage points. The penalty for being female is as much as 0.25 log points.

Sectoral wage-bargaining agreements cover nearly all employees in Austria.[13] Employer (*Wirtschaftskammer*) and employee (*Arbeiterkammer*) representatives bargain over minimum wage levels (*tariffloehne* or contractual wages) and annual wage increases, which will differ according to employee qualifications, seniority and any number of other factors deemed relevant. In the 1960s and early 1970s, periods of high growth and low unemployment, minimum wages and actual wages were subject to the same bargained wage increases. By the early 1980s, with higher unemployment and lower economic growth after the second oil price shock, employees bargained power diminished and bargained wage increases were lower and significantly lower for actual than minimum wages. Increasingly in the 1990s the wage-bargaining agreements began to exclusively focus on the setting of minimum wages.[14]

To measure how binding the centrally bargained wage agreements were I construct a minimum wage for every combination of 4-digit sector (503 industries), year, education category and experience category (a total of 672,000 categories containing an average of 150 observations each). By comparing these minimum wages in each category with those actually paid one can obtain an impression of how binding the *tariffloehne* are. In practice very few workers seem to be receiving wages close to the minimum in their category. It is clear that there is very large wage dispersion, even within very tight categories of workers. the difference between the minimum and the 1st, 5th, 10th, 25th and 50th percentile is on average 0.6, 1.2, 1.5, 2 and 2.6 log points respectively.

4.2.3 Labor mobility:

The Austrian labor market is characterized by a lot of worker mobility. From year to year around 20% of workers switch 1-digit industry; inter-regional mobility is low though, with only around 4% of workers changing regions between years (on the dominant job definition). On average workers in the private sector start or end an employment relationship every 2.3 years, i.e. the fraction of switches to workers is 0.45. This hire and separation rate compares with US quarterly separation rates of 9.4% from the JOLTS or 24.5% from the

[13] There are a few instances where agreements are reached at a firm level, Austrian Airlines is one such.

[14] [ref]

LEHD, see Davis, Faberman and Haltiwanger (2006).[15] Mobility decreases, at a decreasing rate, with age, education and tenure. If we ignore short periods of non-employment around 40% of employment transitions are directly between two firms. A third of workers becomes unemployed (for 40% of which unemployment is short-term, less than 28 days). and one-quarter become unemployed.

The labor mobility numbers outlined here are likely to overstate the amount of actual mobility. Seasonal workers may have stable employment relations but nevertheless change firms twice or more times a year. Workers on short-term contracts may have multiple employment relationships with the same firm in the same year. Correcting for these issues by focusing on switches in dominant jobs, which limits the maximum number of switches per year to one, reduces mobility to around one firm switch in three years, i.e. the fraction of switches to workers is 0.33. This number is comparable to Davis, Faberman and Haltiwanger (2006) full-quarter transitions definition using the LEHD, which is 11.9% per quarter. This suggests perhaps one-third higher mobility in the US than in Austria, keeping in mind that the LEHD generates very high mobility estimates for the US.

4.2.4 Job creation and destruction:

I measure job creation and destruction in two ways: the net number of hours worked that are created or destroyed at a firm; and the net number of employment relations that are created or destroyed by a firm.[16] In each of these categories I distinguish between jobs created / destroyed by expanding or contracting firms and entering or exiting firms. I calculate annual job creation and destruction rates, which, as Pinkston and Spletzer (2004) find, abstracts from the large seasonal fluctuations in employment found in many parts of the economy. The job creation rate is on average 11.8% (8.4% for expanding establishments) per annum when measured in employment relations and 9.4% (6.9%) per annum when measured in hours; the job destruction rates are on average 10.5% (8.4% for contracting establishments) and 9.0% (6.7%) per annum respectively. Over time the series track each other closely, whereby the job series is much more volatile than the hours series, suggesting that part-time employees get hired and fired most frequently. In comparison, using Business Employment Dynamics Data for the US Pinkston and Spletzer (2004) find job creation rates, measured as number of employment relationships, of 14.6% (9.4% from growing establishments) and destruction rates of 13.8% (8.8% from shrinking establishments). Since the laborforce is growing faster in the US economy the destruction rates are probably more easily comparable, suggesting that the US has more job turnover, primarily on account of greater firm entry and exit.

[15] JOLTS is the Job Openings and Labor Turnover Survey. LEHD stands for Longitudinal Employer Household Dynamics. These hire and separation rates can be aggregated, thus JOLTS suggests lower mobility in the US than Austria, whilst LEHD suggests much higher mobility. I quote the average of the hires and separation rates in Davis, Faberman and Haltiwanger (2006).

[16] I use the convention in the literature to measure job creation and destruction from t-1 to t. To turn these numbers into rates I divide by the average employment levels in t-1 and t.

5 Estimation Method

This section shows how, using the data described, the parameters of the model can be recovered. I will follow recent work in this literature by taking a limited information approach.[17] I will estimate transition and unemployment rates using information on worker mobility. The remaining parameters will be identified from individual wages and the relationship between the size of a firm and its productivity. The disadvantage of the limited information approach is that if the model is correct a full information approach will be more efficient. There are three distinct advantages though: this approach is computationally feasible, it provides a clearer sense of which data is identifying which parameters, and the additional restrictions provided by the model can be tested independently. I will show identification under all three assumptions about wage-setting (exogenous sharing parameter, surplus-sharing and Bertrand competition).

The model as described is stationary and non-stochastic. I will assume that at any given point in time (the data is annual from 1972 to 2003) the model is in steady state, and that the observations are independent draws from this steady state distribution. In contrast to the existing literature I will allow firm productivity, as well as person productivity, to be time-varying.

5.1 Transition Rates

The model provides a number of restrictions that the transition rates have to satisfy.

The number of new hires of a firm is given by (15), which can be rearranged to yield:

$$v_{jt} = \frac{\eta_{jt}}{u_t + (1-u_t)\frac{1}{L_t}\int_{f_{\min,t}}^{f_{jt}} m(f')d\Phi(f')} \qquad \text{for all } j,t \tag{40}$$

The random search assumption implies that the aggregate number of job offers, divided by the number of workers in the labor market equals the job offer rate:

$$\lambda_t = \frac{N_{ft}}{N_{wt}} \int_{f_{\min}}^{f_{\max}} v(f_{jt})d\Phi(f) \qquad \text{for all } t \tag{41}$$

where $N_{wt} = \frac{L_t}{1-u_t}$ and L_t is the number of workers actually employed.

The steady-state requirement that the unemployment rate be constant, $\dot{u} = 0$, implies

$$\delta_t = \frac{\lambda_t u_t}{(1-u_t)} \qquad \text{for all } t \tag{42}$$

Finally, the number of workers who leave the firm is given by (11), as well as a time-specific idiosyncratic shock to the firm:

$$\mu_{jt} - \delta_t - \lambda_t\left(1 - \frac{\int_{f_{\min}}^{f} v(f')d\Phi(f')}{\int_{f_{\min}}^{f_{\max}} v(f')d\Phi(f')}\right) = \varepsilon_{jt} \qquad \text{for all } j,t \tag{43}$$

[17] Bontemps, Robin and Van der Berg (2000), Postel-Vinay and Robin (2002) and Christensen et al (2005).

Substituting (40), (41) and (42) into (43) and substituting for N_{wt} yields the following t non-linear equations:

$$\sum_j \left[\mu_{jt} - \frac{N_{ft}}{L_t}\left[\int_{f_{\min}}^{f_{\max}} v(f_{jt})d\Phi(f') \quad - (1-u_t)\int_{f_{\min}}^{f} v(f_{jt})d\Phi(f')\right]\right] = \sum_j \varepsilon_{jt} = 0 \quad \text{forall } t \tag{44}$$

where $v(f_{jt})$ is given by (40):

In the data I observe the number of new hires at a firm η_{jt} and the fraction of workers who exit a firm μ_{jt}. The mean wage at a firm increases monotonely with firm productivity (given the model), so it is possible to calculate $\frac{1}{L_t}\int_{f_{\min,t}}^{f_{jt}} m(f')d\Phi(f')$ directly from the data [condition on industry]. Alternatively I can first estimate f, see below, and then calculate $\frac{1}{L_t}\int_{f_{\min,t}}^{f_{jt}} m(f')d\Phi(f')$.

Hence, (44) describes a non-linear equation in u for each time period, and can be solved using some method of solving non-linear equations. Alternatively, I can add further moments by postulating that $E_j[\varepsilon_{jt}|n] = 0$ for all t, where n is the size of the firm, and use general method of moments.

5.2 Firm and Worker Productivity

Taking the log of the wage-setting equation (4) in each time period and adding an independent and identically distributed error term, yields a linear regression:

$$\ln w_{ijt} = \ln s_t + \ln p_{it} + \ln f_{jt} + \varepsilon_{ijt} \tag{45}$$

for all workers at all firms in all time periods. Clearly the panel dataset used for this paper does not contain enough information to estimate such a regression so additional restrictions have to be imposed. I will assume that an individuals productivity is a function of a person fixed effect and observable time-varying personal characteristics (X) and that the price of these characteristics is time, person and firm invariant, such that:

$$\ln p_{it} = \ln p_i + \beta_x X_{it} \tag{46}$$

I split firm productivity into a time invariant and a mean zero time-varying component:

$$\ln f_{jt} = \ln f_j + (\ln f_{jt} - \ln f_j) \tag{47}$$

where $\ln f_j = \frac{1}{t}\sum_t \ln f_{jt}$. Equation (23) provides a relationship between firm productivity and firmsize and hence a way to measure the time-varying component of firm productivity. Taking logs, rearranging, demeaning and substituting into (47)

$$\ln f_{jt} = \ln f_i + \Delta \ln n_{jt} - \Delta \ln(1-s_t) - \Delta \ln q_{jt} + \Delta \ln \psi_j + \Delta \ln \phi + \Delta(\phi \ln \theta_{jt}) \tag{48}$$

where Δ indicates a demeaned variable, ϕ is taken to be common to the entire labor market and

$$\Delta \ln \theta_{jt} = \Delta \ln \delta_t - \Delta \ln u_t + \Delta \ln \frac{u_t + (1-u_t)(1-\Gamma(f_{jt}))}{u_t + (1-u_t)\int_{f_{\min}}^{f} n(f_{jt}) d\Phi(f_{jt})} \tag{49}$$

I proxy $\frac{u_t + (1-u_t)(1-\Gamma(f_{jt}))}{u_t + (1-u_t)\int_{f_{\min}}^{f} n(f_{jt}) d\Phi(f_{jt})} = n^{\varphi_n}$ as a decreasing power function of n. Substituting these expressions into (45) assuming that ψ is time-invariant, such that $\Delta \ln \psi_{jt} = 0$, and since there is no selection $q_{jt} = q_t$, such that

$$\ln w_{ijt} = \beta_{0t} + \ln p_i + \beta_x X_{it} + \ln f_j + \beta_n \Delta \ln n_{jt} + \varepsilon_{ijt} \tag{50}$$

where β_{0t} is a collection of all the time-varying terms and those common to the entire labor market ($\ln s_t - \Delta \ln(1-s_t) + \Delta \ln \phi - \phi[\Delta \ln \delta_t - \Delta \ln u_t]$), as well as inflation and aggregate productivity growth (which are not in the model, but are in the data), $\beta_n = 1 + \varphi_n$. Once p and f are estimated, Φ and Π can be constructed over the observed support.

The expression for optimal firmsize is given by (23). Given the estimates of f_{jt}, q_{jt} and θ_{jt} from the previous stages and assuming an iid error term leads to the following regression:

$$\ln n_{jt} = \beta_{0t} + d_s + \beta_\theta \ln \hat{\theta}_{jt} + \beta_f \ln \hat{f}_{jt} + \beta_q \ln \hat{q}_{jt} + \varepsilon_{jt} \tag{51}$$

where $\beta_{0t} = \frac{1}{\phi-1}[\ln(1-s_t) - \ln \phi]$, $d_s = -\frac{1}{\phi-1} \ln \psi_s$, and $\beta_f = \beta_q = \frac{1}{\phi-1} = \frac{\beta_\theta}{\phi}$. Finally, by using the estimate of β_{0t} from (50) one can pin down the relative magnitudes of s and ψ.

5.3 Surplus Sharing

Below I outline identification under the Nash-bargaining assumption. Transition and unemployment rates are estimated as before. Nash-bargaining implies a sharing rule, given by (37). I approximate this sharing-rule by:

$$s = \alpha + (1-\alpha)(\varphi_f \ln f + \varphi_p \ln p + \varphi_\lambda \ln \lambda_w) \tag{52}$$

where $\varphi_f, \varphi_\lambda > 0$ and $\varphi_p < 0$. Using a linear approximation $\ln(1-s_t) = s_t$ and

$$\ln s = \ln \alpha + \frac{1-\alpha}{\alpha}(\varphi_f \ln f + \varphi_p \ln p + \varphi_\lambda \ln \lambda_w) \tag{53}$$

where the bargaining share α, as well as φ_f , φ_p and φ_λ are assumed time invariant. Substituting this into the wage equation (45), thereby allowing for the same structure on the firm and person effect, generates the following regression:

$$\ln w_{ijt} = \beta_0 + \beta_p \ln p_i + \beta_{x2} X_{it} + \beta_f \ln f_j + \beta_{n2} \Delta \ln n_{jt} + \beta_\lambda \ln \hat{\lambda}_{wt} + \Delta_{ts} + \varepsilon_{ijt} \tag{54}$$

where $\beta_0 = \ln \alpha$, $\beta_p = \left(1 + \frac{1-\alpha}{\alpha}\varphi_p\right)$, $\beta_{x2} = \left(\beta_x + (1-\alpha)\varphi_p\right)$, $\beta_f = \left(1 + \frac{1-\alpha}{\alpha}\varphi_f\right)$, $\beta_{n2} = (1+\varphi_n)(1 + (1-\alpha)\varphi_f)$ and $\beta_\lambda = \frac{1-\alpha}{\alpha}\varphi_\lambda$, $\Delta_t = (1-\alpha)\varphi_\lambda \Delta \ln \hat{\lambda}_{wt} - \phi[\Delta \ln \delta_t - \Delta \ln u_t]$. I will approximate this last term with a time fixed effect if there is not enough variation in the data. The sharing parameter α and φ_λ are identified from the constant term and the coefficients on $\Delta \ln \lambda_{wt}$ and $\ln \lambda_w$.

To identify the remaining parameters I use the the equation determing firmsize (**??**) and substitute in the (biased) estimates for the firm and productivity effect, $\ln \hat{f}$ and $\ln \hat{p}$:

$$\ln n_{jt} = \frac{1}{\phi(1+\varphi_n)-1}[\alpha + (1+(1-\alpha)\varphi_f)\ln f_j + (1-\alpha)\varphi_p \ln p_i + (1-\alpha)\varphi_\lambda \ln \lambda_w \quad (55)$$
$$-\phi[\ln \delta_t - \ln u_t] - \ln \phi - \ln \psi_s + \ln q_{jt}] \quad (56)$$

which, given an iid error term, results in the regression

$$\ln n_{jt} = \beta_0 + \beta_1(\ln \hat{f}_j + \ln \hat{p}_i) + \beta_2 \ln \lambda_w - \phi[\ln \delta_t - \ln u_t] + d_s + \varepsilon_{jt} \quad (57)$$

where $\beta_0 = -\ln \phi$, $\beta_1 = \frac{1}{\phi(1+\varphi_n)-1}$, $\beta_2 = (1-\alpha)\varphi_\lambda$, $d_s = -\ln \psi_s$.

ϕ and φ_n are identified from the coefficients on $\ln \lambda_w$ and $\ln \delta_t - \ln u_t$ (recall that α and φ_λ are already identified). Then φ_f is identified from the coefficient on $\ln \hat{f}_j$ or from the estimate of β_{n2} in the previous regression. Finally, using the coefficient on $\ln \hat{p}_i$ I can identify φ_p. Thus $\ln f_j$ and $\ln p_i$ can be reconstructed.

5.4 Bertrand Competition

[to be completed]

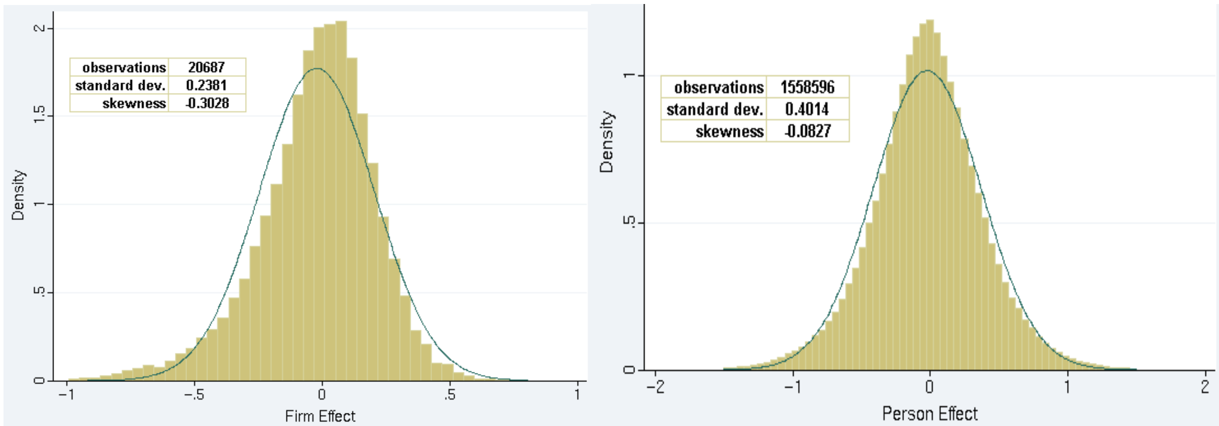
6 Results

In this section I describe the parameter estimates and their relationship to observable worker and firm outcomes. The findings in this section are preliminary and should be taken as indicative. The results are based on data for Vienna, roughly one-quarter of all observations. Estimates are for the exogenous output sharing parameter specification. The labor market specification considered here is all of Vienna, with 20,687 firms and 1,558,596 workers. Transition rate estimates are to be completed and not outlined here.

6.1 Estimates

6.1.1 Firm and Person Effects:

The firm and person effects have a standard deviation of 0.16 and 0.36 respectively. They are both close to normally distributed, though somewhat negatively skewed. The coefficient of correlation between the two is 0.15, the covariance is 0.0087.



(58)

Over time there has been an slow upwards trend in the variance of the firm effect and a slow downwards trend in the variance of the person effect.

6.1.2 Transition Rates:

[to be completed]

6.2 Wage Level Differentials

6.2.1 Cross-Sectional Wage Dispersion:

Observable worker characteristics, education, gender, experience and tenure, can explain around 40% of the variation in log wages observed in the data, see Table This compares with around 30% in the usual Mincer wage regressions, typically somewhat higher for men and somewhat lower for women. Partly this is explained by the fact that the measures of experience and tenure I use are superior to those usually available. Sample selection, recall that observations of workers at small firms have not been used, may also play a role. Firm effects explain around one-quarter of the remaining variation in wages, and the unobservable component of the person effects the remaining three-quarters, see Table 1.

Table 2 shows that the explanatory variables are highly positively correlated with each other, hence the variance of wages, 0.33, is half again that of the explanatory combined, 0.22. As predicted by the model, tenure and experience are both positively correlated with the firm effect. The firm and person effect are also positively correlated, contradicting the no sorting prediction of the model. However, the positive correlation is much reduced once I control for the observed part of the person effect (the net person effect is the residual from regression (5) in Table 1). Similarly, much of the positive correlation between the person effect and experience and tenure can be explained by education and gender.

Table 1: Cross-section regressions on log wages, person effect, and log wages net of observables

Dependent Var	Log Wages				Person Effect	Net Log Wages		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Experience	0.0588	0.0532	0.0704	0.0648				
Experience sq.	-0.0012	-0.0011	-0.0016	-0.0014				
Tenure	0.0186	0.0205	0.0063	0.0084				
Tenure sq.	-0.0003	-0.0003	0.0001	0.0000				
Female	-0.2919	-0.2497			-0.2557			
Compulsory	0.0624	0.0326			0.0360			
Apprenticeship	0.1596	0.1052			0.1097			
Vocational	0.3275	0.2356			0.2459			
High school	0.3807	0.2654			0.2760			
University	0.4145	0.2946			0.2995			
Firm effect		1.1686		1		1.1943		1.0414
Person effect			1.0667	1				
Net person							1.0342	0.9952
Constant	1.9552	2.0232	2.0179	2.0393	-0.0447	2.0722	2.0834	2.0736
R-squared	0.399	0.5019	0.7442	0.8208	0.1724	0.2573	0.7929	0.9874

Table 2: Summary correlations in the cross-section

	Correlation					
	Log Wage	Experience	Tenure	Firm Effect	Person Effect	Net Person
Log wage	1					
Experience	0.5302	1				
Tenure	0.3986	0.5338	1			
Firm effect	0.4321	0.1193	0.0534	1		
Person effect	0.6875	0.0198	0.0972	0.1484	1	
Net person	0.5851	-0.0263	0.0615	0.0758	0.9097	1

6.2.2 Wage Dispersion by Gender and Educational Category:

Table 3 shows mean and variance of the explanatory variables by worker characteristic. Around three-quarters of the gender wage gap is explained by lower person effects. Lower firm effects and labor market experience account for most of the remaining differentials.

Differences in the average person effect account for most of the wage differentials between education groups, average firm effect, labor market experience and tenure also increase substantially with educational attainment level. Interestingly the variance of firm effects decreases and the variance of person effects increases with education. The ratio of firm to person variance is 0.3 for workers with no qualifications, whilst it is 0.14 for those with a university degree, suggesting that firms play a larger role in explaining wage dispersion for low-skilled workers. The results look similar when decomposing by occupation.

Table 3: Means and variances of variables by gender and education

	Mean (relative)			Variance			Covariance
	Wage	Firm	Person	Wage	Firm	Person	Firm & Person
Male	0	0	0	0.3253	0.0246	0.1163	0.0055
Female	-0.3319	-0.0376	-0.2466	0.2674	0.0282	0.1155	0.0082
No qualifications	0	0	0	0.2502	0.0340	0.1109	0.0079
Compulsory	0.0916	0.0264	0.0343	0.2780	0.0308	0.1187	0.0061
Apprenticeship	0.2822	0.0565	0.1615	0.3110	0.0251	0.1142	0.0064
Vocational	0.3575	0.0752	0.2074	0.3311	0.0220	0.1313	0.0067
High school	0.4935	0.1034	0.2873	0.3346	0.0203	0.1354	0.0055
University	0.5884	0.1140	0.3490	0.3416	0.0209	0.1519	0.0057
Population Avg.	2.5019	0.0094	-0.0010	0.3297	0.0262	0.1302	0.0087

6.2.3 Inter-Industry Wage Differentials:

The earliest literature on the relative importance of firm versus person characteristics analyzed inter-industry wage differentials, with papers by Krueger and Summers (1988), Murphy and Topel (1987) and Gibbons and Katz (1992). Abowd, Kramarz et al (1999, 2000, 2002) find that unobserved firm and person effects each account for around half the inter-industry wage differentials found in France and Washington State.

Firms are categorized at the 2-digit industry level, a total of 38 sectors. The average experience, tenure, education and sex by industry are calculated by weighting them by the market prices of these attributes, using the regressions in Table 1. For example, a year of experience is weighted by 0.065, and a university degree by 0.3. The standard deviation of average log wages, is 0.22; the interquartile range is 0.29. Table 4 shows that all explanatory variables are highly positively correlated, suggesting a high degree of sorting and therefore making it difficult to disentangle the various determinants of average log wages.

Either firm or person effects can explain around 80% of the variation in wages on their own. Only 38% of the total variance in average log wages can be attributed unambiguously to

Table 4: Correlations at 2-digit industry level

	Correlation of 2-Digit Industry Means							
	Wage	Firm	Person	Net Person	Exp.	Tenure	Education	Sex
Log Wage	1							
Firm Effect	0.8833	1						
Person Effect	0.9151	0.7341	1					
Net Person	0.8159	0.5940	0.9092	1				
Experience	0.5725	0.3177	0.3338	0.3375	1			
Tenure	0.4876	0.3066	0.3658	0.4144	0.4515	1		
Education	0.6862	0.5481	0.7725	0.7144	0.2201	0.2322	1	
Sex	0.6240	0.5833	0.6397	0.3413	0.200	0.1656	0.1556	1

individual components (calculated as the sum of the variances of the observable components divided by the variance of average log wages). 45% of this variance is due to the differences in the average firm effect, 20% due to differences in unobservable (net) person effects, and 15% due to both differences in total experience and gender composition.

6.2.4 Firmsize Wage Effect:

There is a large literature related to the observation that larger firms pay higher wages, with an average premium of around 30% when going from a firm with 25-99 employees to one with over 1000 employees, and controlling for 1-digit industry, see Oi and Idson (1999). I find a somewhat lower elasticity of 0.026 and 0.016 in my data, when controlling for 1-digit and 2-digit industry fixed effects respectively.

The literature suggests a number of explanations for this phenomenon, monitoring costs (Oi, 1983), to prevent shirking (Shapiro and Stiglitz, 1984 for example) and rent-sharing (Katz and Summers, 1989), whereby larger firms may have higher rents for a number of reasons. The explanation provided by the model in this paper is most similar to the last of these theories, conditional on the industry more productive firms pay both higher wages and are larger. Hence, once I control for firm fixed effects the positive effect of firmsize on wages should disappear.

The firm fixed effect reduces the elasticity of the wage with respect to firm size by two-thirds, and accordingly can explain two-thirds of the firmsize wage effect. Adding additional controls entirely eliminates the firm size wage effect, see Table 5.

6.3 Wage Growth Differentials

Wage growth is infrequently discussed in the literature, Abowd, Kramarz and Margolis (1999) for example do not discuss it at all, and Mincer regressions are nearly always done in levels. Once I have obtained firm fixed effect estimates I can run the regression (50) in first differences, thereby providing insights on role of firms in determining life-cycle growth patterns.

Table 5: Firmsize wage effect

Dependent Variable: Log Wage	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Log firmsize	0.0158	0.0069	0.0101	0.0043	-0.0060	-0.0040	0.0004
Experience		0.0581	0.0690		0.0536	0.0536	0.0648
Experience sq.		-0.0012	-0.0015		-0.0011	-0.0011	-0.0014
Tenure		0.0159	0.0042		0.0201	0.0203	0.0083
Tenure sq.		-0.0024	0.0001		-0.0003	-0.0003	0.0000
Female		-0.2663				-0.2454	
Compulsory		0.0576				0.0304	
Apprenticeship		0.1459				0.1020	
Vocational		0.2815				0.2091	
High school		0.3084				0.2344	
University		0.3545				0.2707	
Firm effect				1.3513	1.2264	1.1289	0.9955
Person effect			1.0230				0.9974
Constant	2.4196	1.947	1.8909	2.4671	2.0810	2.0524	1.9851
R-squared	0.1020	0.4299	0.7608	0.2161	0.4560	0.5075	0.8209
Controls: all specifications control for 2-digit industry							

It is worth noting that a first difference to a traditional Mincer regression, see Table 6 column (1), has very low predictive power with an R-squared of 0.04. In part I suspect that is on account of measurement error in measured hours, nevertheless there is clearly much room for improvement. It turns out that the change in the firm effect alone has more than twice the predictive power of the observable variables and is only slightly correlated with changes in tenure and experience. Reassuringly, the coefficients on experience and tenure are very close to those from the regression in wage levels. However, the coefficient on the change in the firm effect is below 0.8, whilst we would expect it to be equal to one.

On average the firm effect's contribution to wage growth over the life-cycle is around 10% of total wage growth, with gains to experience accounting for around half of wage growth. This number may understate the contribution of firm changes to wage growth though since the standard deviation in workers' first job is already 0.18.

Workers with higher levels of education switch jobs less frequently, but benefit more from job-switching than those with less education. In contrast their gains to experience are lower. Gender differences are not pronounced.

6.4 Worker Mobility

A central feature of the model, and indeed most models of job mobility, is that workers switch in response to wage changes. Moreover, they switch from firms with lower to those with higher firm effects. It has been observed that workers seem to frequently take wage cuts, even when switching jobs, see for example [ref]. This observation has been taken to provide

Table 6: Wage regression in first differences

Dependent variable: Difference Log Wage						
	(1)	(2)	(3)	(4)	(5)	(6)
Diff. Experience	0.0698				0.0645	0.0653
Diff. Experience sq.	-0.0020				-0.0018	-0.0018
Diff. Tenure	0.0064				0.0109	0.0111
Diff. Tenure sq.	-0.0001				-0.0002	-0.0002
Diff. Firm effect			0.7956	0.7944	0.7856	0.7819
Controls		sector		sector		sector
Constant	0.0232		0.0458	0.0458	0.0206	0.0202
R-squared	0.0402		0.0878	0.0889	0.1250	0.1263

Table 7: Wagegrowth summary statistics by sex and education category

	Mean Diff. Wage	Variance Diff. Wage	Switches per Year	Mean Diff. Firm	Variance Diff. Firm
Male	0.0482	0.0515	0.1981	0.0137	0.0356
Female	0.0482	0.0578	0.2039	0.0002	0.0360
No qualifications	0.0314	0.0686	0.2696	-0.0132	0.0386
Compulsory	0.0392	0.0575	0.2369	-0.0013	0.0368
Apprenticeship	0.0493	0.0525	0.2048	0.0113	0.0352
Vocational	0.0545	0.0536	0.1674	0.0303	0.0353
High school	0.0548	0.0506	0.1557	0.0431	0.0329
University	0.0532	0.0516	0.1535	0.0457	0.0338
Population avg.	0.0482	0.0538	0.2002	0.0135	0.0357

Table 8: Regression on the probability of leaving a firm

Dependent variable: Indicator Variable for Exit from Firm						
	(1)	(2)	(3)	(4)	(5)	(6)
Experience		-0.0199	-0.0093	-0.0094	-0.0099	-0.0503
Experience sq.		0.0005	0.0003	0.0003	0.0003	-0.0095
Tenure			-0.0298	-0.0296	-0.0291	0.0003
Tenure sq.			0.0010	0.0010	0.0010	-0.0291
Female				0.0115		0.0010
Compulsory schooling				-0.0163		
Apprenticeship				-0.0340		
Vocational education				-0.0576		
High school				-0.0601		
University				-0.0689		
Firm effect	-0.0904	-0.0475	-0.0703	-0.0569	-0.0573	-0.0503
Person effect					-0.0586	-0.0516
Log Wage						-0.0069
Constant	0.2102	0.3320	0.3368	0.3681	0.3370	0.3511
R-squared	0.0226	0.0519	0.0728	0.0744	0.0752	0.0752
Controls: all specifications control for 2-digit industry fixed effects						

strong support for a large match-specific component of wages and wage growth. Unless the model is extended to include a match-specific effect I can not disentangle to what degree observed wage cuts are due to such factors and to what degree they are a statistical artifice on account of a lot of measurement error in the hours worked variable.

The model also predicts that the probability of a worker exiting a firm is decreasing and convex in the firm productivity, which is a more easily testable prediction since in the data switches between firms are well measured. A linear regression[18] on an indicator variable for when a worker exits the firm, see Table 8, consistently shows that the probability of exit is decreasing in the firm effect, even when controlling for the wage. As predicted by the model mobility is also decreasing and concave in both experience and tenure. Mobility is also decreasing in education and women are slightly more likely to move than men.

6.5 Turnover

The model predicts that the number of workers leaving a firm, conditional on the size of the firm, is decreasing in firm productivty. Moreover the fraction of hiring form the pool of unemployed should be decreasing in firm productivity. Both these predictions are confirmed by the data and robust to conditioning on 2-digit industry and year fixed effects, see Table 9. However, the coefficient on log firmsize and log inflow should be equal to one. Furthermore when controlling for other firm characteristics, such as average person effect, the magnitude

[18]Probit results give very similar marginal effects to those found in the linear regression.

Table 9: Regression on exit rate and fraction entering from unemployment

Dependent: Log outflow of workers			Dependent: Log inflow from unemployment		
	(1)	(2)		(3)	(4)
Firm effect	-0.0437	-0.0682	Firm effect	-0.2889	-0.2538
Log firmsize	0.8791	0.8796	Log inflow	0.8756	0.8871
Year effect		Yes	Year effect		Yes
R-squared	0.6477	0.6514	R-squared	0.8059	0.8276
Controls: all specifications control for 2-digit industry.					

of the firm effect becomes positively related to the outflow of worker.

6.6 Job Creation and Destruction

[to be completed

- How good are idiosyncratic/aggregate shocks to f at explaining job creation and destruction?
- How good are aggregate shocks to δ at explaining job creation and destruction?]

6.7 Economy Aggregates

[to be completed

Show various aggregate relationships and compare with U.S. results:

- Beveridge curve
- Matching function
- Relative volatility of parameters f, δ, λ_w, u and N_f.]

6.8 Simulations

[to be completed]

7 Conclusions

Given that this is a proposal in conclusion I will outline the main aspects in which I intend to extend this paper. So as to ensure that the estimation technique fully reflects the model, I intend to extend the model to allow for idiosyncratic shocks to productivity and match-productivity. This will allow for estimation of transition dynamics, will rationalize the estimation of time-varying firm productivity and explain the tenure-wage and tenure-exit probability profiles observed in the data. Sorting could be incorporated by allowing for directed search by firms; I suspect, however, that such an extension will merit a separate paper.

I need to work out the identification under the Bertrand competition wage-setting assumption, whereby, unlike Postel-Vinay and Robin (2002), I intend to use the panel aspect of the data to achieve identification of the model. Empirically a lot of work remains, estimating transition rates and productivity parameters with the full set of data, under all three wage-setting assumption and various labor market specifications. Furthermore, the transition dynamics need to be tested, as well as the macroeconomic implications.

8 Bibliography

Abowd, Creecy and Kramarz (2002): "Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data". Cornell university working paper.

Abowd, Finer and Kramarz (1999): "Individual and Firm heterogeniety in Compensation." In Haltiwanger, Lane, Spletzer, Theeuwes, eds., *The Creation and Analysis of Employer-Employee Matched Data.* Amsterdam: North-Holland.

Abowd, Kramarz and Margolis (1999): "High Wage Workers and High Wage Firms", *Econometrica*, 67, 251-334.

Acemoglu and Shimer (1999): "Wage and Technology Dispersion." *Review of Economic Studies* 68: 585-608.

Bartelsman and Doms (2000): "Understanding Productivity: Lessons from Longitudinal Microdata." *Journal of Economic Literature*, 38, 569-595.

Bontemps, Robin and van den Berg (2000): "Equilibrium Search with Continuous Productivity Dispersion." *International Economic Review* 41: 305-358.

Bowlus, Kiefer and Neumann (1995): "Estimation of Equilibrium Wage Distributions with Heterogeneity." *Journal of Applied Econometrics* 10: S119-S131.

Bronars and Famulari (1997)."Wage, Tenure, and Wage Growth Variation within and across Establishments" *Journal of Labor Economics*, 15(2), 285-317.

Burdett and Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39: 257-273.

Christensen, Lentz, Mortensen, Neumann and Werwatz (2005): "On-the-Job Search and the Wage Distribution." *Journal of Labor Economics*, 23, 31-57.

Davis, Faberman and Haltiwanger (2006): "The Flow Approach to Labor Markets: New Data Sources and Micro-Macro Links." NBER working paper 12167.

Davis, Haltiwanger and Schuh (1996): *Job Creation and Destruction.* Cambridge, MA: The MIT Press.

Diamond (1982): "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies* 49: 217-227.

Dickens and Katz (1987): "Inter-industry Wage Differentials and Industry Characteristics" In *Unemployment and the Structure of Labor Markets*, edited by Lang and Leonard. New York, Blackwell.

Eckstein and Wolpin (1990): "Estimating a Market Search Equilibrium Model from Panel Data on Individuals. *Econometrica*, 58, 783-808.

Farber and Gibbons (1996): "Learning and Wage Dynamics." *The Quarterly Journal of Economics*, 111(4), 1007-1047.

Gibbons and Katz (1992): "Does Unmeasured Ability Explain Inter-Industry Wage Differentials?" *Review of Economic Studies* 59: 515-535.

Heckman and Honoré (1990): "The Empirical Contents of the Roy Model." *Econometrica*, 58, 1121-1149.

Heckman, Lochner and Todd (2005): "Earning Functions, Rates of Return and Treatment Effects: the Mincer Equation and Beyond." mimeo.

Heckman and Sedlacek (1985):"Heterogeneity, Aggregation and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market" *The Journal of Political Economy,* 93(6), 1077-1125.

Jovanovic (1979a): "Job Matching and the Theory of Turnover" *The Journal of Political Economy,* 87(5), 972-990.

Jovanovic (1979b): "Firm-specific Capital and Turnover" *The Journal of Political Economy,* 87(6),1246-1260.

Jovanovic, Boyan and Moffitt, Robert. "An Estimate of a Sectoral Model of Labor Mobility" *The Journal of Political Economy*, 98(4) (Aug., 1990), 827-852.

Krueger and Summers (1988): "Effficiency Wages and the Inter-Industry Wage Structure." *Econometrica* 56: 259-294.

Neal Derek (1995): "Industry-Specific Human Capital: Evidence from Displaced Workers" *Journal of Labor Economics*, 13(4), 653-677.

Neal, Derek (1999): "The Complexity of Job Mobility Among Young Men" *Journal of Labor Economics*, 17(2), 237-261.

Mortensen (2003): *Wage Dispersion.* Cambridge, MA: The MIT Press.

Moscarini (2005): "Job Matching and the Wage Distribution." *Econometrica*, 73(2), 481-516.

Murphy and Topel (1987): "Unemployment Risk, and Earnings: Testing for Equalizing Wage Differences in the Labor Market." In Land and Leonard, eds., *Unemployment and the Structure of the Labor Market.* Oxford: Blackwell.

Oi and Idson (1999): "Firm Size and Wages." In Aschenfelter and Card, eds., *Handbook of Labor Economics*, vol. 3B, 2165-2214. Amsterdam: Elsevier.

Pinkston and Spletzer (2004): "Annual measures of gross job gains and gross job losses." *Monthly Labor Review*, 127(11), 3-13.

Pissarides (1985): "Short-Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages." *American Economic Review* 75: 676-690.

Pissarides (2000): *Equilibrium Unemployment Theory.* 2nd ed. Cmbridge, MA: The MIT Press.

Postel-Vinay and Robin (1999): "The Distribution of Earnings in an Equilibrium Search Model with State-Dependent Offers and Counter-Offers." INRA_LEA working paper.

Postel-Vinay and Robin (2002): "An Equilibrium Search Model for Matched Employer-Employee Data." Econometrica

Roy (1951): "Some Thoughts on the Distribution of Earning." *Oxford Economic Papers* (New Series) 3: 135-146.

Shapiro and Stiglitz (1984): "Equilibrium Unemployment as a Worker Discipline Device." *American Economic Review* 74: 433-444.

Shimer (2005a): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 95(1): 25-49.

Shimer (2005b): :On-the-Job Search and Strategic Bargaining" Forthcoming, European Economic Review and Conference Volume on Structural Labour Market Models in honor of Dale Mortensen.

Syverson (2004): "Productivity Dispersion and Product Substitutability." *Review of Economics and Statistics*, 86(2), 534-550.

Topel (1991): "Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority" *The Journal of Political Economy*, 99(1),145-176.

Topel and Ward (1992): "Job Mobility and the Careers of Young Men" *The Quarterly Journal of Economics*, 107(2), 439-479.

Woodcock (2005): "Heterogeneity and Learning in Labor Markets" mimeo.

9 Appendix A: Details of Some Theoretical Results

9.1 Appendix A.1

Workers' wage profile is in expectation increasing and concave in labor market tenure. Wage growth is in expectation decreasing and convex. Workers accept offers from firms with higher productivity, therefore the expected wage of a worker after having received n wage offers is (if we assume draws are independent):

$$\begin{aligned} M_n &\equiv spE\max(f_1, ..f_n) && (59) \\ &= \int_{f_{\min}}^{f_{\max}} f d\Phi(f)^n && (60) \\ &= \int_{f_{\min}}^{f_{\max}} f n\Phi(f)^{n-1}\Phi'(f)df && (61) \\ &= [f\Phi(f)^n]_{f_{\min}}^{f_{\max}} - \int_{f_{\min}}^{f_{\max}} \Phi(f)^n df && (62) \\ &= sp(f_{\max} - \int_{f_{\min}}^{f_{\max}} \Phi(f)^n df) && (63) \end{aligned}$$

Notice that M_n is strictly increasing and strictly concave in n.

$$\begin{aligned} \frac{\partial M_n}{\partial n} &= -sp\int_{f_{\min}}^{f_{\max}} \Phi(f)^n \ln\Phi(f)df > 0 && (64) \\ \frac{\partial^2 M_m}{\partial^2 n} &= -sp\int_{f_{\min}}^{f_{\max}} \Phi(f)^n (\ln\Phi(f))^2 df < 0 && (65) \end{aligned}$$

The undirected search assumption implies that all workers receive job offers at a Poisson arrival rate of λ, so that the number of job offers any worker receives follows a Poisson distribution, i.e.:

$$\pi_n(t) \equiv \Pr(n \text{ offers in time } t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \tag{66}$$

So then the expected wage for a worker continuously employed for t periods is given by:

$$Ew(t) \equiv \sum_{n=0}^{\infty} \pi_n(t) M_n \tag{67}$$

The first derivative is positive since M_n is increasing in n:

$$\begin{aligned}
\frac{\partial Ew(t)}{\partial t} &= \frac{\partial}{\partial t}\left[\sum_{n=1}^{\infty} \pi_n(t) M_n + e^{-\lambda t} M_0\right] &(68)\\
&= \sum_{n=1}^{\infty} M_n \frac{\lambda n(\lambda t)^{n-1} e^{-\lambda t} - \lambda(\lambda t)^n e^{-\lambda t}}{n!} - \lambda e^{-\lambda t} M_0 &(69)\\
&= \sum_{n=1}^{\infty} M_n \lambda(\pi_{n-1} - \pi_n) - \lambda e^{-\lambda t} M_0 &(70)\\
&= \sum_{n=0}^{\infty} \lambda \pi_n (M_{n+1} - M_n) > 0 &(71)
\end{aligned}$$

The second derivative is negative since M_n is increasing and concave in n:

$$\begin{aligned}
\frac{\partial^2 Ew(t)}{\partial t^2} &= \frac{\partial}{\partial t}\left[\sum_{n=1}^{\infty} \lambda \pi_n (M_{n+1} - M_n) + \lambda e^{-\lambda t}(M_1 - M_0)\right] &(72)\\
&= \sum_{n=1}^{\infty} \lambda^2 (\pi_{n-1} - \pi_n)(M_{n+1} - M_n) - \lambda^2 e^{-\lambda t}(M_1 - M_0) &(73)\\
&= -\sum_{n=0}^{\infty} \lambda^2 \pi_n (M_{n+1} - M_n) + \sum_{n=1}^{\infty} \lambda^2 \pi_{n-1} M_{n+1} - \sum_{n=1}^{\infty} \lambda^2 \pi_{n-1} M_n &(74)\\
&= \sum_{n=0}^{\infty} \lambda^2 \pi_n (M_{n+2} - 2M_{n+1} + M_n) < 0 &(75)
\end{aligned}$$

The third derivative is positive since M_n is increasing and concave in n:

$$\frac{\partial^3 Ew(t)}{\partial t^3} = \frac{\partial}{\partial t}\left[\sum_{n=1}^{\infty}\lambda^2\pi_n(M_{n+2}-2M_{n+1}+M_n)+\lambda^2 e^{-\lambda t}\left(M_2-2M_1+M_0\right)\right] \tag{76}$$

$$= \sum_{n=1}^{\infty}\lambda^3(\pi_{n-1}-\pi_n)(M_{n+2}-2M_{n+1}+M_n)-\lambda^3 e^{-\lambda t}\left(M_2-2M_1+M_0\right) \tag{77}$$

$$= -\sum_{n=0}^{\infty}\lambda^3\pi_n(M_{n+2}-2M_{n+1}+M_n)+\sum_{n=1}^{\infty}\lambda^3\pi_{n-1}M_{n+2}-\sum_{n=1}^{\infty}2\lambda^3\pi_{n-1}M_{n+1}+\sum_{n=1}^{\infty}\lambda^3\pi_{n-1}M_n \tag{78}$$

$$= -\sum_{n=0}^{\infty}\lambda^3\pi_n(M_n-M_{n+1}-M_{n+2}+M_{n+3})>0 \tag{79}$$

Workers job switch rate is in expectation decreasing and convex in labor market tenure. I have shown that workers switch jobs when they receive a job offer from a more productive firm. The probability that one of n job offers is higher than the current is (given the independence assumption) is:

$$P_n = 1-\Phi(f)^n \tag{80}$$

P_n is strictly increasing and convex in f, with a negative third derivative:

$$\frac{\partial P_n}{\partial n} = -\Phi(f)^n\ln\Phi(f)>0 \tag{81}$$

$$\frac{\partial^2 P_n}{\partial n^2} = -\Phi(f)^n\ln\Phi(f)^2<0 \tag{82}$$

Given that the number of job offers in a given time follows a Poisson distribution (66) the probability of having switched at least once in a given period of time, $\sigma(t)$, is:

$$\pi_n(t)\equiv \Pr(n \text{ offers in time } t)=\frac{(\lambda t)^n e^{-\lambda t}}{n!} \tag{83}$$

$$\sigma(t)=\sum_{n=0}^{\infty}\pi_n(t)P_n \tag{84}$$

Notice that the derivatives of P_n have the same sign as those of M_n, so that using the same procedure as before it can be shown that:

$$\frac{\partial\sigma(t)}{\partial t}>0,\frac{\partial^2\sigma(t)}{\partial t^2}<0,\frac{\partial^2\sigma(t)}{\partial t^2}>0$$

9.2 Appendix A.2

Average tenure at a firm is increasing in firm productivity. [see proof on p.91 in Ljunquist and Sargent]

More productive firms employ, in expectation, a more experienced workforce, i.e. a workforce with higher labor market tenure. [to be completed]

The firm's value function, first-order conditions and optimal size: Solve for the firm's value function as follows:

$$\rho V(f,n) = \max_v \{(1-s)nfp - c(v) + \dot{n}V_n\} \tag{85}$$
$$= \max_v \{(1-s)nfp - c(v) + (\eta - \mu n)\, V_n\} \tag{86}$$

Guess that the solution takes the form $V = x + by$, substitute into the value function for the firm () and collect terms

$$\rho yn = (1-s)nfp - \mu ny \tag{87}$$
$$y = \frac{(1-s)fp}{\rho+\mu} \tag{88}$$
$$\rho x = -c(v) + \eta y \tag{89}$$
$$x = \frac{-c(v) + \eta\frac{(1-s)fp}{\rho+\mu}}{\rho} \tag{90}$$

which is verified by substituting the expressions for x and y into $V = x + yn$:

$$V = \frac{-c(v)}{\rho} + \frac{\eta}{\rho}\frac{(1-s)fp}{\rho+\mu} + \frac{(1-s)fp}{\rho+\mu}n \tag{91}$$
$$\rho V = -c(v) + (\eta + \rho n)\frac{(1-s)fp}{\rho+\mu} \tag{92}$$
$$= -c(v) + \eta V_n + (\rho + \mu - \mu)\frac{(1-s)fp}{\rho+\mu}n \tag{93}$$
$$= (1-s)nfp - c(v) + (\eta - \mu n)\, V_n \tag{94}$$

So the firm problem is to maximize V by choice of v:

$$V = \frac{(1-s)fp}{\rho+\mu}n + \frac{1}{\rho}\left[\eta\frac{(1-s)fp}{\rho+\mu} - c(v)\right] \tag{95}$$
$$\frac{\partial V}{\partial v} = \frac{(1-s)fp}{\rho+\mu}\frac{\partial n}{\partial v} - \frac{1}{\rho}c'(v) = 0 \tag{96}$$

Imposing the steady state condition that $\dot{n} = 0$ implies $n = \frac{\eta}{\mu}$ and from () $\frac{1}{\theta} = \frac{n}{v} = \frac{\eta}{\mu}\frac{1}{v}$ resulting in:

$$\frac{(1-s)fp}{\rho+\mu}\frac{1}{\theta} = \frac{1}{\rho}c'(v) \tag{97}$$

which given the functional form for $c(v)$ () and recalling that $v = n\theta$ yields:

$$n(f,u) = \left(\frac{\rho}{\rho+\mu}\frac{1}{\psi\theta^2}\right)(1-s)fp \tag{98}$$

The more general functional form for the cost of making job offers:

$$c(v) = \psi v^{\phi} \tag{99}$$

yields

$$\frac{(1-s)fp}{\rho+\mu}\frac{1}{\theta} = \frac{1}{\rho}\phi\psi\,(n\theta)^{\phi-1} \tag{100}$$

$$n = \left[\frac{\rho}{\rho+\mu}\frac{1}{\phi\psi\theta^{\phi}}(1-s)fp\right]^{\frac{1}{\phi-1}} \tag{101}$$

The value function of a firm is strictly increasing in firm productivity. Define

$$y = \frac{(1-s)fp}{\rho+\mu} \tag{102}$$

$$\frac{\partial y}{\partial f} = \frac{(1-s)p}{\rho+\mu} - \frac{(1-s)p}{(\rho+\mu)^2}\frac{\partial\mu}{\partial f} > 0 \tag{103}$$

since $\frac{\partial\mu}{\partial f} < 0$. Hence we can rewrite the value function as

$$V = yn + \frac{1}{\rho}\left(y - c(v)\right) \tag{104}$$

$$\frac{\partial V}{\partial f} = \frac{\partial y}{\partial f}(n+\frac{1}{\rho}) + y\frac{\partial n}{\partial f} - \frac{1}{\rho}c'(v)\frac{\partial v}{\partial f} \tag{105}$$

Imposing FOC ($y\frac{1}{\theta} = \frac{1}{\rho}c'(v)$):

$$\frac{\partial V}{\partial f} = (n+\frac{1}{\rho})\frac{\partial y}{\partial f} + y(\frac{\partial n}{\partial f} - \frac{1}{\theta}\frac{\partial v}{\partial f}) \tag{106}$$

In steady state:

$$v^* = n^*\frac{\mu}{\pi} \tag{107}$$

$$\frac{\partial v}{\partial f} = \frac{\mu}{\pi}\frac{\partial n}{\partial f} + \frac{n}{\pi}\frac{\partial\mu}{\partial f} - \frac{\mu}{\pi^2}\frac{\partial\pi}{\partial f} \tag{108}$$

so that

$$\frac{\partial n}{\partial f} - \frac{1}{\theta}\frac{\partial v}{\partial f} = \frac{\partial n}{\partial f} - \frac{\pi}{\mu}\left(\frac{\mu}{\pi}\frac{\partial n}{\partial f} + \frac{n}{\pi}\frac{\partial\mu}{\partial f} - \frac{\mu}{\pi^2}\frac{\partial\pi}{\partial f}\right) \tag{109}$$

$$= -\frac{n}{\mu}\frac{\partial\mu}{\partial f} + \frac{1}{\pi}\frac{\partial\pi}{\partial f} \tag{110}$$

and

$$\frac{\partial V}{\partial f} = (n + \frac{1}{\rho})\frac{\partial y}{\partial f} + y\left(\frac{1}{\pi}\frac{\partial \pi}{\partial f} - \frac{n}{\mu}\frac{\partial \mu}{\partial f}\right) > 0 \tag{111}$$

since $\frac{\partial \pi}{\partial f} > 0$ and $\frac{\partial \mu}{\partial f} < 0$.

9.3 Appendix A.3

To find the minimum productivity level at which, in steady state, an entrepreneur is willing to start a firm, set

$$\begin{aligned} V(\zeta(u), u) &= U(u) && (112)\\ \frac{(1-s)fp}{\rho+\mu}n + \frac{1}{\rho}\left(\frac{(1-s)fp}{\rho+\mu} - c(v)\right) &= \frac{bp + \lambda E(W)}{\rho+\lambda} && (113)\\ (1-s)fp(\rho n + 1) - (\rho+\mu)\,c\left(n\frac{\mu}{\pi}\right) - \rho\,(\rho+\mu)\,\frac{bp+\lambda E(W)}{\rho+\lambda} &= 0 && (114) \end{aligned}$$

By the implicit function theorem:

$$\frac{\partial f_{\min}}{\partial u} = -\frac{k_1\frac{\partial n}{\partial u} - k_2\frac{\partial \mu}{\partial u} - k_3 c'(v)\frac{\partial v}{\partial u} - k_4\frac{\partial \mu}{\partial u} - k_5\frac{\partial U}{\partial u}}{k_6 + k_1\frac{\partial n}{\partial f} - k_2\frac{\partial \mu}{\partial f} - k_3 c'(v)\frac{\partial v}{\partial f} - k_4\frac{\partial \mu}{\partial f} - k_5\frac{\partial U}{\partial f}} \tag{115}$$

where $k_i > 0$ for all $i = \{1, ..6\}$. Now, in steady state the probability of a worker exiting a firm is decreasing in the unemployment rate (since higher unemployment is associated with a lower job offer rate), and the probability of a worker accepting a job offer is increasing in the unemployment rate. The combined effect implies that higher unemployment is associated with a lower θ for all firms:

$$\begin{aligned} \frac{\partial \mu}{\partial u} &= \frac{\partial \lambda}{\partial u}(1 - \Gamma(f)) < 0 && (116)\\ \frac{\partial \pi}{\partial u} &= 1 - \frac{1}{N_w}\int_{f_{\min}}^{f} m(f', \varpi)d\Phi(f') > 0 && (117)\\ \frac{\partial \theta}{\partial u} &= \frac{1}{\pi}\frac{\partial \mu}{\partial u} - \frac{\mu}{\pi^2}\frac{\partial \pi}{\partial u} < 0 && (118) \end{aligned}$$

which implies that

$$\frac{\partial n}{\partial u} = \left(-\frac{\rho}{(\rho+\mu)^2}\frac{1}{\psi\theta^2}\frac{\partial \mu}{\partial u} - \frac{\rho}{\rho+\mu}\frac{2}{\theta}\frac{\partial \theta}{\partial u}\right)(1-s)fp > 0 \tag{119}$$

and

$$\frac{\partial v}{\partial u} = \frac{\mu}{\pi}\frac{\partial n}{\partial u} + \frac{1}{\pi}\frac{\partial \mu}{\partial u} - \frac{\mu}{\pi^2}\frac{\partial \pi}{\partial u} \tag{120}$$

[....]
Hence

$$\frac{\partial \zeta(u)}{\partial u} < 0 \tag{121}$$

9.4 Appendix A.4: Surplus-Sharing

The Bellman equations for a filled vacancy, an employed and an unemployed worker are:

$$\rho J(p,f) = (1-s)fp - \lambda(1-\Omega(W))J - \delta J \tag{122}$$

$$\rho W(p,\Omega,U) = sfp + \delta(U-W) + \lambda \int_{W}^{W_{\max}} (W'-W)d\Omega(W') \tag{123}$$

$$\rho U = bp + \lambda(E(W')-U) \tag{124}$$

The Nash-bargaining condition is

$$w_{ij} = \arg\max (W_{ij}-U_i)^{\alpha} J_{ij}^{1-\alpha} \tag{125}$$

The first-order condition, assuming that the wage policy does not affect turnover, satisfies

$$\alpha(W_{ij}-U_i)^{\alpha-1}J_{ij}^{1-\alpha} = (1-\alpha)(W_{ij}-U_i)^{\alpha}J_{ij}^{-\alpha} \tag{126}$$

$$\alpha J_{ij} = (1-\alpha)(W_{ij}-U_i) \tag{127}$$

$$\alpha[(1-s)fp - \lambda_w(1-\Omega(W))J - \delta J] = (1-\alpha)[sfp - \delta(W-U) \tag{128}$$

$$+\lambda \int_{W}^{W_{\max}} (W'-W)d\Omega(W') - bp - \lambda(E(W')-U)] \tag{129}$$

so

$$\alpha fp - \alpha\lambda(1-\Omega(W))J = sfp + (1-\alpha)[\lambda \int_{W}^{W_{\max}} W'd\Omega(W') - bp - \lambda(E(W)-\Omega(W)U)] \tag{130}$$

$$-(1-\alpha)\lambda(1-\Omega(W))(W-U) \tag{131}$$

$$\alpha fp + (1-\alpha)bp = sfp + (1-\alpha)\lambda[\int_{W}^{W_{\max}} W'd\Omega(W') - E(W') + \Omega(W)U] \tag{132}$$

$$\alpha fp + (1-\alpha)bp = sfp - (1-\alpha)\lambda[\int_{W_{\min}}^{W} W'd\Omega(W') + \Omega(W)U] \tag{133}$$

$$\alpha fp + (1-\alpha)bp = sfp - (1-\alpha)\lambda \int_{W_{\min}}^{W} (W'-U)d\Omega(W') \tag{134}$$

$$w = \alpha fp + (1-\alpha)[bp + \lambda \int_{W_{\min}}^{W} (W'-U)d\Omega(W')] \tag{135}$$

Taking the derivative of the worker's Bellman equation with respect to f:

$$(\rho+\delta)\frac{\partial W}{\partial f} = \frac{\partial w}{\partial f} - \lambda(1-\Omega(W))\frac{\partial W}{\partial f} \tag{136}$$

$$\frac{\partial W}{\partial f} = \frac{1}{\rho+\delta+\lambda(1-\Omega(W))}\frac{\partial w}{\partial f} \tag{137}$$

and

$$\frac{\partial w}{\partial f} = \alpha p + \alpha\lambda\Omega'(W)(W-U) > 0 \tag{138}$$

so then

$$\frac{\partial W}{\partial f} > 0 \tag{139}$$

and workers will switch to firms with higher productivity and implying that we can replace Ω with Φ, such that

$$w = \alpha f p + (1-\alpha)[bp + \lambda \int_{f_{\min}}^{f} (W' - U) d\Phi(f')] \tag{140}$$

and using the first-order condition that $\alpha J = (1-\alpha)(W - U)$

$$w = \alpha f p + (1-\alpha) bp + \alpha\lambda \int_{f_{\min}}^{f} J' d\Phi(f') \tag{141}$$

$$s = \alpha + (1-\alpha)\frac{b}{f} + \alpha\lambda \int_{f_{\min}}^{f} \frac{J'}{fp} d\Phi(f') \tag{142}$$

Recall that, replacing Ω with Φ:

$$\rho J(p,f) = (1-s) f p - \lambda(1-\Phi(f))J - \delta J \tag{143}$$

$$J = \frac{(1-s) f p}{\rho + \delta + \lambda(1-\Phi(f))} \tag{144}$$

Hence we can write

$$s = \alpha + (1-\alpha)\frac{b}{f} + \alpha\lambda \int_{f_{\min}}^{f} \frac{(1-s) f'}{[\rho + \delta + \lambda(1-\Phi(f'))] f} d\Phi(f') \tag{145}$$

which is independent of the productivity of the worker.

9.5 Appendix A.5: Bertrand Competition:

[see Postel-Vinay and Robin (2002)]