

Relational Contracts with On-the-Job Search

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Abstract

This paper characterises the optimal self-enforcing contracts in a large anonymous labour market. We show that the introduction of on-the-job search leads otherwise identical firms to offer different contracts. High-wage firms rarely lose workers and demand a high level of effort. Low-wage firms frequently lose workers and demand little effort. We also endogenise the number of firms, showing that free-entry leads to full employment. In equilibrium, incentives are provided through underemployment rather than through unemployment.

1 Introduction

When employers wish to encourage leadership, teamwork, and other subjective tasks, measurement problems prevent the use of complete contracts. In such environments, firms can motivate workers through an ongoing relationship, subject to the constraint that all punishments and rewards are credible. That is, firms are restricted to contracts that are self-enforcing.

The optimal self-enforcing contracts in large anonymous markets have been analysed by Shapiro and Stiglitz (1984) and MacLeod and Malcomson (1998). We extend these models to allow for on-the-job search. This is important for two reasons. First, on-the-job search plays a major role in the economy, accounting for more than 70% of all job transitions (Nagypal (2004)). Second, the addition of on-the-job search substantially changes the qualitative predictions of the model.

The economy is as follows. There is measure 1 of workers and measure $n < 1$ of firms, where all workers and all firms are identical, and each firm has one job. Within a relationship, a firm pays fixed wage w_t , the worker exerts effort e_t , and the firm responds with a voluntary bonus b_t . At the end of each period, either the firm or worker can terminate the relationship. Separation also occurs with exogenous probability, $1 - \alpha$.

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After termination, a firm with a vacancy chooses a contract $\langle w_t, e_t, b_t \rangle$ and is matched with a worker. The matching rule has two properties. First, matching is efficient, in that every firm finds a worker. Second, matching is anonymous, in that job offers do not depend on a worker's current employment status. Economically, this means that on-the-job search and off-the-job search are equally productive. This matching rule can also be implemented by a version of the Gale-Shapley algorithm.

Given this setup, the paper characterises the optimal self-enforcing contracts. First, consider an individual relationship. As shown by MacLeod and Malcomson (1989, 1998), firms can restrict themselves to contracts that specify a constant wage and effort level (w, e) . Intuitively, bonuses and seniority-based pay are infeasible, since the firm would renege and match with another worker. In addition, any shirking worker is immediately fired. Intuitively, termination is the harshest possible punishment and is therefore the optimal penal code (Abreu (1988)).

Next, we show that there is no equilibrium where all firms offer the same contract. To see why, suppose all firms initially offer the same wages and never lose their workers to each other. Firm i can then increase its profits by offering a contract with low wages and low required effort. Such a firm regularly loses its workers to other firms but, because of on-the-job search, does not have to compensate its agents for forgoing search activity. Essentially, firm i is able to opt-out of the competition for workers by acting like a temp-agency, employing unemployed workers until they find more profitable opportunities elsewhere.

The model exhibits a unique equilibrium in which firms offer a nondegenerate distribution of wages. High-wage firms rarely lose workers and require a high level of effort. Low-wage firms frequently lose workers and require little effort. The introduction of on-the-job search reduces the probability of retaining a worker, and therefore reduces effort and welfare. In addition, on-the-job search raises firms' profits. Intuitively, on-the-job search allows a firm to differentiate itself by posting a low wage, opting out of direct competition. The increase in profits is substantial: firms' payoffs equal those obtained in the collusive equilibrium of a model without on-the-job search.

We next endogenise the number of firms. Somewhat surprisingly, we find that free entry leads to full employment. In equilibrium, incentives are maintained through the threat of under-employment, rather than the threat of unemployment. This result can be contrasted to Shapiro and Stiglitz (1984) who find that, without on-the-job search, there must be unemployment in equilibrium.

1.1 Empirical Regularities

The model helps explain a number of empirical regularities.

First, the model predicts that different firms will pay different wages to similar workers. This is consistent with the large unexplained inter-industry and cross-employer wage differentials

found by labour economists (e.g. Krueger and Summers (1988), Mortensen (2003)). The model also helps to explain why industries without piece-rates seem to have more wage dispersion (Groschen (1991)).

Second, the model predicts that higher wage firms will demand more effort, and will therefore have higher productivity. This matches the finding that high-wage plants have fewer disciplinary actions (Raff and Summers (1987), Cappelli and Chauvin (1991)), and that high-wage industries have higher output (Huang et al. (1998)).

Third, the model predicts that agents will work their way up a job ladder. This is consistent with the findings of Topel and Ward (1992) who show that a large fraction of wage growth occurs at job transitions, and that these jumps are both more frequent and larger at the start of an agent's career.

As with all job ladders, the model predicts that high-wage firms will have lower turnover. This is supported by evidence at the industry level (Krueger and Summers (1988)) and individual level (Viscusi (1980), Royalty (1998)). For example, Topel and Ward (1992) find that when an agent changes job, a larger wage jump leads to a lower subsequent probability of quitting. Similarly, higher wages tend to increase the number of number of applications (Raff and Summers (1987), Holzer, Katz, and Krueger (1991)). The model is also consistent with the finding that agents with longer tenure have a lower probability of quitting (Topel and Ward (1992)).

The model is particularly applicable to managerial and professional occupations. In such positions, workers have more discretion, and effort tends to be more subjective (Milgrom and Roberts (1992, p. 258)). In addition, tenure rates are longer, further increasing the importance of relational contracts.¹ These occupations also exhibit a particularly high rate of on-the-job search: it is now common for such employees to have their resume's constantly posted online (e.g. monster.com). As a result, job-to-job transitions are more important for workers with high levels of experience or education (Topel and Ward (1992), Nagypal (2005)).

The model helps explain a number of empirical regularities about professional occupations. First, workers with high levels of education and experience have higher levels of residual wage inequality (Lemieux (2006)). Second, the unemployment rate for professional occupations is much lower than in rest of the population.² Third, agents strive to climb up a job ladder. Fourth, those higher up the ladder work harder than those at the bottom. These last two properties are illustrated by the market for economists, where a poor initial placement leads to a lower later placement and to lower productivity (Oyer (2006)). Interestingly, these results were anticipated by Bulow and Summers (1986, p. 387) who observed that, "job ladders appear in parts of the economy where individual performance is difficult to disentangle".

¹In 2006, the median years of tenure was 5.2 years for professionals, compared to 4.0 years for average workers. Source: Bureau of Labour Statistics.

²In 2006, the unemployment rate was 2.1% for professionals, compared to 4.6% for average workers. Source: Bureau of Labour Statistics.

1.2 Related Literature

This paper builds on the relational contracting models of Shapiro and Stiglitz (1984) and MacLeod and Malcomson (1998). These models assume firms only recruit from the pool of unemployed, ruling out the possibility of on-the-job search. They show that workers can be motivated by sufficiently high wages, and that unemployment is a necessary component of any equilibrium.³ Intuitively, agents only work if shirking is punished by a spell of unemployment. Of course, this logic depends on all jobs being equally valuable: with on-the-job search, firms offer different jobs, with incentives in a ‘good job’ maintained by fear being demoted to a ‘bad job’.

It has been previously argued that relational contracts can help explain the existence of wage differentials. For example, Bulow and Summers (1986) show that wage differentials can be generated by differences in monitoring costs. However, there has been the view that

“If all firms were identical, one would not expect to see different firms paying different wages even if efficiency wages were important.” Krueger and Summers (1988, p. 261)

This observation has undermined confidence in the ability of the efficiency wage model to explain wage dispersion. For example, it seems unrealistic for large cross-firm variations to be explained by the use of different monitoring technology (Mortensen (2003)). This hypothesis also lacks empirical support: Leonard (1987) shows there is no relation between the number of supervisors and wage premia in the high-tech sector. In the same study, however, Leonard does find that high-wage firms have less turnover, as the current model predicts.

The paper is closely linked to the on-the-job search model of Burdett and Mortensen (1998). In this model, a firm can employ many workers, but meets agents infrequently. Burdett and Mortensen find that, in equilibrium, identical firms post different wages. Intuitively, in a fixed wage equilibrium, one firm can steal extra workers by paying a little more than its rivals.

Our analysis compliments that of Burdett and Mortensen. Both formulations predict a nondegenerate wage distribution, and both yield an endogenous job ladder. However, the driving force behind the results is quite different. Indeed, the current model is expressly designed to parse out the size effects that generate the Burdett–Mortensen results. In addition, the models deliver different testable implications: for example, the current paper predicts high-wage firms have higher productivity; while, under decreasing returns, the Burdett–Mortenson model predicts the opposite.

The paper is also related to models of turnover. Salop (1979) argues that firms will pay efficiency wages in order to reduce quits and save turnover costs (e.g. training costs). One can

³MacLeod and Malcomson (1998) exhibits two types of equilibria. In the first type, some workers are unemployed; in the second type, some firms are unemployed.

therefore view the current paper as endogenising both the turnover costs (caused by shirking) and the responsiveness of turnover to wages. From the perspective of a worker, the paper is similar to Burdett’s (1978) model of on–the–job search, albeit with an endogenous distribution of wage offers.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 considers the benchmark case without on–the–job search. Section 4 analyses on–the–job search, characterising the optimal distribution of wages and the establishing the full employment result. Section 5 allows for firms to differ in innate productivity. Section 6 examines the robustness of the main results. Section 7 concludes.

2 Model

Basics. The economy consists of measure 1 of workers and measure $n < 1$ of firms. Each firm has one job. Time t is discrete and infinite, $t \in \{0, 1, \dots\}$. Both firms and workers have discount rate $\delta \in (0, 1)$.

Matching. At the start of each period, every firm with a vacancy chooses a contract $\langle w_t, e_t, b_t \rangle$ and is matched with a worker. Matching is efficient, in that every vacancy is filled. Matching is also anonymous in the following sense. Without on–the–job search, job offers are made to unemployed workers, independent of their history. With on–the–job search, job offers are made to all workers, independent of their history and current employment status.

Jobs. The employment game consists of three stages. First, the firm pays the worker wage $w_t \in [0, \infty)$. Next, the worker chooses observable but noncontractible effort $e_t \in [0, \infty)$ and produces output pe_t . Third, the firm pays a noncontractible bonus $b_t \in [0, \infty)$. The firm’s profit is

$$\pi_t := pe_t - w_t - b_t$$

where p is the firm’s productivity. The worker’s utility is

$$u_t := w_t + b_t - c(e_t)$$

where the cost function $c(\cdot)$ is strictly increasing, strictly convex and is normalised so that $c(0) = 0$. In addition, $c(\cdot)$ is continuously differentiable and satisfies $c'(0) = 0$ and $\lim_{e \rightarrow \infty} c'(e) = \infty$. The subgame perfect equilibrium of the stage game is $\langle w, e, b \rangle = \langle 0, 0, 0 \rangle$.

Relational Contracts. A relational contract $\langle w_t, e_t, b_t \rangle$ specifies a wage, effort and bonus as a function of the history of the relationship (i.e. previous wages, efforts and bonuses). At the

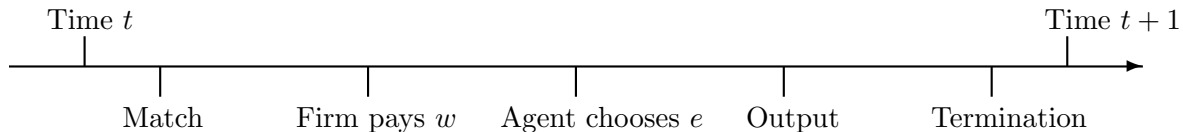


Figure 1: **Timeline.** At the start of the period, firms with vacancies choose their contract $\langle w, e \rangle$ and are matched with workers. The firm then pays wage w , and the worker chooses his effort level. The relationship is terminated at the end of the period if (a) either party has defected, or (b) an exogenous shock occurs. Termination may also occur at the start of the next period, in the matching phase.

end of each period either party may terminate the relationship. In addition, the relationship is exogenously terminated with probability $1 - \alpha$. Denote the vacancy–unemployment ratio by

$$\theta := \frac{(1 - \alpha)n}{(1 - n) + (1 - \alpha)n}. \quad (2.1)$$

Equilibrium Concept. Within a given relationship, we allocate the bargaining power to the firm, since there is an excess supply of workers. We then look at the set subgame perfect equilibria which are stationary in that a relational contract consists of a repeated one–period contract $\langle w, e, b \rangle$.⁴

2.1 Relational Contracts

Consider firm i , and suppose firms $j \neq i$ use stationary contracts. We claim that firm i 's optimal relational contract $\langle w_t, e_t, b_t \rangle$ takes the form of a stationary contract $\langle w, e \rangle$, where any defection is punished by termination. The argument follows the lines of MacLeod and Malcomson (1998).

First, observe that a contract $\langle w_t, e_t, b_t \rangle$ is self–enforcing if and only if any defection is punished by termination. Intuitively, termination minimises both parties payoffs, so is the optimal penal code (Abreu (1988)).

Second, bonuses must be zero in any equilibrium. Let Π_t denote the maximal discounted profits in period t of a relationship. The environment is stationary, so a contract is self–enforcing at time 1 if and only if it is self–enforcing at time t , implying $\Pi_1 = \Pi_t$. A firm can quit a relationship and immediately start a new one, so the firm's dynamic enforcement constraint requires

$$\beta(\Pi_{t+1} - \Pi_1) \geq b_t$$

where β is the discounted probability the relationship continues. Hence the optimal contract cannot contain bonuses.

⁴While we look for an equilibrium which is stationary, we do not impose stationarity off the equilibrium path. If firms $j \neq i$ use stationary contracts, then firm i 's optimal contract is also stationary.

Third, the optimal contract $\langle w_t, e_t \rangle$ is stationary. Since profits are stationary, the optimal contract can be implemented by repeating the same one-period contract.

The resulting game is summarised in Figure 1.

3 Relational Contracts without On-the-Job Search

In this section we analyse the optimal relational contracts when firms with vacancies are matched with unemployed workers. Section 3.1 describes the matching process. Sections 3.2–3.3 analyse the competitive model. Finally, Section 3.4 supposes that firms collude. These cases serve as useful benchmarks for the analysis with on-the-job search.

3.1 Matching

The matching rule has two properties.

1. *Efficiency.* Each firm with a vacancy is matched with a worker.
2. *Anonymity.* Each unemployed worker is treated equally.

These two properties are sufficient to pin down workers' value functions.

This matching rule can be implemented by the following game. First, each firm i endowed with a vacancy approaches a random unemployed worker and offers them a contract $\langle w^i, e^i \rangle$. If the worker accepts, both the firm and worker are taken out of the pool. If the worker rejects, the firm makes an offer to another unemployed worker. The process continues until (almost) all firms are matched.⁵

3.2 Value Functions

Suppose firm i offers the relational contract $\langle w^i, e^i \rangle$ inducing rent u^i . Let V^i be the value of an employed worker immediately after the matching stage. Similarly, let V_0 be the value of an unemployed worker. We thus have

$$V^i := u^i + \alpha\delta V^i + (1 - \alpha)V_0 \tag{3.1}$$

Rearranging,

$$V^i = \frac{1}{1 - \alpha\delta} [u^i + (1 - \alpha)V_0] \tag{3.2}$$

⁵Given this process, good jobs will tend to go before bad jobs. A worker can choose to shirk, so a job always dominates unemployment. Since $n < 1$, all jobs will go eventually.

For the unemployed worker,

$$V_0 := 0 + \delta [\theta E[V^i] + (1 - \theta)V_0] \quad (3.3)$$

Rearranging,

$$V_0 = \frac{\delta\theta}{1 - \delta(1 - \theta)} E[V^i] \quad (3.4)$$

Taking expectations over (3.2) and substituting into (3.4),

$$V_0 = \frac{\theta}{1 - \alpha\delta(1 - \theta)} \frac{\delta}{1 - \delta} E[u^i] \quad (3.5)$$

3.3 Competitive Wage Setting

Firm i chooses a relational contract $\langle w^i, e^i \rangle$ to solve the following problem:

$$\max_{w^i, e^i} \pi^i = pe^i - w^i \quad (3.6)$$

$$\text{s.t. } V^i \geq w^i + V_0 \quad (\text{DEA}^i)$$

The worker's dynamic enforcement constraint (DEA^{*i*}) states that the utility from cooperating exceeds the utility from putting in zero effort and being terminated. Using equation (3.2), the dynamic enforcement constraint becomes

$$\alpha\delta w^i \geq c(e^i) + \alpha(1 - \delta)V_0 \quad (3.7)$$

Substituting (3.7) into profits (3.6), firm i maximises

$$\pi^i = pe^i - \frac{1}{\alpha\delta} c(e^i) - \frac{1 - \delta}{\delta} V_0 \quad (3.8)$$

Profit is concave in e^i , so the optimal effort satisfies

$$c'(e^i) = \alpha\delta p \quad (3.9)$$

for each firm.

Wages are determined as follows. Taking expectations over (3.7) and using equation (3.5) to substitute for V_0 yields the aggregate dynamic enforcement constraint,

$$E[w^i] \geq \frac{1}{\alpha\delta(1 - \theta)} E[c(e^i)] \quad (3.10)$$

In the optimal contract, (DEAⁱ) binds with equality, so that (3.10) must also bind. Since all firms choose the same effort, the equilibrium wage is given by

$$w^i = \frac{1}{\alpha\delta(1-\theta)}c(e^i) \quad (3.11)$$

The optimal relational contract $\langle w^i, e^i \rangle$ is thus characterised by equations (3.9) and (3.11).

One should note that this analysis assumes that profit π^i is positive, since the firm can always choose to shut down. This is satisfied if n is sufficiently small. The number of firms is endogenised in Section 4.6.

3.4 Collusive Wage Setting

As a benchmark, consider the collusive wage setting problem. The firms' joint problem is

$$\begin{aligned} \max_{w^i, e^i} E[\pi^i] &= E[pe^i - w^i] & (3.12) \\ \text{s.t.} \quad V^i &\geq w^i + V_0 & (\text{DEA}^i) \end{aligned}$$

The problem is easy to solve. Substituting the aggregate dynamic enforcement constraint (3.10) into profits (3.12), the firms choose efforts $\{e^i\}$ to maximise

$$E[\pi^i] = E \left[pe^i - \frac{1}{\alpha\delta(1-\theta)}c(e^i) \right] \quad (3.13)$$

Optimal effort then satisfies

$$c'(e^i) = \alpha\delta(1-\theta)p. \quad (3.14)$$

Since every firm chooses the same effort, wages are given by (3.11).

Proposition 1. *Wages, effort and rents are higher under competition than under collusion.*

Proof. Comparing (3.9) and (3.14), effort is greater under competition. Wages, as determined by (3.11), are increasing in e and thus larger under competition. Subtracting $c(e)$ from wages, rents are also increasing in e and are thus larger under competition. \square

Proposition 1 says that when firms collude, they choose to reduce the workers' effort and pay. Intuitively, when firm i wishes to have its worker exert more effort, it must increase the agent's rents to prevent defection. However, by making job i more valuable, the firm also raises the value of unemployment, making other firms' workers more expensive. When the firms collude, they internalise this negative externality. For future reference, Table 1 summarises the results of this section under quadratic costs.

	Firms collude	Firms compete
Effort	$\alpha\delta(1-\theta)p$	$\alpha\delta p$
Wage	$\alpha\delta(1-\theta)p^2/2$	$\alpha\delta(1-\theta)^{-1}p^2/2$
Profit	$\alpha\delta(1-\theta)p^2/2$	$\alpha\delta(1-2\theta)(1+\theta)^{-1}p^2/2$

Table 1: **Outcomes without On-the-Job Search.** This table summarises the outcomes under collusion and competition without on-the-job search, assuming quadratic cost of effort, $c(e) = e^2/2$.

4 Relational Contracts with On-the-Job Search

We now analyse the optimal relational contracts with on-the-job search. Section 4.1 describes the matching rule. Section 4.2 shows that, in any equilibrium, firms must offer different contracts. Section 4.3 characterises job-to-job transitions for a given distribution of wages. Sections 4.4–4.5 analyse the optimal distribution of wages. Section 4.6 endogenises the number of firms.

4.1 Matching

The matching rule has two properties.

1. *Efficiency.* Each firm with a vacancy is matched with a worker.
2. *Anonymity.* Each worker is treated equally, independent of employment status.

We make two further assumptions. First, a worker sticks to their current firm if another firm makes an identical offer. Second, new offers are not observed by the worker's current employer.⁶

Let $\Lambda(u)$ be the distribution of rents at time $t+1$, conditional on being unemployed at time t . Similarly, let $\Lambda(u|u_t)$ be the distribution of rents at time $t+1$, conditional on having job u_t at time t . Anonymity then implies that

$$\begin{aligned}
 1 - \Lambda(u|u_t) &= 1 - \Lambda(u) && \text{if } u > u_t \\
 &= 1 && \text{if } u \leq u_t
 \end{aligned}
 \tag{4.1}$$

Equation (4.1) says that a worker in a bad job has the same probability of obtaining a good job as a worker in a middling job. That is, on-the-job and off-the-job search are equally productive.

The above matching rule can be implemented by the following game. Suppose firm i separates from its worker. Firm i then makes an offer $\langle w^i, e^i \rangle$ to a random worker, who may or

⁶These two assumptions are not crucial: see Section 6.

may not be unemployed. There are three possibilities. If the offer is turned down, then firm i instantly makes an offer to another random worker. If the offer is accepted by an unemployed worker, then the game ends. Finally, if the offer is accepted by an employed worker, then the displaced firm instantly makes an offer to a random worker. While this procedure is only stated for one firm, one can easily extend it to many firms (see Appendix A.1). One can also imagine other equivalent games where, for example, workers apply to firms who are better than their current employer.

4.2 Nonexistence of Fixed–Wage Equilibria

Consider firm i , and suppose firms $j \neq i$ offer contracts inducing common utility u . With on–the–job search, firm i 's contract choice will affect the probability with which it retains its worker. If i offers utility $u^i < u$ then, when firm j loses a worker, they may match with firm i 's worker. However, if firm i offers $u^i \geq u$ then, when firm j loses a worker, they will only match with unemployed workers.

First, we can calculate the aggregate value functions (V_0, V) . Since almost all firms are offering the same utility, the value of this common job is given by equation (3.1). Similarly, the value of unemployment is given by (3.3).

Next, we can consider firm i 's profits from deviating. If firm i offers a contract yielding utility $u^i \geq u$, then it will never lose a worker to another firm. As a result, the value function of a worker is given by equation (3.1). Firm i 's problem is to maximise profits (3.6) subject to (DEA^i) . As in Section 3.3, this reduces to maximising profits,

$$\pi^+(e^i) = pe^i - \frac{1}{\alpha\delta} c(e^i) - \frac{1-\delta}{\delta} V_0 \quad (4.2)$$

where the '+' superscript indicates an upwards deviation. The optimal effort, e^+ , is then determined by $c'(e^+) = \alpha\delta p$, as in Section 3.3.

Instead, suppose firm i considers a downwards deviation, $u^i < u$. In this case, firm i will always lose workers to competing offers from other firms. This yields the value function:

$$V^i = u^i + \alpha\delta [(1-\theta)V^i + \theta V] + (1-\alpha)V_0 \quad (4.3)$$

Rearranging,

$$V^i = \frac{1}{1-\alpha\delta(1-\theta)} [u^i + \alpha\delta\theta V + (1-\alpha)V_0]$$

Firm i 's problem is then to maximise profits (3.6) subject to (DEA^i) . Using equation (3.4) to

substitute for V_0 , (DEA^{*i*}) becomes

$$w^i \geq \frac{1}{\alpha\delta(1-\theta)}c(e^i). \quad (4.4)$$

Crucially, the aggregate value functions (V_0, V) do not affect the agent's incentives. Since the worker leaves firm i in exactly those states when an unemployed worker obtains a new job, an increase in firm j 's offer raises the value of working and the value of shirking by exactly the same amount. In this sense, firm i is able to opt-out of the competition for workers. Profits are then

$$\pi^-(e^i) = pe^i - \frac{1}{\alpha\delta(1-\theta)}c(e^i) \quad (4.5)$$

The optimal effort, e^- , is given by $c'(e^-) = \alpha\delta(1-\theta)p$, as in Section 3.4.

Proposition 2. *There exists no equilibrium where all firms offer contracts that provide agents with the same utility.*

Proof. Special case of Proposition 3. □

The intuition behind Proposition 2 is as follows. Suppose firms ignore the possibility of losing a worker through on-the-job search. In this case, firms have a high probability of retention and require the worker to put in a lot of effort (in particular, the competitive level). This high level of effort raises the firm's profits but also increases the worker's rents. Consequently, V_0 is very high, making it expensive to prevent a worker from cheating.

At this point, it is profitable for firm i to deviate to a lower wage and a lower level of effort (in particular, the collusive level). Such a firm regularly loses workers to other firms, but does not have to increase its compensation in response to wage rises of the other firms. Intuitively, firm i 's worker benefits from the wage increases of firms $j \neq i$ whether he shirks or not. Hence, by opting out of the 'rat race' and letting its employees go, firm i can increase its profits.

This idea is illustrated in Figure 2. First, suppose firms require the competitive level of effort, $e = 1/4$, which is optimal if workers are never bid away. Panel A shows that this is not an equilibrium: the profit from undercutting the current wage, $\pi^-(e)$, exceeds that from matching the current wage, $\pi^+(e)$. Conversely, suppose firms require the collusive level of effort, $e = 3/16$, which is optimal if workers are always bid away. Panel B then shows that this is also not an equilibrium: a firm can increase its profit by raising the current wage, attaining $\pi^+(e)$. In general, the overall profit function is given by the upper envelope of $\pi^-(e)$ and $\pi^+(e)$ so, for any initial effort, there will always be a profitable deviation.

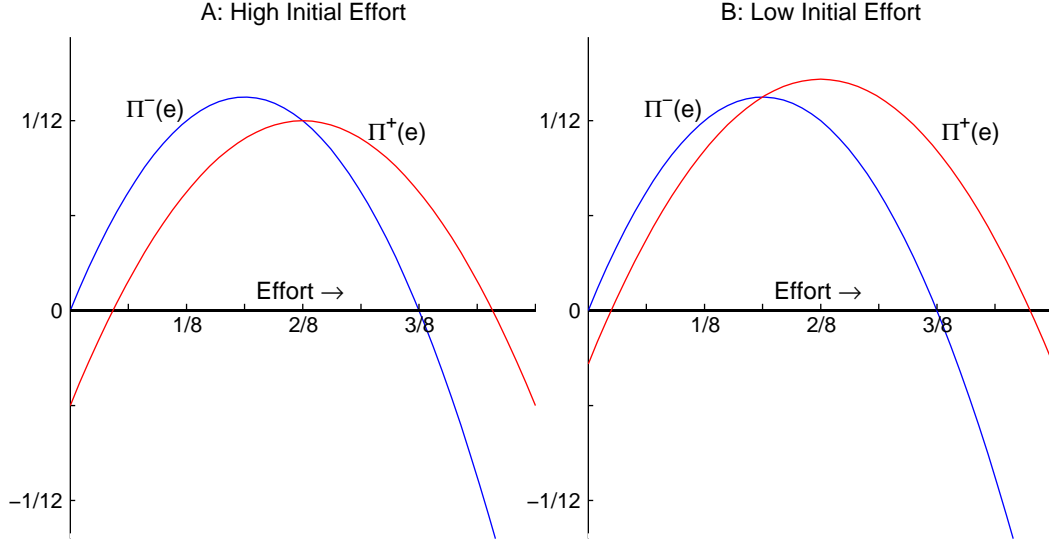


Figure 2: **Nonexistence of Fixed Wage Equilibria.** This figure illustrates the profit when deviating from the candidate equilibrium. It assumes the cost of effort is $c(e) = e^2/2$, marginal productivity is $p = 1$, the mass of firms is $n = 2/5$, the mass of labour is 1, the breakup rate is $1 - \alpha = 1/2$, yielding $\theta = 1/4$, and the discount rate is $\delta = 1/2$. The two panels are explained in the text.

4.3 Job Transitions

Suppose that firms post utilities according to some distribution function, $u \sim F(u)$ on $[\underline{u}, \bar{u}]$. Let $\Phi(u)$ be the distribution of rents of an unemployed worker, conditional on having a job. Anonymity means that, for an unemployed worker, we have,

$$\Lambda(u) = 1 - \theta(1 - \Phi(u)) \quad \text{if } u \geq \underline{u} \quad (4.6)$$

Using equation (4.1), anonymity also implies that, for a worker with initial rent u_t ,

$$\begin{aligned} \Lambda(u|u_t) &= 1 - \theta(1 - \Phi(u)) & \text{if } u \geq u_t \\ &= 0 & \text{if } u < u_t \end{aligned} \quad (4.7)$$

Lemma 1. *Suppose rents are distributed according to $F(u)$. Then,*

$$\Phi(u) = \left[\frac{1 - \alpha + \alpha\theta}{1 - \alpha + \alpha\theta F(u)} \right] F(u) \quad (4.8)$$

Proof. Using equation (4.6), the probability an unemployed agent obtains a job offering utility

$u_{t+1} > u$ is,

$$\theta(1 - \Phi(u)) = \frac{(1 - \alpha)n(1 - F(u))}{(1 - n) + nF(u) + (1 - \alpha)n(1 - F(u))} \quad (4.9)$$

The numerator on the right hand side of equation (4.9) is the number of available jobs offering utility more than u . The denominator equals the number of people competing for jobs offering more than u , consisting of the unemployed and those with a job paying less than u . Rearranging yields (4.8). \square

Lemma 1 tells us how workers move between firms each period. Figure 3 provides an illustration. As a corollary, the retention probability for a firm with job u , conditional on forced separation not occurring, is

$$\Lambda(u) = 1 - \theta(1 - \Phi(u)) = \frac{(1 - \alpha)(1 - \theta) + \theta F(u)}{1 - \alpha + \alpha \theta F(u)} \quad (4.10)$$

If there are no atoms in the distribution of rents (see Proposition 3), we have $\lim_{u \rightarrow \bar{u}} \Lambda(u) = 1$ and $\lim_{u \rightarrow \underline{u}} \Lambda(u) = 1 - \theta$, as one would predict.

Table 2 summarises the mass of agents moving between different employment states. For example, without atoms, the number of agents who make direct job-to-job transitions is

$$\begin{aligned} n\alpha \int_{\underline{u}}^{\bar{u}} [1 - \Lambda(u)] dF(u) &= n\alpha \int_0^1 [1 - \Lambda(F^{-1}(q))] dq \\ &= n\alpha \int_0^1 \left[\frac{(1 - \alpha)\theta(1 - q)}{1 - \alpha + \alpha\theta q} \right] dq \\ &= n(1 - \alpha) \left[\frac{(1 - \alpha + \alpha\theta)}{\alpha\theta} \ln \left(\frac{1 - \alpha + \alpha\theta}{1 - \alpha} \right) - 1 \right] \end{aligned} \quad (4.11)$$

This flow is independent of the distribution of utilities because the transition probabilities depend on u only through the rank of the job, $F(u)$. Comparative statics are then straightforward: for example, equation (4.11) implies that the number of transitions is hump-shaped in α . Intuitively, a direct job-to-job transition requires both new vacancies ($\alpha < 1$) and workers to avoid layoffs ($\alpha > 0$).

4.4 Optimal Wage Dispersion

Suppose firms offer contracts inducing rents $u \sim F(u)$ on $[\underline{u}, \bar{u}]$. Let V_0 be the value of unemployment, and $V(u)$ be the value of a job with rents u . Then

$$V_0 = 0 + \delta \left[(1 - \theta)V_0 + \theta \int_{\underline{u}}^{\bar{u}} V(u) d\Phi(u) \right] \quad (4.12)$$

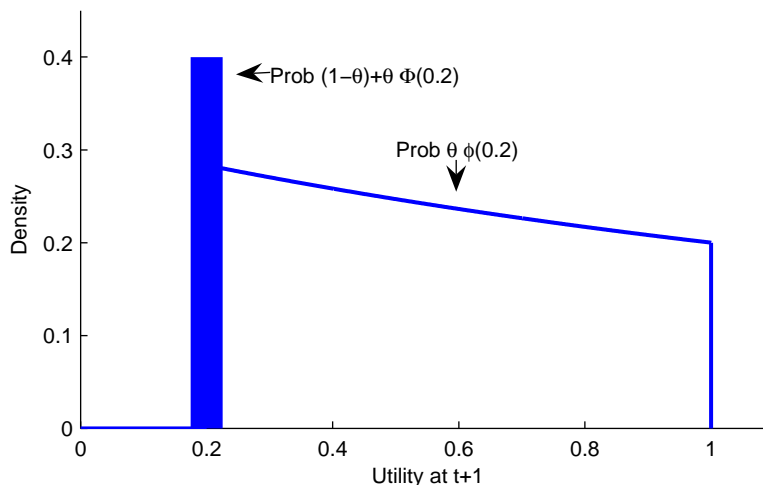


Figure 3: **Distribution of Rents with On-the-Job Search.** This figure illustrates the density of period $t + 1$ rents, given that the workers is in a job paying rents $u_t = 0.2$. It assumes the distribution of rents $f(u)$ is uniform on $[0, 1]$, the mass of firms is $n = 2/5$, the mass of labour is 1, and the breakup rate is $1 - \alpha = 1/2$, yielding $\theta = 1/4$. In this example, the worker stays in the same job with 81% probability.

Transition		Mass
Not laid off	Find new job	$n\alpha \int_{\bar{u}} [1 - \Lambda(u)] dF(u)$
	Stay in same job	$n\alpha \int_{\bar{u}} \Lambda(u) dF(u)$
Laid off	Find new job	$n(1 - \alpha)\theta$
	Stay unemployed	$n(1 - \alpha)(1 - \theta)$
Unemployed	Find new job	$(1 - n)\theta$
	Stay unemployed	$(1 - n)(1 - \theta)$

Table 2: **Mass of Transitions between Jobs.** The left column describes the worker's employment status at the start of the period. The middle column shows the worker's status after the matching process. The right column provides the mass of workers in each category.

Similarly,

$$\begin{aligned} V(u) &= u + (1 - \alpha)V_0 + \alpha\delta \left[\int_{\underline{u}}^{\bar{u}} V(s) d\Lambda(s|u) \right] \\ &= u + (1 - \alpha)V_0 + \alpha\delta \left[\left((1 - \theta) + \theta\Phi(u) \right) V(u) + \theta \int_u^{\bar{u}} V(s) d\Phi(s) \right] \end{aligned} \quad (4.13)$$

Denote the (discounted) probability the relationship continues by

$$\beta(u) := \alpha\delta((1 - \theta) + \theta\Phi(u)) \quad (4.14)$$

If there are no atoms (see below), then we have $\beta(\underline{u}) = \alpha\delta(1 - \theta)$ and $\beta(\bar{u}) = \alpha\delta$ for the lowest and highest firms. Observe that the value function, $V(u)$, is increasing and convex.⁷ As a result, it is continuous and differentiable almost everywhere. Where the derivative exists, it is given by

$$V'(u) = 1 + \beta(u)V'(u) \quad (4.15)$$

Consequently, $V(u)$ is linear where $\beta(u)$ is flat, such as outside $[\underline{u}, \bar{u}]$. The intuition behind equation (4.15) is as follows. Raising u increases the utility in inframarginal states, where the worker obtains a bad job offer, and prevents the agent from quitting in marginal states. With on-the-job search, the latter effect is irrelevant: $V(u)$ does not depend on whether a worker takes a job with firm i or with a competitor when indifferent. Hence an increase in u only raises $V(u)$ through the inframarginal states.

We can now calculate the optimal effort levels. Firm i 's problem is to choose $\langle u, e \rangle$ to maximise profits (3.6) subject to (DEA ^{i}). Observing that $u = w - c(e)$, firm i 's problem is

$$\max_{u, e} pe - u - c(e) \quad \text{s.t.} \quad V(u) \geq u + c(e) + V_0$$

Solving, for any optimal u , yields the following result.

Proposition 3. *There are no atoms in the distribution of utilities. Moreover, any optimal contract $\langle u, e \rangle$ satisfies*

$$c'(e) = p\beta(u) \quad (4.16)$$

Proof. Fix u , and consider the optimal effort. Denote the first-best effort by e^* , so that $c'(e^*) = p$. Denote the necessary utility for first-best by u^* , so $V(u^*) = u^* + c(e^*) + V_0$.

⁷Monotonicity is obvious. For convexity, see Appendix A.2.

The profit-maximising effort is then given by

$$\begin{aligned} e(u) &= c^{-1}(V(u) - u - V_0) && \text{if } u \leq u^* \\ &= e^* && \text{if } u \geq u^* \end{aligned}$$

Intuitively, effort is raised until (DEAⁱ) binds or the firm hits the first-best level.

First, observe that (DEAⁱ) binds in any optimal contract. If $u > u^*$, then the firm can lower u without changing effort, therefore increasing profits. For any $u \leq u^*$, we can write profits as

$$\pi(u) = pe(u) - c(e(u)) - u$$

where $e(u) = c^{-1}(V(u) - u - V_0)$. Moreover, the derivative of $e(u)$, where it exists, is given by

$$e'(u) = \frac{1}{c'(e(u))} \frac{\beta(u)}{1 - \beta(u)}.$$

Second, suppose firm i offers utility $u_0 > 0$, and consider a downwards deviation. Since $\pi(u)$ is continuous, a necessary condition for optimality is that the left derivative,

$$D_u^- \pi(u) \Big|_{u=u_0} = \frac{\beta^-(u_0)}{1 - \beta^-(u_0)} \left[\frac{p - c'(e(u_0))}{c'(e(u_0))} \right] - 1, \quad (4.17)$$

is positive, where $\beta^-(u) := \lim_{\epsilon \rightarrow 0} \beta(u - \epsilon)$. Rearranging, equation (4.17) implies

$$c'(e(u_0)) \leq p\beta^-(u_0). \quad (4.18)$$

Third, suppose firm i offers utility $u_0 < u^*$, and consider an upwards deviation. Since $\pi(u)$ is continuous, a necessary condition for u_0 to be optimal is that the right derivative,

$$D_u^+ \pi(u) \Big|_{u=u_0} = \frac{\beta(u_0)}{1 - \beta(u_0)} \left[\frac{p - c'(e(u_0))}{c'(e(u_0))} \right] - 1, \quad (4.19)$$

is negative. Rearranging, equation (4.19) implies

$$c'(e(u_0)) \geq p\beta(u_0). \quad (4.20)$$

Fourth, we show that $u_0 \in (0, u^*)$. If $u_0 = u^*$, then $c'(e(u_0)) = p > p\beta^-(u_0)$, which contradicts (4.18) and yields a downward deviation. If $u = 0$, then $c'(e(u_0)) = 0 < p\beta(u_0)$, which contradicts (4.20) and yields an upward deviation. As a result, at any optimal u_0 , both (4.18) and (4.20) must hold.

Finally, if there is an atom at u_0 , then $\beta(u_0) > \beta^-(u_0)$, and either (4.18) or (4.20) fails to

hold, yielding a profitable deviation. If there is no atom at u_0 , then (4.18) and (4.20) yield equation (4.16), as required. □

Proposition 3 shows that the optimal effort in any given relationship is determined by the probability the worker remains at the firm. Intuitively, when the retention probability is low, a wage increase has little effect of the future value of the relationship, and only induces a small rise in effort. To be a slightly more formal, suppose firm i increases its wage by ϵ . Since the worker can steal one period's wages, this increases lifetime rents by ϵ and, using equation (4.15), increases per-period rents by $(1 - \beta)\epsilon$. The remainder of the wage increase, $\beta\epsilon$, is spent on extra effort, raising the level by $\beta\epsilon/c'(e)$. Equating marginal costs and benefits, the firm therefore chooses effort so that $p\beta = c'(e)$.

Proposition 3 has the corollary that the highest ranked firm, offering rents \bar{u} , chooses the same effort as a competitive firm without on-the-job search (Section 3.3). In addition, the lowest ranked firm, offering rents \underline{u} , chooses the same effort as a collusive firm without on-the-job search (Section 3.4).

We can now calculate the optimal distribution of wages in three steps.⁸

1. Calculate profits. As in Section 4.2, the dynamic enforcement constraint of the lowest firm is independent of the rents of other firms. Using equation (4.5), profits are

$$\pi(\underline{u}) = \max_e \left\{ pe - \frac{1}{\alpha\delta(1-\theta)} c(e) \right\} \quad (4.21)$$

In equilibrium, $\pi(u) = \pi(\underline{u})$ for all $u \in [\underline{u}, \bar{u}]$.

2. Calculate the effort associated with each quantile of the distribution of rents. Suppose a firm is at quantile $q \in [0, 1]$ of the distribution. Using equation (4.8) and the definition of β ,

$$\beta(q) = \alpha\delta \left[\frac{(1-\alpha)(1-\theta) + \theta q}{1-\alpha + \alpha\theta q} \right] \quad (4.22)$$

Applying Proposition 3, effort is given by $c'(e(q)) = p\beta(q)$.

3. Calculate the distribution of wages using $w(q) = pe(q) - \pi(\underline{u})$. In addition, one can then derive rents from $u(q) = w(q) - c(e(q))$.

In the next section we present an example where the wage distribution is explicitly calculated using these three steps. We first assess the effect of on-the-job search more generally.

⁸There are many other methods to derive wages. We present a related approach in Appendix A.3. One can also use (DEAⁱ) or the second order condition of the profit-maximisation problem, although these techniques require one to solve a differential equation.

Proposition 4. *Suppose firms set wages competitively. Then on-the-job search:*

- (a) *Reduces welfare;*
- (b) *Increases all firms' profits; and*
- (c) *Reduces utility in any job.*

Proof. (a) Under on-the-job search, effort is lower for every agent (Proposition 3). Since welfare is maximised when $c'(e) = p$, welfare is also lower.

(b) Under on-the-job search, profits (4.21) equal those under collusion (3.13).

(c) On-the-job search reduces welfare and increases profits in every job, and therefore reduces utility. \square

Proposition 4 shows that the introduction of on-the-job search reduces the longevity of relationships. This reduces effort and welfare. Proposition 4 also states that on-the-job search increases firms' profits. This is somewhat surprising since on-the-job search increases the outside option of workers, seemingly increasing the competition between firms. However, on-the-job search enables firms to differentiate themselves by posting different wages, opting out of direct competition.

4.5 Example: Quadratic Costs

Suppose that the cost of effort is quadratic, $c(e) = e^2/2$. First, observe that profit of the lowest firm, and hence all firms, is given by

$$\pi(\underline{u}) = \frac{p^2}{2} \alpha \delta (1 - \theta) \quad (4.23)$$

Second, under quadratic costs, the optimal effort is $e(q) = p\beta(q)$.

We can now derive wages. Using the fact that $\beta(0) = \alpha\delta(1 - \theta)$ and $\beta(1) = \alpha\delta$, the highest and lowest wages are:

$$\bar{w} = \frac{p^2}{2} \alpha \delta (1 + \theta) \quad \text{and} \quad \underline{w} = \frac{p^2}{2} \alpha \delta (1 - \theta)$$

Remarkably, the wage spread only depends on market tightness,

$$\frac{\bar{w}}{\underline{w}} = \frac{1 + \theta}{1 - \theta}$$

This means that, as the number of firms increases and the unemployment rate shrinks, job-to-job transitions become more frequent and the spread of wages increases.⁹ The distribution of

⁹In practice the male residual 90–10 spread has varied between 2.63 and 2.95 over the last 30 years (Lemieux (2006, Figure 4)). While we have not specified the length of a period, an unemployment rate and separation rate of 5% implies that $\theta = 19/39$, yielding a wage spread of 2.9.

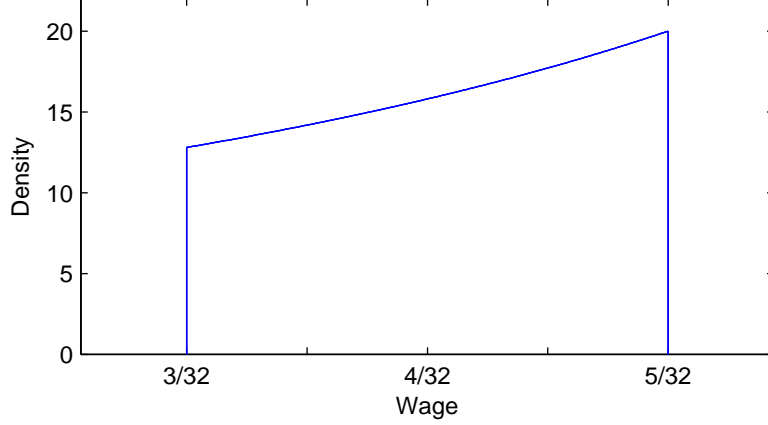


Figure 4: **Distribution of Wages.** This figure illustrates the density of wages in the economy. It assumes the cost of effort is $c(e) = e^2/2$, marginal productivity is $p = 1$, the mass of firms is $n = 2/5$, the mass of labour is 1, the breakup rate is $1 - \alpha = 1/2$, and the discount rate is $\delta = 1/2$.

wages is given by

$$\begin{aligned}
H(w) &= \Pr(pe(q) - \pi(\underline{u}) \leq w) \\
&= \Pr(\beta(q) \leq (w + \pi(\underline{u}))/p^2) \\
&= \left[\frac{1 - \alpha}{\alpha\theta} \right] \left[\frac{2w - p^2\alpha\delta(1 - \theta)}{p^2\delta(2 - \alpha(1 - \theta)) - 2w} \right] \\
&= \left[\frac{w - \underline{w}}{\bar{w} - \underline{w}} \right] \left[\frac{(1 - \alpha)(\bar{w} + \underline{w})}{(\bar{w} + \underline{w}) - \alpha(w + \underline{w})} \right]
\end{aligned} \tag{4.24}$$

where the third line comes from inverting (4.22). Figure 4 provides an illustration.

We can also solve for the distribution of rents.¹⁰ This is given by

$$F(u) = \frac{1 - \alpha}{\alpha\theta} \left[\frac{(1 - \alpha\delta(1 - \theta)) - \sqrt{1 - \alpha\delta(1 - \theta) - 2u/p^2}}{\sqrt{1 - \alpha\delta(1 - \theta) - 2u/p^2} - (1 - \delta)} \right] \tag{4.25}$$

with support

$$\bar{u} = \frac{p^2}{2} \alpha\delta(1 + \theta - \alpha\delta) \quad \text{and} \quad \underline{u} = \frac{p^2}{2} \alpha\delta(1 - \theta)(1 - \alpha\delta(1 - \theta))$$

¹⁰The derivation is easy to do using the approach in Appendix A.3.

4.6 Free Entry of Firms

So far we have taken the measure of firms, n , to be exogenous. Instead, suppose there is free entry. For simplicity, assume costs are quadratic, $c(e) = e^2/2$.¹¹

First, suppose there is no on-the-job search. Using Table 1, each firm's profit is given by

$$\pi = \alpha \delta p^2 \left[\frac{1 - 2\theta}{2 - 2\theta} \right]$$

Firms enter until profits are driven to zero. Free entry thus implies $\theta = 1/2$. That is, mass $n = 1/(2 - \alpha)$ firms enter the industry, implying that mass $(1 - \alpha)/(2 - \alpha)$ of workers are unemployed.

Next, suppose there is on-the-job search. For a fixed θ , profits are given by (4.23). Free entry therefore implies $\theta = 1$. That is, mass $n = 1$ firms enter the industry. The introduction of on-the-job search thus replaces an unemployment problem with an underemployment problem. Indeed, when $n = 1$, the worst-paying firm loses its employee every period and chooses effort $e(\underline{u}) = 0$. Using equation (4.24), wages are then distributed on $[0, \alpha \delta p]$ with distribution and density,

$$H(w) = \frac{(1 - \alpha)w}{\bar{w} - \alpha w} \quad \text{and} \quad h(w) = \frac{(1 - \alpha)\bar{w}}{(\bar{w} - \alpha w)^2}.$$

Proposition 5. *Suppose firms set wages competitively and $c(e) = e^2/2$. Then, under free-entry, average welfare is lower with on-the-job search.*

Proof. See Appendix A.4 □

Even when the number of firms is determined endogenously, the introduction of on-the-job search leads to a reduction in average welfare and, since profits are zero, in average utilities. However, on-the-job search also leads to a decrease in inequality. With on-the-job search utilities are distributed on $[\underline{u}, \bar{u}] = [0, \frac{1}{2}\alpha\delta p^2(2 - \alpha\delta)]$. Without on-the-job search, unemployed workers receive utility \underline{u} , while employed worker receive utility \bar{u} .

5 Productivity Differences

In this section we analyse the impact of different firm productivity on the distribution of wages. Suppose the distribution of productivities, $G(p)$, is atomless and has support $[\underline{p}, \bar{p}]$. Section 5.1 derives the distribution of wages without on-the-job search. Section 5.2 analyses on-the-job search.

¹¹The full employment result does not depend on this assumption.

5.1 Relational Contracts without On-the-Job Search

Firm p chooses a relational contract $\langle w(p), e(p) \rangle$ to solve the following problem:

$$\max_{w(p), e(p)} \pi(p) = pe(p) - w(p) \quad (5.1)$$

$$\text{s.t. } V(u(p)) \geq w(p) + V_0 \quad (\text{DEA}^p)$$

where $V(u)$ is defined by (3.1). As in Section 3.3, profits equal

$$\pi(p) = pe(p) - \frac{1}{\alpha\delta} c(e(p)) - \frac{1-\delta}{\delta} V_0$$

Assuming firm p 's profit is positive, equilibrium effort is given by

$$c'(e(p)) = \alpha\delta p$$

As a result, workers in higher productivity firms exert more effort.

To derive wages, observe that the aggregate dynamic enforcement constraint (3.10) binds at the optimum and $w(p) = u(p) + c(e(p))$. Rearranging,

$$E[u(p)] = \frac{1 - \alpha\delta(1 - \theta)}{\alpha\delta(1 - \theta)} E[c(e(p))]$$

Using the individual dynamic enforcement constraint (3.7) and the value of unemployment (3.5),

$$w(p) = \frac{1}{\alpha\delta(1 - \theta)} \left[(1 - \theta)c(e(p)) + \theta E[c(e(p))] \right] \quad (5.2)$$

Effort in firm i only depends on i 's productivity. In contrast, the wage paid by firm i depends both on i 's productivity and the average productivity in the industry. Intuitively, if firm j experiences an increase in productivity, they pay their workers more in order to work harder. In doing so, firm j raises the value of unemployment, forcing firm i to pay its workers more.

One can easily extend the analysis to allow for free entry of firms. Suppose there is mass 1 of potential entrants, with $p \sim G(p)$ on $[\underline{p}, \bar{p}]$. Also suppose costs are quadratic, $c(e) = e^2/2$. The marginal firm, p_0 , is then uniquely defined by $\pi(p_0) = 0$, where

$$\pi(p_0) = \frac{\alpha\delta}{2(1 - \theta)} \left[(1 - \theta)p_0^2 - \theta E[p^2 | p \geq p_0] \right]$$

In this equation, $n = 1 - G(p_0)$ and market tightness, θ , is defined by (2.1).

5.2 Relational Contracts with On-the-Job Search

Firm p 's problem is to choose contract $\langle w(p), e(p) \rangle$ to maximise profits (5.1) subject to (DEA^p) , where $V(u(p))$ is given by (4.13). Observing that $u = w - c(e)$, firm p 's problem is

$$\max_{u(p), e(p)} pe(p) - u(p) - c(e(p)) \quad \text{s.t.} \quad V(u(p)) \geq u(p) + c(e(p)) + V_0$$

Observe that (DEA^p) will bind in any optimal contract. The resulting profit function, $\pi(u, p)$, is strictly supermodular in (u, p) , so any optimal selection, $u(p)$, is increasing (Topkis (1998, Theorem 2.8.4)). Moreover, $u(p)$ is strictly increasing since Proposition 3 implies that the distribution of rents has no atoms.

We can now derive the distribution of wages in three steps.

1. Calculate effort associated with each productivity level. Using equation (4.8) and the definition of β ,

$$\beta(p) = \alpha\delta \left[\frac{(1-\alpha)(1-\theta) + \theta G(p)}{1-\alpha + \alpha\theta G(p)} \right]$$

since the distribution of rents is given by $F(u(p)) = G(p)$. Applying Proposition 3, effort is determined by $c'(e(p)) = p\beta(p)$

2. Derive profits. Let firm p 's profit under the optimal contract be given by $\pi(p)$. The profits of the lowest productivity firm are given by (4.21). Applying the envelope theorem,

$$\pi(p) = \int_0^p e(s) ds$$

where $e(p) := e(\underline{p})$ for $p \in [0, \underline{p}]$.

3. Calculate the distribution of wages using $w(p) = pe(p) - \pi(p)$. In addition, one can derive rents from $u(p) = w(p) - c(e(p))$.

Figures 5–6 illustrate the wage function $w(p)$ for different distributions of productivities and the resulting wage distribution. An increase in the lowest productivity, \underline{p} , has two effects. First, the average wage level rises, increasing the value of unemployment and forcing any given firm to pay more. For this reason, the highest wage is increasing in \underline{p} . Second, the increase in competition lowers the productivity ranking of any given firm, lowering the probability of retention and reducing effort. Consequently, any given firm pays less when they are the bottom of the distribution than when they are in the middle. Together, these two effects cause the wage function $w(p)$ to become steeper as \underline{p} rises. In the limit, as $\underline{p} \rightarrow \bar{p}$, the distribution converges to that in Section 4.

Finally, when we allow for free entry of firms, we once again obtain full employment. In

equilibrium, incentives are maintained through underemployment rather than through unemployment.

6 Extensions

This section considers several basic extensions to the on-the-job search model in Section 4.

6.1 Restricting On-the-Job Search

The matching process is both efficient and anonymous, implying that on-the-job search is just as effective as off-the-job search. In practice, firms may be able to limit their employees' search activities by banning email and phone calls, and refusing to grant time off for interviews. Firms may also use non-compete clauses to directly limit a worker's movement.

In this model, it is a dominant strategy for each firm to refuse to use such blocking tactics. The reason is as follows. Whether or not firm i allows on-the-job search, its problem is to choose $\langle w, e \rangle$ to maximise profits (3.6) subject to (DEA ^{i}). When firm i allows on-the-job search, the value associated with job $\langle w, e \rangle$ is given by,

$$V(u) = u + (1 - \alpha)V_0 + \alpha\delta \left[(1 - \theta)V(u) + \theta \int_{\underline{u}}^{\bar{u}} \max\{V(u), \hat{V}\} d\hat{\Phi}(\hat{V}) \right]$$

where \hat{V} is the value of another job, which may block on-the-job search, and $\hat{\Phi}(\hat{V})$ is the corresponding distribution function. When firm i bans on-the-job search, the value associated with job $\langle w, e \rangle$ is given by (3.1), which we can write as

$$\tilde{V}(u) = u + (1 - \alpha)V_0 + \alpha\delta\tilde{V}(u)$$

Since $V(u) \geq \tilde{V}(u)$, on-the-job search reduces the cost of eliciting effort and raises firm i 's profits, independent of other firms' decisions to block their employees.

6.2 Competition between Firms

The matching game in Section 4.1 made two assumptions about the form of competition between firms. First, a worker's current employer does not observe, and therefore cannot respond to, a competing offer. Second, a competing firm cannot condition its wage offer on a worker's current employment state. Neither of these assumptions affect the results.

First, suppose when firm i 's worker receives an offer from firm j , then i has the option to match the offer and keep its employee.¹² It is then an equilibrium for firms to offer contracts

¹²One can allow for other protocols, such as a second price auction.

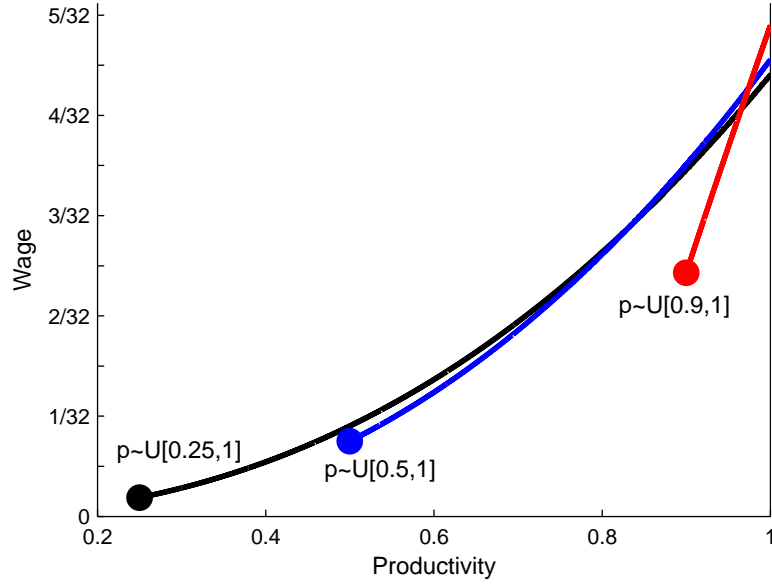


Figure 5: **Wage as a Function of Productivity.** This figure shows $w(p)$ for $p \sim [0, 1]$, $p \sim [0.5, 1]$ and $p \sim [0.9, 1]$. It assumes the cost of effort is $c(e) = e^2/2$, the mass of firms is $n = 2/5$, the mass of labour is 1, the breakup rate is $1 - \alpha = 1/2$, and the discount rate is $\delta = 1/2$.

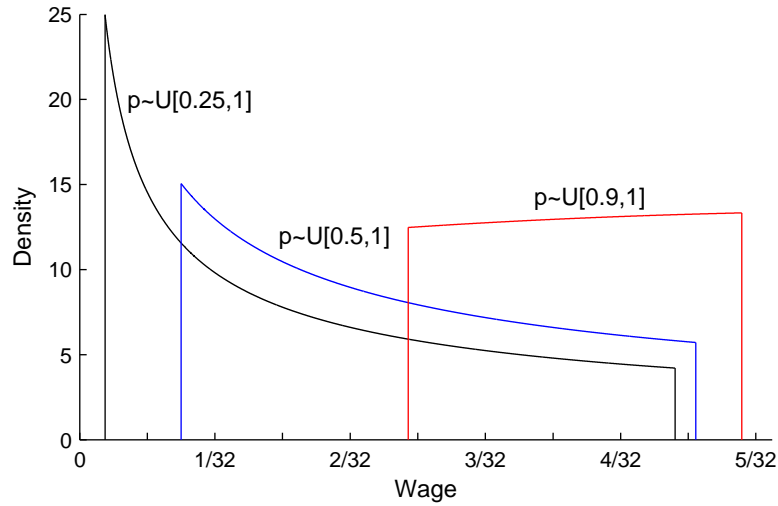


Figure 6: **Distribution of Wages.** This figure shows the distribution of wages for $p \sim [0.25, 1]$, $p \sim [0.5, 1]$ and $p \sim [0.9, 1]$. It assumes the cost of effort is $c(e) = e^2/2$, the mass of firms is $n = 2/5$, the mass of labour is 1, the breakup rate is $1 - \alpha = 1/2$, and the discount rate is $\delta = 1/2$.

as in Section 4, and not to respond to outside offers. The reason is that firms make per-period profits $\pi(\underline{u})$ from any optimal contract. Matching an outside offer thus moves the firm up the distribution of wages, but does not increase profits.

Second, suppose firm j could condition its offer on firm i 's current wage. It is then an equilibrium for firm j to offer a contract from Section 4, ignoring the worker's current status. Again, firm j 's profits are independent of which optimal contract is offered, so there is no need to condition the job offer on a worker's current contract.

The crucial point is that every firm always obtains one worker each period, so there is no need to fight over any single worker. This result can be contrasted to Burdett and Mortensen (1998), where firms make the same profit in expectation, but not ex-post. Consequently, the predictions of the model are very different under complete information (Postel-Vinay and Robin (2002)).

6.3 Tie-Breaking

The matching game in Section 4.1 assumes that a worker stays with his current employer if a competing firm makes an identical offer. Changing the tie-breaking rule does not change the results. In Section 4.2, payoffs from deviating are the same, except that the profit from matching the other firms is given by some combination of $\pi^-(u)$ and $\pi^+(u)$. In Section 4.4, the equilibrium is unaffected since there are no ties to be broken.

7 Conclusion

This paper has studied the optimal self-enforcing contracts in a large anonymous labour market. We have seen that the equilibrium with on-the-job search matches a number of empirical regularities. First, identical firms offer different wages. Second, high-wage firms require high effort, and therefore have higher productivity. Third, high-wage jobs induce higher rents and job satisfaction. This leads to an endogenous job ladder which workers strive to climb. Fourth, incentives are maintained by the fear of underemployment as well as the fear of unemployment. In addition to these major qualitative features, the model makes precise quantitative predictions about the distribution of rents, the wage spread, the mass of employment transitions, and many other economic variables.

The model is very simple, and can be extended in a number of ways. One could introduce size effects, as analysed by Burdett and Mortensen (1998) in the absence of moral hazard. It would be interesting to take account of job-specific human capital accumulation, either paid for by the firm, paid for by the worker, or as a byproduct of tenure. Finally, the model could be extended to allow for different worker abilities or imperfect observability of effort. These topics are left for future work.

A Omitted Material

A.1 Matching with On-the-Job Search

Here we formalise the extensive form game sketched in Section 4.1. This game is analogous to the Gale–Shapley algorithm. First denote the set of firms who initially separate from their workers by A_0 and set $k = 0$.

1. Firms in the set A_k make offers to randomly picked workers.
2. Workers accept or reject the offer. If a worker accepts, he leaves his old firm. If he rejects, he stays with his old firm (or stays unemployed). Let A_{k+1} be the set of firms who have a vacancy at the end of the step.
3. Set $k = k + 1$. Return to step 1.

Under this matching protocol, a worker in job i accepts an offer from firm j if and only if $u_j > u_i$. Since unemployed workers always accept, the measure of unmatched firms, A_k , declines monotonically in k , converging to 0. The limiting outcome is anonymous since every worker receives offers with the same probability.

A.2 Convexity of $V(u)$

Let $u' > u$. Pick $\lambda \in [0, 1]$ and let $\tilde{u} := \lambda u' + (1 - \lambda)u$. The definition of $V(u)$, given by equation (4.13), implies

$$V(u)[1 - \beta(\tilde{u})] = u + (1 - \alpha)V_0 + [\beta(u) - \beta(\tilde{u})]V(u) + \alpha\delta\theta \int_u^{\tilde{u}} V(s) d\Phi(s)$$

Plugging in,

$$\begin{aligned} & \left[\lambda V(u') + (1 - \lambda)V(u) - V(\tilde{u}) \right] \left[1 - \beta(\tilde{u}) \right] \\ &= \lambda \left[[\beta(u') - \beta(\tilde{u})]V(u') - \alpha\delta\theta \int_{\tilde{u}}^{u'} V(s) d\Phi(s) \right] + (1 - \lambda) \left[[\beta(u) - \beta(\tilde{u})]V(u) + \alpha\delta\theta \int_u^{\tilde{u}} V(s) d\Phi(s) \right] \end{aligned}$$

Both terms in square brackets are positive since

$$- \int_{\tilde{u}}^{u'} V(s) d\Phi(s) \geq -V(u')[\Phi(u') - \Phi(\tilde{u})] \quad \text{and} \quad \int_u^{\tilde{u}} V(s) d\Phi(s) \geq V(u)[\Phi(\tilde{u}) - \Phi(u)]$$

As a result, $V(u)$ is convex.

A.3 Alternative Derivation of the Equilibrium

In Section 4.4, we present a three-step approach to derive the distribution of wages. Here we present an alternative approach that expresses variables in terms of utilities rather than quantiles. This method simplifies the derivation of the distribution of rents.

1. Calculate profits. As before, calculate $\pi(\underline{u})$ and observe that $\pi(u) = \pi(\underline{u})$ for all $u \in [\underline{u}, \bar{u}]$.
2. The optimal education level, $e(u)$, is given by the smallest root to the equation:¹³

$$\pi(\underline{u}) = pe(u) - c(e(u)) - u \quad (\text{A.1})$$

Using Proposition 3, the relationship continues with probability $\beta(u) = c'(e(u))/p$.

3. Derive the optimal distribution of rents, $F(u)$. Using equation (4.8) and the definition of β ,

$$F(u) = \left[\frac{1 - \alpha}{\alpha\theta} \right] \left[\frac{\beta(u) - \alpha\delta(1 - \theta)}{\delta - \beta(u)} \right]$$

4. Derive wages using $w(u) = u + c(e(u))$ or $w(u) = pe(u) - \pi(\underline{u})$. Given the distribution of rents, we can thus calculate the distribution of wages.

A.4 Proof of Proposition 5

Without on-the-job search, $\theta = 1/2$ and $e = \alpha\delta p$. Aggregate welfare is thus given by

$$\begin{aligned} W^{NS} &= n[pe - c(e)] \\ &= \frac{1}{2 - \alpha} \left[\alpha\delta p^2 - \frac{1}{2}\alpha^2\delta^2 p^2 \right] \end{aligned} \quad (\text{A.2})$$

With on-the-job search, $\theta = 1$. Using equation (4.25), rents are thus distributed according to

$$F(u) = \frac{1 - \alpha}{\alpha} \left[\frac{1 - \sqrt{1 - 2u/p^2}}{\sqrt{1 - 2u/p^2} - (1 - \delta)} \right]$$

Inverting,

$$F^{-1}(q) = \frac{\alpha\delta q(1 - \alpha + \alpha q - \alpha\delta q/2)}{(1 - \alpha + \alpha q)^2} p^2$$

¹³Given the assumptions on $c(\cdot)$, equation (A.1) has two roots. Only the smaller has the property that $c'(e) \leq p$, which is a necessary condition of the optimum, by Proposition 3.

Aggregate welfare with on-the-job search is then

$$W^S = \int_{\underline{u}}^{\bar{u}} u dF(u) = \int_0^1 F^{-1}(q) dq$$

Observe that,

$$\begin{aligned} \int_0^1 \frac{\alpha \delta q (1 - \alpha + \alpha q)}{(1 - \alpha + \alpha q)^2} dq &= \delta \left[q - \left(\frac{1 - \alpha}{\alpha} \right) \ln(1 - \alpha + \alpha q) \right]_0^1 \\ &= \frac{\delta}{\alpha} \left[\alpha + (1 - \alpha) \ln(1 - \alpha) \right] \end{aligned} \quad (\text{A.3})$$

And,

$$\begin{aligned} \int_0^1 \frac{(\alpha \delta q)^2}{2(1 - \alpha + \alpha q)^2} dq &= \frac{\delta^2}{2\alpha} \left[-2(1 - \alpha) \ln(1 - \alpha + \alpha q) - \frac{(1 - \alpha)^2}{(1 - \alpha + \alpha q)} + \alpha q \right]_0^1 \\ &= \frac{\delta^2}{2\alpha} \left[1 - (1 - \alpha)^2 + 2(1 - \alpha) \ln(1 - \alpha) \right] \end{aligned} \quad (\text{A.4})$$

Subtracting (A.4) from (A.3) and multiplying by p^2 ,

$$W^S = \frac{\delta p^2}{\alpha} \left[(1 - \delta)(1 - \alpha) \ln(1 - \alpha) + (1 - \delta)\alpha + \frac{\alpha^2 \delta}{2} \right] \quad (\text{A.5})$$

Subtracting (A.5) from (A.2) and rearranging, the difference is

$$W^{NS} - W^S = \frac{\delta(1 - \delta)(1 - \alpha)p^2}{\alpha} \left[-\frac{2\alpha}{2 - \alpha} - \ln(1 - \alpha) \right] \geq 0$$

The inequality comes from the fact that

$$\begin{aligned} -\ln(1 - \alpha) &= \alpha + \frac{1}{2}\alpha^2 + \frac{1}{3}\alpha^3 + \frac{1}{4}\alpha^4 + \dots \\ &\geq \alpha + \frac{1}{2}\alpha^2 + \frac{1}{4}\alpha^3 + \frac{1}{8}\alpha^4 + \dots = \frac{2\alpha}{2 - \alpha} \end{aligned}$$

as required.

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