

Optimal Fiscal and Monetary Policy: Equivalence Results

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ABSTRACT

In this paper, we analyze the implications of price setting restrictions for the conduct of cyclical fiscal and monetary policy. We consider standard monetary economies that differ in the price setting restrictions imposed on the firms. We show that independently of the degree or type of price stickiness it is possible to implement the same efficient set of allocations, and that each allocation in that set is implemented with policies that are also independent of the price stickiness. In this sense, environments with different price setting restrictions are equivalent.

Key words: Optimal fiscal and monetary policy; sticky prices; state-contingent debt.

JEL classification: E31; E40; E52; E58; E62; E63.

I. Introduction

How relevant are sticky prices in conducting monetary policy? This question has been at the center of the stabilization policy debate for decades. Recently there has been a renewed interest in it (see Woodford, 2003). We address this question and conclude that, in contrast with the conventional wisdom, once fiscal policy is taken into account, the extent of the nominal rigidity does not matter for the conduct of monetary policy. We show that neither the optimal allocation, nor the optimal policy, depend on the price rigidity. This is the main result of our paper.

We analyze the optimal policy problem in a dynamic general equilibrium model with money and taxes, following the dynamic Ramsey literature after Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991). This approach allows us to study the distortions created by price stickiness in a model where other distortions are present, in particular the need to raise distortionary taxation to finance government expenditures. This is important, since, in general, the optimal way to deal with a distortion depends on other existing distortions.

As a benchmark, we consider a model that is very similar to the perfectly competitive model of Lucas and Stokey (1983) where prices are fully flexible. They consider a stochastic production economy without capital, with cash and credit goods. In their model, government finances exogenous expenditures with revenue from labor income taxes and seigniorage and issues state contingent debt. We depart from Lucas and Stokey by assuming that the fiscal instruments are both income and consumption taxes and that government debt is noncontingent nominal debt. Because we want to consider firms that are price setters, we assume that final consumption is a composite good, aggregating over a continuum of goods using the Dixit-Stiglitz aggregator. Each good is produced by a monopolist and they all share the same labor-only linear technology.

This standard flexible price economy is compared to economies where firms are restricted in

setting prices, but are otherwise identical. In most of the analysis we will consider that a fraction of firms set prices one period in advance but we extend the analysis to other price-setting restrictions such as Calvo (1983), where the effects of monetary policy shocks are persistent. In general, for arbitrarily given monetary and fiscal policies, the equilibrium allocations depend on the extent of price stickiness. As we show, however, this dependence vanishes once optimal policy is considered. In particular, under the optimal policy the allocations are invariant to the extent of the price stickiness. Furthermore, there is a sense in which the optimal policy is also invariant to the extent of the rigidity: as we vary the degree of price stickiness the same policy can implement the optimal allocation.

The set of fiscal policy instruments is crucial for the results. In related work, Siu (2004), Schmitt-Grohe and Uribe (2004), and Benigno and Woodford (2003), find a non-trivial dependence of optimal policy on the extent of price stickiness. They make similar assumptions on the fiscal and debt instruments, except that they only consider one tax, on consumption or labor income. Once we combine the state contingent labor income taxes (available in the models of Siu and Schmitt-Grohe and Uribe) with the consumption taxes (available in the model of Benigno and Woodford), then optimal policy will be independent of the extent of the price stickiness.

With state-contingent consumption and labor income taxes, it is possible to neutralize the effect of the price stickiness, so that any allocation which is implementable under flexible prices is also implementable under sticky prices. This is clear once we show that under flexible prices it is possible to implement each equilibrium allocation with policies that induce a constant producer price level. Under those policies, if restrictions on the setting of prices were to be imposed, such as prices set in advance or staggered prices as in Calvo (1983), they would not have any impact. Those policies can therefore implement under sticky prices the same set of allocations as under

flexible prices.

Under sticky prices, the set of implementable allocations includes the set under flexible prices but it also includes allocations other than those. In particular, because firms may be restricted in different ways in their setting of prices, there are equilibrium allocations such that otherwise identical firms may set different prices and therefore supply different quantities. We show that such allocations are dominated in welfare terms by the allocations under flexible prices, so that the optimal allocation under sticky prices is the same as that under flexible prices.

A more precise way to understand this result is as follows. In the model there is a continuum of firms that use labor to produce imperfectly substitutable goods. These goods can be interpreted as intermediate goods that are aggregated into two composite final goods, a cash and a credit good. Production is efficient when a marginal increase in labor in the production of each of the intermediate goods has the same marginal impact on the production of the final good. In equilibrium this will be the case if the price of each of the intermediate goods is the same. Under flexible prices that will have to be the case, so that the government can only implement production efficient allocations. Under sticky prices, however, it is also possible to implement many production inefficient allocations as well. It is easy to show that the Ramsey allocation under sticky prices must satisfy production efficiency, using an argument similar to one used in Diamond Mirrlees (1971) to prove the optimality of zero taxes on intermediate goods. It follows that the optimal allocations under sticky and flexible prices coincide.

The main policy lesson from our analysis is that, when state-contingent fiscal and monetary policy are decided jointly, (producer) price stability is a requirement of optimality. This is a normative statement that appears to be consistent with the recent practice of central banks in developed economies.

One implication of our results is that the results in the literature on the behavior of optimal allocations under flexible prices, as in Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991), immediately extend to environments with nominal rigidities. The results from the flexible price literature on policies also extend as long as one subtlety is respected. Under flexible prices there are typically a variety of ways to decentralize the optimal allocation. One of these has constant prices while others do not. A policy that, under sticky prices, decentralizes the optimal allocation coincides with the particular one with constant prices that, under flexible prices, decentralizes the optimal allocation.

The paper also extends the literature on optimal monetary policy under sticky prices by explicitly considering both fiscal and monetary policy. The benchmark in that literature (e.g. Clarida, Gali and Gertler (1999), Rotemberg and Woodford 1997) is to assume that the government can tax in a lump sum fashion to finance a subsidy to production that eliminates the markup distortion. It is also common to abstract from the money demand distortion by assuming that the economy is the cashless limit of a sequence of monetary economies. By replicating flexible prices it is possible to eliminate the only remaining distortion, the nominal rigidity, and achieve the first best. In contrast, in our paper we have a number of distortions that cannot be overcome, so that the first best allocations are not feasible.

Another class of papers, aiming to abstract from fiscal policy altogether, allow for lump sum taxes but not for distortionary taxes or subsidies (see Ireland (1996), Adao, Correia and Teles (2003), Khan, King and Wolman, 2003). Since it is no longer possible to eliminate the markup distortion, the problem is then a second best problem¹. For example, Adao, Correia and Teles (2003) show that under sticky prices, with such restrictions on the class of fiscal policies, the optimal flexible price allocation is implementable but not optimal.² As we show in this paper, the

joint consideration of both fiscal and monetary policy reverses this result.

As already mentioned, the most closely related work is by Siu (2004), Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2003). They address the same issues that we do but they impose the restriction that only one tax can be used. With that restriction on the fiscal instruments, it is no longer possible to implement under sticky prices the set of allocations under flexible prices. The result is a third best situation in which the extent of the nominal rigidity matters both for optimal policies and allocations.

The paper proceeds as follows: In Section II, we describe the model economy. We assume that a share of firms set prices one period in advance. In Section III we characterize the sets of implementable allocations and policies. We show that the degree of rigidity is irrelevant in determining both the optimal allocation and the policies that implement it. In Section IV, we interpret the results by allowing for good specific taxes, that affect the wedges between the differentiated goods in the same way as policy under sticky prices. We relate the results to the standard result of optimality of production efficiency in Diamond and Mirrlees (1971). In Section V we show that the results are robust to alternative price setting restrictions. We also show that, if restrictions on the tax instruments were to be imposed, the results would not hold, as in related work by Siu (2004) and others. Section VI contains concluding remarks.

II. The economy

The economy is inhabited by identical households, a continuum of firms indexed by $i \in [0, 1]$, and a government. Time is discrete and in each period $t = 1, 2, \dots$, one of finitely many events $s_t \in S_t$ occurs. The history of events up to period t , (s_0, s_1, \dots, s_t) , is denoted by $s^t \in S^t$ and the initial realization s_0 is given. Let $\pi(s^t)$ be the probability of occurrence of state s^t .

Each firm uses labor $n_i(s^t)$ to produce a distinct perishable good $y_i(s^t)$ that can be used for private consumption as a cash good $c_{1i}(s^t)$ or as a credit good $c_{2i}(s^t)$, or for public consumption $g_i(s^t)$. The technology is given by

$$c_{1i}(s^t) + c_{2i}(s^t) + g_i(s^t) = y_i(s^t) = A(s^t)n_i(s^t) \quad (1)$$

where $A(s^t)$ is the technology parameter that is common across goods.

Households draw utility from composite cash, $C_1(s^t)$, and credit goods, $C_2(s^t)$; and disutility from labor $N(s^t)$, according to the following function that has the standard properties:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C_1(s^t), C_2(s^t), N(s^t)), \quad (2)$$

with

$$C_1(s^t) = \left[\int_0^1 c_{1i}(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \quad (3)$$

$$C_2(s^t) = \left[\int_0^1 c_{2i}(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \quad (4)$$

and

$$N(s^t) = \int n_i(s^t) di. \quad (5)$$

Aggregate government purchases $G(s^t)$,

$$G(s^t) = \left[\int_0^1 g_i(s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \quad (6)$$

are exogenous and must be financed with state contingent consumption taxes, $\tau^c(s^t)$, taxes on labor

income, $\tau^n(s^t)$, on profits, $\tau^d(s^t)$ and by printing money, $M(s^t)$. We will restrict public nominal debt to be of one period maturity and to be state-noncontingent.

Note that each good enters with the same weight in the Dixit-Stiglitz aggregators. In addition, the technology shock is the same for all goods. These symmetry assumptions are standard in the literature.

For simplicity only, we assume that profits are fully taxed, $\tau^d(s^t) = 1$, and that initial wealth is zero, which is equivalent to assuming that it is also fully taxed. Our results are unchanged if we assume that there are bounds, which may be zero, on both the tax rates on profits and on initial wealth. With such bounds, however, the proofs follow logic similar to that used here but are more cumbersome.

Briefly, here the revenue for the full taxation of profits equals the revenue needed to finance the implicit subsidy to labor needed to eliminate the monopoly distortion. This implicit subsidy takes the form of having taxes on labor being lower than they otherwise would be without it. It turns out that with such taxes and implicit subsidies the optimal allocations in this economy with imperfect competition equal those in an otherwise identical economy with perfect competition. Without such profit taxation, this equivalence breaks down but our results do not. (See Correia, Nicolini and Teles, 2002, for details.)

Finally, it is worth noting that even with our simplifying assumption that profits and initial wealth are fully taxed, there is still the need to raise revenues through distortionary taxes.

Households The households start period t with nominal wealth $\mathbb{W}(s^t)$. They decide to buy money balances $M(s^t)$, riskless nominal bonds $\bar{B}(s^t)$ that pay $R(s^t)\bar{B}(s^t)$ units of money one period later, and $B(s^{t+1})$ units of state-contingent nominal securities. Here the only role of this

state-contingent debt is to define state-contingent prices. We assume that the state-contingent bonds are only traded among consumers so that they play no role in allocations. These bonds pay one unit of money at the beginning of period $t + 1$ in state s^{t+1} , and cost $Q(s^{t+1}|s^t)$ in units of money in state s^t .

Thus, the purchases of assets by the household must satisfy

$$M(s^t) + \bar{B}(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) B(s^{t+1}) \leq \mathbb{W}(s^t) \quad (7)$$

At the end of the period, the households receive labor income $W(s^t)N(s^t)$, where $W(s^t)$ is the nominal wage. Note that households do not receive profits from the firms because they are fully taxed. If we let $p_i(s^t)$ be the producer price of good i in units of money and

$$p_i^c(s^t) \equiv (1 + \tau^c(s^t))p_i(s^t)$$

be the price of good i gross of consumption taxes, the evolution of nominal wealth is governed by

$$\begin{aligned} \mathbb{W}(s^{t+1}) = & R(s^t)\bar{B}(s^t) + B(s^{t+1}) + M(s^t) - \int_0^1 p_i^c(s^t)c_{1i}(s^t)di \\ & - \int_0^1 p_i^c(s^t)c_{2i}(s^t)di + (1 - \tau^n(s^t))W(s^t)N(s^t), \text{ for } t \geq 0. \end{aligned} \quad (8)$$

Money, $M(s^t)$, is used to purchase consumption of the cash good, $C_1(s^t)$, according to the cash-in-advance constraint

$$P^c(s^t)C_1(s^t) \leq M(s^t), \quad (9)$$

where $P^c(s^t)$ is

$$P^c(s^t) = \left[\int (p_i^c(s^t))^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad (10)$$

which is the money cost to buy one unit of the composite goods.

Households choose the sequence $\{c_{1i}(s^t), c_{2i}(s^t), N(s^t)\}_{t=0}^{\infty}$ that maximizes utility, (2), satisfying (3), (4), (7), (8) and (9). The following are necessary marginal conditions:

$$\frac{c_{1i}(s^t)}{C_1(s^t)} = \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta}, \quad j = 1, 2, t \geq 0. \quad (11)$$

$$\frac{c_{2i}(s^t)}{C_2(s^t)} = \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta}, \quad j = 1, 2, t \geq 0. \quad (12)$$

that determine the demand for each good as a function of the relative price and the consumption of the composite good;

$$\frac{u_{C_1}(s^t)}{u_{C_2}(s^t)} = R(s^t) \geq 1, \quad t \geq 0, \quad (13)$$

that set the marginal rate of substitution between cash and credit goods equal to its relative price distorted by the nominal interest rate. The inequality $R(s^t) \geq 1$ must hold in any equilibrium, since otherwise households could make arbitrarily large profits by issuing bonds and holding money.

The marginal conditions also include the intratemporal condition

$$-\frac{u_{C_2}(s^t)}{u_N(s^t)} = \frac{P^c(s^t)}{(1 - \tau^n(s^t))W(s^t)}, \quad t \geq 0 \quad (14)$$

where it is apparent how consumption and labor income taxes affect the marginal choice between

labor and the credit good as well as the intertemporal conditions

$$\frac{u_{C_1}(s^t)}{P^c(s^t)} = \beta R(s^t) E_t \left[\frac{u_{C_1}(s^{t+1})}{P^c(s^{t+1})} \right], t \geq 0, \quad (15)$$

$$Q(s^{t+1}|s^t) = \beta \pi(s^{t+1}|s^t) \frac{u_{C_1}(s^{t+1})}{u_{C_1}(s^t)} \frac{P^c(s^t)}{P^c(s^{t+1})}, t \geq 0, \quad (16)$$

for the optimal choice of the one-period-ahead state-contingent and noncontingent nominal assets.

Clearly, these last two equations imply that holding a risk free nominal bond must be equivalent to holding a full array of one period state contingent bonds, or

$$\frac{1}{R(s^t)} = \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t), t \geq 0,$$

Let $Q(s^r|s^t) = Q(s^{t+1}|s^t) \cdots Q(s^r|s^{r-1})$ be the price of one unit of money at s^r in units of money at s^t . Optimality also requires that the following transversality condition

$$\lim_{T \rightarrow \infty} \sum_{s^{T+1}|s^t} Q(s^{T+1}|s^t) [M(s^{T+1}) + \bar{B}(s^{T+1})] = 0$$

holds. Then, the budget constraints can be written with equality as

$$\begin{aligned} & \sum_{r=t}^{\infty} \sum_{s^r} \frac{Q(s^r|s^t)}{R(s^r)} [P^c(s^r) (C_1(s^r) + C_2(s^r))] + \\ & \sum_{r=t}^{\infty} \sum_{s^r} \frac{Q(s^r|s^t)}{R(s^r)} [M(s^r) (R(s^r) - 1) - (1 - \tau^n(s^r)) W(s^r) N(s^r)] \\ & = \mathbb{W}(s^t). \end{aligned} \quad (17)$$

We can replace in these budget constraints the intertemporal prices $Q(s^r|s^t)$, and use the

intertemporal conditions (15), the intratemporal conditions (14) and (13), and the cash-in-advance constraints (9) with equality to write the budget conditions as

$$\begin{aligned}
E_t \sum_{r=t}^{\infty} \beta^{r-t} \{u_{C_1}(s^r) C_1(s^r) + u_{C_2}(s^r) C_2(s^r) + u_N(s^r) N(s^r)\} \\
= u_{C_1}(s^t) \frac{\mathbb{W}(s^t)}{P^c(s^t)}, t \geq 0
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\mathbb{W}(s^t) = & M(s^{t-1}) + R(s^{t-1}) \bar{B}(s^{t-1}) \\
& - P^c(s^{t-1}) \left[C_1(s^{t-1}) + C_2(s^{t-1}) - \frac{u_N(s^{t-1})}{u_{C_2}(s^{t-1})} N(s^{t-1}) \right]
\end{aligned}$$

for $t \geq 1$, and $\mathbb{W}_0 = 0$. Note that in the expression for $\mathbb{W}(s^t)$, the level of state-contingent assets $B(s^t)$ is set equal to zero.

It is worth noting that the implementability conditions (18) do not depend on the price setting restrictions. See Correia, Nicolini and Teles, (2002) for the case where profits are not fully taxed.

Note also that equation (18) is the standard implementability constraint. When the government is allowed to issue state contingent debt, like in Lucas and Stokey (1983), the constraints from period one on are satisfied by the choice of the supply of state-contingent assets, for all s^t . Here instead that supply is zero.

The Government Given the exogenous evolution of aggregate expenditures, $G(s^t)$, and the prices, $p_i^c(s^t)$, the government minimizes expenditure $\int_0^1 p_i^c(s^t) g_i(s^t) di$ needed to obtain $G(s^t)$

given by (6) by deciding according to

$$\frac{g_i(s^t)}{G(s^t)} = \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta}. \quad (19)$$

Given $\tau^d(s^t) = 1$ for all s^t , a government policy consists of public consumption on each good, $g_i(s^t)$, money supplies, $M(s^t)$, taxes, $\tau^c(s^t)$, $\tau^n(s^t)$, the nominal interest rates $R(s^t)$ and debt supplies, $\overline{B}^g(s^t)$ for $t \geq 0$, all states $s^t \in S^t$.

If the budget constraint of the households and the market clearing conditions hold, the budget constraint of the government also holds.

Firms Each good $i \in [0, 1]$ is produced by a monopolist that faces a constant elasticity demand function

$$y_i(s^t) = \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta} Y(s^t), \quad (20)$$

obtained from the demand functions for the private and public goods, (11), (12) and (19), where $Y(s^t) = C_1(s^t) + C_2(s^t) + G(s^t)$. There are two types of firms: *sticky price firms* $i \in [0, \alpha]$, with $\alpha \in [0, 1)$, set prices one period in advance; the remaining $(1 - \alpha)$ *flexible price firms* $i \in (\alpha, 1]$ choose prices contemporaneously. The flexible price firms choose prices to maximize the nominal value of profits at each date $t \geq 0$,

$$p_i(s^t) y_i(s^t) - W(s^t) n_i(s^t)$$

given the technology (1) and the demand function (20) where $p_i^c(s^t) = (1 + \tau^c(s^t)) p_i(s^t)$.

As all monopolists face the same demand curve and have the same technologies, all set a common price

$$p_i(s^t) = p_f(s^t) = \frac{\theta}{\theta - 1} \frac{W(s^t)}{A(s^t)}, \text{ for all } i \in [0, \alpha] \quad (21)$$

equal to a constant markup over marginal cost.

The firms that set prices one period in advance, sell the output on demand at the pre-set prices. In period 0 they charge an exogenously given price p_{-1} . For $t \geq 1$, they choose a period t price $p_i(s^{t-1})$ that does not depend on contemporaneous shock, to maximize profits³

$$\sum_{s^{t+1}|s^{t-1}} Q(s^{t+1}|s^{t-1}) [p_i(s^{t-1}) y_i(s^t) - W(s^t) n_i(s^t)], \quad (22)$$

subject to (1) and (20) where $p_i^c(s^t) = (1 + \tau^c(s^t)) p_i(s^{t-1})$.

The following condition characterizes the solution of this problem

$$\sum_{s^t} \left[\frac{Q(s^t|s^{t-1}) Y(s^t)}{R(s^t)} \left(\frac{P^c(s^t)}{(1 + \tau^c(s^t))} \right)^\theta \left(p_i(s^{t-1}) - \frac{\theta}{\theta - 1} \frac{W(s^t)}{A(s^t)} \right) \right] = 0 \quad (23)$$

for $t \geq 1$.

Using (16) and (13), this condition can be rearranged so that the expression for the common price chosen by the sticky firms for period t , $p_s(s^{t-1})$, is

$$p_i(s^{t-1}) = p_s(s^{t-1}) = \frac{\theta}{\theta - 1} E_{t-1} \left[v(s^t) \frac{W(s^t)}{A(s^t)} \right], \quad t \geq 1, \quad (24)$$

for all $i \in (\alpha, 1]$, where

$$v(s^t) = \frac{u_{C^2}(s^t) (1 + \tau^c(s^t))^{-\theta} (P^c(s^t))^{\theta-1} A(s^t) N(s^t)}{E_{t-1} \left[u_{C^2}(s^t) (1 + \tau^c(s^t))^{-\theta} (P^c(s^t))^{\theta-1} A(s^t) N(s^t) \right]}.$$

so that the sticky firms charge a mark-up over the expected value of a weighted marginal cost. This condition implies that the price set by the firms one period in advance is a weighted conditional average of the price set by the flexible price firms,

$$p_s(s^{t-1}) = E_{t-1} [v(s^t) p_f(s^t)], \quad t \geq 1. \quad (25)$$

Market clearing Demand must be equal to supply for each good i

$$c_{1i}(s^t) + c_{2i}(s^t) + g_i(s^t) = A(s^t) n_i(s^t) \quad (26)$$

and for labor

$$N(s^t) = \int n_i(s^t) di. \quad (27)$$

The market clearing conditions for the nominal debt markets are

$$\bar{B}(s^t) = \bar{B}^g(s^t), \quad (28)$$

$$B(s^{t+1}) = 0. \quad (29)$$

Equilibria An equilibrium in an economy with $0 \leq \alpha < 1$, given p_{-1} , is an allocation $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$, all s^t , $\{c_{1i}(s^t), c_{2i}(s^t), n_i(s^t)\}_{t=0}^{\infty}$, all $s^t, i \in [0, 1]$, debt levels $\{\bar{B}(s^t), B(s^{t+1})\}_{t=0}^{\infty}$, all

s^t , prices $\{p_i(s^t), P^c(s^t), W(s^t), Q(s^{t+1}|s^t)\}_{t=0}^{\infty}$, all s^t , all i , and policy variables $\{g_i(s^t), G(s^t), M(s^t), \tau^c(s^t), \tau^n(s^t), \bar{B}$, s^t , all i , that solve the problems of the households, the firms, and the government and such that markets clear.

The set of equilibria is characterized by the households marginal conditions, (11), (12), (13), (14), (15), (16), the cash in advance constraints, (9), together with the nonnegativity constraint on the nominal interest rates that can be written as

$$u_{C_1}(s^t) \geq u_{C_2}(s^t)$$

Given p_{-1} , the price setting conditions, (21) and (24), characterize the optimal behavior of the firms. The government purchases public goods according to (19), and chooses the other policy variables, satisfying the budget constraints for every s^t , which, given the market clearing conditions, can be written as the households budget constraints, (18). Finally, the market clearing conditions (26), (27), (28) and (29) must hold.

As we show in the following lemma, in any equilibrium the quantities produced by the flexible price firms are equal across firms and similarly for the sticky price firms.

Lemma 1. *a) In any equilibrium with $\alpha = 0$,*

$$c_{1i}(s^t) = C_1(s^t), \quad c_{2i}(s^t) = C_2(s^t), \quad n_i(s^t) = N(s^t) \text{ for all } i \in [0, 1]$$

b) In any equilibrium with $\alpha \in (0, 1)$,

$$c_{1i}(s^t) = C_1^s(s^t), c_{2i}(s^t) = C_2^s(s^t), n_i(s^t) = N^s(s^t) \text{ for all } i \in [0, \alpha]$$

$$c_{1i}(s^t) = C_1^f(s^t), c_{2i}(s^t) = C_2^f(s^t), n_i(s^t) = N^f(s^t) \text{ for all } i \in (\alpha, 1]$$

Proof:

a) The pricing equation of the monopolists (21) implies that all firms set the same price.

The demand functions (11), (12) imply that quantities will be the same across goods for all $i \in [0, 1]$ and equal to the aggregate.

b) Conditions (21) and (24) imply that all flexible price firms will set a common price and that all sticky price firms also set a common price. Therefore, the demand functions imply that the quantities will be the same across flexible price firms and across sticky price firms, where $C_j^f(s^t)$ and $C_j^s(s^t)$, $j = 1, 2$ denote those common values of consumption of the goods produced by the flexible and sticky price firms, respectively. ■

In the flexible price economies (part a)), as all firms set the same price, the quantities will be the same, so that one unit of labor applied to the production of any of the intermediate goods has the same marginal impact on the production of the final good. This is the condition that guarantees *production efficiency*, meaning that production takes place along the *production possibilities frontier*.

When there are sticky price firms (part b)), there may be equilibria where production of the flexible price firms will be different from production of the sticky price firms. In this case production of the aggregates will be in the interior of the production possibilities set. To see this, notice that if we add up the market clearing conditions for each good i , (26), and use the demand

functions (11), (12), and (19), as well as resource constraints (27), we obtain

$$(C_1(s^t) + C_2(s^t) + G(s^t)) \left[\alpha \frac{C_1^s(s^t)}{C_1(s^t)} + (1 - \alpha) \frac{C_1^f(s^t)}{C_1(s^t)} \right] = A(s^t) N(s^t), \quad (30)$$

where

$$C_1(s^t) = \left[\alpha C_1^s(s^t)^{\frac{\theta-1}{\theta}} + (1 - \alpha) C_1^f(s^t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

It can be shown that $\left[\alpha \frac{C_1^s(s^t)}{C_1(s^t)} + (1 - \alpha) \frac{C_1^f(s^t)}{C_1(s^t)} \right] > 1$ whenever $C_1^s(s^t) \neq C_1^f(s^t)$. When this is the case, production is inefficient.

If, in equilibrium, the flexible price firms were to set the same price as the one set by the sticky price firms, then $C_1^s(s^t) = C_1^f(s^t)$. In this case, also in the sticky price economy there would be production efficiency, and production would be along the production possibilities frontier, described by

$$C_1(s^t) + C_2(s^t) + G(s^t) = A(s^t) N(s^t). \quad (31)$$

III. Implementable allocations and policies

We now characterize the set of implementable allocations under flexible and sticky prices. It turns out that with flexible prices the set of implementable allocations is the same as in Lucas and Stokey (1983). With our set of policy instruments there are multiple ways to decentralize a given allocation in that set. We show that it is always possible to decentralize an allocation with constant prices. This result will be instrumental in proving a major result in the paper, namely, that every allocation under flexible prices can be implemented under sticky prices.

Under flexible prices the set of implementable allocations can be characterized as restrictions on allocations only, independently of prices and taxes. Under sticky prices such a characterization

is not possible, and instead this set is simply defined by all the equilibrium conditions stated above. Because of this we find it convenient to take an indirect route in proving our main propositions.

We show that the set of flexible price allocation is contained in the set of sticky price allocations. We also show that any allocation that is both a flexible price allocation and a sticky price allocation can be implemented by the same policies.

We then consider an artificial Ramsey problem designed to make our proofs simple. This Ramsey problem is defined over the set of all allocations that can be obtained with any relative prices of the differentiated goods. This large set clearly contains the set of sticky price allocations. We establish our result by showing that the optimal allocation in this large set is a flexible price allocation. It follows that the optimal allocation under sticky prices is the same as under flexible prices.

Flexible Prices In the following proposition we characterize the set of implementable allocations and policies under flexible prices.

Proposition 1. (1) *Under flexible prices, the set of implementable allocations for the consumption goods and labor, $\{C_1(s^t), C_2(s^t), N(s^t)\}$, is characterized by the implementability conditions*

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u_{C_1}(s^t) C_1(s^t) + u_{C_2}(s^t) C_2(s^t) + u_N(s^t) N(s^t)\} = 0 \quad (32)$$

$$u_{C_1}(s^t) \geq u_{C_2}(s^t) \quad (33)$$

and the feasibility conditions

$$C_1(s^t) + C_2(s^t) + G(s^t) = A(s^t) N(s^t). \quad (34)$$

(2) Each allocation $\{C_1(s^t), C_2(s^t), N(s^t)\}$ is implemented with a unique path for $\left\{R(s^t), \frac{(1+\tau^c(s^t))}{(1-\tau^n(s^t))}, \frac{W(s^t)}{p_f(s^t)}\right\}_{t=0}^\infty$. If the cash in advance constraint, (9), holds with equality, given the initial money supply, $M(s_0)$, there is also a unique solution for $\{P^c(s^t), \bar{B}^g(s^t), M(s^t)\}_{t=0}^\infty$.

Proof: Consider an allocation $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$, that satisfies the conditions of the proposition. We need to show that there exist prices and policies such that it is an equilibrium allocation. By Lemma 1, consumption and labor input are the same for every good, so (26) and (27) become (34). The households marginal conditions on the choice of cash and credit goods, (13), determine uniquely the nominal interest rates, $\{R(s^t)\}_{t=0}^\infty$, which will be nonnegative because of the constraints (33). For a given $M(s_0)$, $P^c(s_0)$ is determined using the cash-in-advance constraint (9) with equality. Condition (32) is (18) for $t = 0$. Let Φ_t be the number of states in period t , with $\Phi_0 = 1$. For any $t \geq 1$, given the values for $P^c(s^{t-1})$ and $M(s^{t-1})$, there are Φ_{t-1} intertemporal constraints, (15), and Φ_t implementability constraints, (18), to determine $\Phi_t + \Phi_{t-1}$ variables, the price levels gross of consumption taxes and the state-noncontingent debt levels, $P^c(s^t)$ and $\bar{B}^g(s^{t-1})$. The money supply in period t and state s^t can be determined using the cash-in-advance constraint, (9), with equality. Given an initial value for the money supply $M(s_0)$, the whole sequence $\{P^c(s^t), \bar{B}^g(s^t), M(s^t)\}_{t=0}^\infty$ is uniquely determined.

The price setting equations, (21), determine uniquely the real wages, $\{W(s^t)/p_f(s^t)\}_{t=0}^\infty$. From the price level condition, (10), $P^c(s^t) = (1 + \tau^c(s^t))p_f(s^t)$. The households intratemporal conditions, (14), given $\{W(s^t)/p_f(s^t)\}_{t=0}^\infty$, determine, also uniquely, the proportionate tax distortions, $\{(1 + \tau^c(s^t))/(1 - \tau^n(s^t))\}_{t=0}^\infty$. Finally, the prices of the state-contingent debt, $Q(s^{t+1}|s^t)$, will be given by (16). ■

In comparing economies with and without price stickiness, we maintain the same objective function for the government. Thus, the relevant object to compare is the set of implementable allocations in each case. Let Ω^f be the set of implementable allocations with flexible prices, characterized in the previous proposition. In order to compare the case of $\alpha = 0$ with the case in which $\alpha \in (0, 1)$, the natural next step would be to characterize the set of implementable allocations in the second case $\Omega^s(\alpha)$ ⁴ and compare it with the one just characterized. It turns out, however, that it cannot be characterized in terms of allocations only, so that the comparison becomes cumbersome. We therefore take an alternative approach. First, we prove that any allocation under flexible prices can be characterized by a constant price level.

Corollary 1. Each allocation in the set of implementable allocations in Proposition 1 can be implemented with policies such that the price set by the flexible price firms is constant over time and equal to the exogenous price of the sticky firms at time 0, $p_f(s^t) = p_{-1}$.

Proof: Consider an implementable allocation $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$ in the set defined by the implementability and feasibility conditions, (32), (33) and (34). From part (2) of Proposition 1, we know that, given $M(s_0)$, the four variables, $\tau^c(s^t)$, $\tau^n(s^t)$, $p_f(s^t)$ and $W(s^t)$, are restricted by only three restrictions, the values of $(1 + \tau^c(s^t))/(1 - \tau^n(s^t))$, $(1 + \tau^c(s^t))p_f(s^t)$ and $W(s^t)/p_f(s^t)$, for each date and state. The path for one of the variables is unrestricted, and therefore it can be set equal to an arbitrary sequence, e.g. $p_f(s^t) = p_{-1}$ for all s^t . ■

A. Sticky prices

We begin by using Corollary 1 to prove that any implementable allocation under flexible prices can also be implemented under sticky prices, for any value of $\alpha \in (0, 1)$.

Proposition 2. (1) *The set of implementable allocations under flexible prices is a subset of the implementable set under sticky prices for any degree of price stickiness $0 \leq \alpha < 1$.*

(2) *Each allocation in that common set can be implemented with policies that are independent of the degree of price stickiness $0 \leq \alpha < 1$.*

Proof: Let $p_f(s^t) = p_{-1}, t \geq 0^5$. In this case, from (25), the prices of the sticky and flexible price firms coincide, $p_s(s^{t-1}) = p_f(s^t) = p_{-1}, t \geq 1$. The equilibrium conditions, irrespective of $\alpha > 0$, collapse to the ones under flexible prices, in addition to the constraint that the producer price level, $p_f(s^t)$, is constant over time. As stated in Corollary 1, given an initial money supply, $M(s_0)$, there are policies under flexible prices that implement each allocation with a constant price level. Those policies clearly do not depend on the degree of price stickiness $0 \leq \alpha < 1$ ■

The proposition implies that

$$\Omega^f \subset \Omega^s(\alpha).$$

It immediately follows that a Ramsey government under sticky prices cannot do worse than a Ramsey government under flexible prices. We now show that it cannot do better.

As we already mentioned, characterizing the set $\Omega^s(\alpha)$ is cumbersome. We consider a larger set Ω^R of allocations, referred to as the *relaxed set* which contains $\Omega^s(\alpha)$, so that we have

$$\Omega^f \subset \Omega^s(\alpha) \subset \Omega^R.$$

We then show that the optimal allocation in Ω^R is contained in Ω^f . This result obviously implies that the optimal allocation in all three sets is the same.

We define the relaxed Ramsey problem as one in which the choice set is the relaxed set, where relative prices of the differentiated goods are left unrestricted. As we show in Section 4, the relaxed set Ω^R can be interpreted as the set of implementable allocations when good specific consumption taxes can be levied.

By allowing for any configuration of relative prices across differentiated goods, we are considering allocations with very general configurations of equilibrium quantities across goods. When some relative prices are different from one, the aggregation of market clearing conditions (26) is not as straightforward as in the case of flexible prices where all the quantities of the differentiated goods are equal to the aggregate. It is possible, however, to obtain a relationship between aggregate demand and aggregate supply similar to (34) that depends on the relative prices. If we add up the market clearing conditions for each good i , (26), and use the demand functions (11), (12), and (19), as well as resource constraints (27), we obtain

$$(C_1(s^t) + C_2(s^t) + G(s^t)) \int_0^1 \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta} di = A(s^t) N(s^t), \quad (35)$$

The *relaxed set* is defined as the set $\{C_1(s^t), C_2(s^t), G(s^t), N(s^t)\}$ of aggregate allocations such that there exist positive prices $\{p_i^c(s^t), P^c(s^t)\}$ that satisfy the following conditions: *i*) the resource constraint (35) holds, *ii*) the aggregate price is consistent with the individual prices

$$P^c(s^t) = \left[\int (p_i^c(s^t))^{1-\theta} di \right]^{\frac{1}{1-\theta}} \quad (36)$$

and *iii*) the implementability constraint (32) holds.

We then have the following proposition.

Proposition 3. *The Ramsey allocation under sticky prices is the same as the Ramsey allocation under flexible prices, for any degree of price rigidity $0 \leq \alpha < 1$.*

Proof: Consider the problem of choosing a sequence of allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$ and relative prices $\{p_i^c(s^t)/P^c(s^t)\}_{t=0}^{\infty}$ for $i \in [0, 1]$, that maximizes welfare in the relaxed set characterized by the implementability conditions (32), (33) and (35), together with (36).

Clearly, the set of implementable under sticky prices is contained in the relaxed set, $\Omega^s(\alpha) \subset \Omega^R$. The implementability condition (32) was derived using the households' conditions that are the same under flexible and sticky prices. The condition that the nominal interest rate must be positive, (33), must also be satisfied under sticky prices while the resource constraints (27) and (26) and the restrictions on the relative prices (36) are also satisfied under sticky prices.⁶ Hence,

$$\Omega^f \subset \Omega^s(\alpha) \subset \Omega^R.$$

It is straightforward, at this point, to show that the optimal allocation in the relaxed set is contained in the flexible price set. Since $\theta > 1$, the term in the resource constraints

$$D(s^t) = \int_0^1 \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta} di \geq 1$$

and it is minimized subject to (36) when $p_i^c(s^t) = P^c(s^t)$. The Ramsey planner will always minimize this term which is the measure of the resource cost due to production inefficiency. The resource constraints become the ones under flexible prices. Since the other constraints are common to the flexible and relaxed set, the planner will face the same constraints in the two problems and the optimal solutions coincide. Thus, the Ramsey allocation under flexible prices maximizes welfare

in the set of allocations in Ω^R . As $\Omega^s(\alpha) \subset \Omega^R$ for any value of α , it must be the case that the optimal allocation under flexible prices also maximizes welfare in the set of sticky price allocations, $\Omega^s(\alpha)$, for any value of α . ■

We next state a proposition on the irrelevance of sticky prices for the policies that implement the Ramsey allocation.

Proposition 4. The Ramsey allocation can be implemented by policies that do not depend on the degree of price rigidity $0 \leq \alpha < 1$. Under flexible prices, the set of policies that implement the Ramsey allocation is a larger set including policies that would not implement that allocation under sticky prices, for $0 < \alpha < 1$.

Proof: The first statement in the proposition is a direct implication of the previous two propositions. For $0 < \alpha < 1$ the Ramsey allocation is implemented with policies that are associated with a constant price level. Those policies obviously also implement the Ramsey allocation under flexible prices. However under flexible prices, there are other policies, such that the price level moves with contemporaneous information, that also implement the Ramsey allocation. But for these other policies the allocation would violate productive efficiency if some firms were constrained in the setting of prices. ■

So far, we followed the Ramsey tradition in assuming that the government is benevolent and aims at maximizing the utility function of the representative agent. However, our results are more general, as the proof of Proposition 3 makes clear.

Assume government preferences on the final goods are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t F(C_1(s^t), C_2(s^t), N(s^t)) \tag{37}$$

where F is increasing in aggregate consumption and decreasing in aggregate labor and differentiable. Then, the optimal problem can also be solved in two steps, where the first is choosing relative prices to minimize the resource cost due to production inefficiency. The solution, again, will exhibit production efficiency. As a special case, F can coincide with the preferences of the households, u .

IV. Interpreting the results: Optimality of zero taxation of intermediate goods.

Here we relate our results to the work of Diamond and Mirrlees (1971) on the optimal taxation of intermediate goods. To make this connection, we first interpret the individual goods as intermediate inputs in the production function of aggregate consumption. We then allow for different taxes on those goods. In particular, let $\tau_i^c(s^t)$ be the consumption tax levied on good i at state s^t , so that the price of good i gross of consumption taxes is $p_i^c(s^t) = (1 + \tau_i^c(s^t))p_i(s^t)$.

In our benchmark economy without good specific taxes, sticky prices distort the relative prices across goods. The sticky price firms, namely those with $i \leq \alpha$, have a different price from that of the flexible price firms, with $i > \alpha$. Under flexible prices we can replicate these allocations with one tax, $\tau_s^c(s^t)$, for the firms with $i \leq \alpha$, and another, $\tau_f^c(s^t)$, for the remaining firms.

Our result that the optimal allocations under flexible and sticky prices coincide is equivalent to the result in this alternative economy that it is optimal to tax the two sets of goods at the same rate. This latter result is in turn a special case of Diamond and Mirrlees (1971).

In general, if we were to consider good specific taxes under flexible prices, and no restrictions were imposed on those taxes, the set of implementable allocations would be the relaxed set, Ω^R , used in proving Proposition 3. If those same taxes were available under sticky prices, the set of implementable allocations would also be the relaxed set, Ω^R . We show both these results in the Appendix. As we showed in Proposition 3, the optimal allocation in the relaxed set is in the set

under flexible prices without good specific taxes, Ω^f . The optimality of a constant tax across goods, $\tau_i^c(s^t) = \tau^c(s^t)$ for all i , and our equivalence results are applications of the same principle that production efficiency is optimal, under quite general conditions, even in a distorted second best world, as shown by Diamond and Mirrlees (1971).

The good specific taxes are redundant instruments, in the sense that a Ramsey government would not use them as part of the optimal policy. Similarly, when prices are sticky, a Ramsey government that is not allowed to use good specific taxes can implement allocations that are not production efficient, but will choose not to do it. Policy in a sticky price environment partially completes the set of instruments, in the sense made explicit in Chari and Kehoe (1999). However, the extra instrument provided by price stickiness is redundant, the same way the good specific taxes are.

V. Robustness

A. *Alternative price setting restrictions*

We have established the irrelevance of the degree of price stickiness for the optimal choice of allocations and policies in a model with prices set one period in advance. This particular form of price stickiness has obvious limitations for the lack of persistence of a monetary policy shock. In this section we argue that our results are robust to the consideration of alternative forms of price setting restrictions.

Consider, for example, introducing in our model the price setting restrictions suggested by Calvo (1983)⁷. Calvo assumes that firms are restricted from revising prices with some probability $\gamma < 1$. Since there is a continuum of firms, that probability is also the share of firms that are not able to revise the price in a given period. The smaller the probability, the larger the expected

number of periods that a price will remain unchanged. These price setting restrictions introduce heterogeneity between firms, given by the period in which they get the chance to optimally decide on the price. This is the only asymmetry between firms.

Suppose the economy started from a steady state where all the firms charged the same price, p_{-1} . Every period, a fraction of firms get the chance to change the price, so as time passes, the heterogeneity may increase over time. In fact, a result similar to Lemma 1 can be stated, where, given that we assumed all firms start with the same price, in each period t there can be, at most, as many different equilibrium prices as number of periods. In any time period t , the γ fraction of firms that get the chance to change the price will choose the same price, since technologies, demand functions and information sets are the same.

The number of periods t is the maximum number of different prices that can be observed in equilibrium in any period t . It does not imply that, in period t , there will always be t different prices. Policy may be such that the firms that are able to change prices, choose to set the same price as the firms that are restricted not to change prices. Indeed, if the policy was the one described in Corollary 1, such that under flexible prices the price level would be constant and equal to that exogenous initial price, p_{-1} , then all the firms would want to set that same price, when given the chance. The price setting restrictions would not be binding, and therefore under staggered price setting it would be possible to implement the flexible price set of allocations, so that

$$\Omega^f \subset \Omega^{Stag}(\gamma).$$

As in the proof of Proposition 3, for the case with prices set in advance, it is also straightforward to show that the set of implementable allocations in this case of staggered price setting,

$\Omega^{Stag}(\gamma)$ is contained in the relaxed set,

$$\Omega^{Stag}(\gamma) \subset \Omega^R.$$

for any γ^8 . To see this, notice, again, that the implementability condition (32) was derived using the households' conditions that are the same under flexible and sticky prices, whether these are set in advance or staggered. The condition that the nominal interest rate must be positive, (33), must also be satisfied under staggered prices while the resource constraints (27) and (26) and the restrictions on the relative prices (36) are also satisfied under staggered prices⁹.

Since as shown in the proof of Proposition 3, the optimal allocation in Ω^R is in Ω^f , and $\Omega^f \subset \Omega^{Stag}(\gamma) \subset \Omega^R$, it must be that the optimal allocation in $\Omega^{Stag}(\gamma)$ must be in Ω^f . Thus, the result in Proposition 3, that the optimal allocations coincide under flexible and sticky prices, for any degree of price rigidity, follows through. The optimal allocation under staggered price setting is implemented with a policy that must induce a constant price.

These results under Calvo (1983) staggered price setting obviously generalize to other forms of price setting restrictions, such as staggered prices as in Taylor (1980), costly adjustment of prices as in Rotemberg (1982), or state dependent pricing as in Dotsey, King and Wolman (1999).

If the source of nominal rigidity was on the setting of wages, rather than prices, the arguments we make in this paper would follow through, as well. Those restrictions would also be irrelevant for optimal allocations and policies. The key point is that in a world with flexible prices and wages the set of implementable allocations can be decentralized with policies that keep the wages constant over time. This is an implication of Proposition 1, part (2) that establishes that in a flexible environment one of the paths for either the price level, the nominal wage or one of the

taxes, on consumption or labor income, is not pinned down. This implies that each allocation can be implemented with a policy that sets the nominal wage equal to some exogenous constant value, and therefore if restrictions on the setting of wages were to be imposed they would not be binding. The optimality result is also straightforward.

B. Restrictions on fiscal policy instruments.

In order to obtain the equivalence results in this paper it is crucial that both consumption and labor income taxes are used. With only one of those taxes we would not be able to obtain the results, as is the case in the related literature by Siu (2004), Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2003). The optimal policy, and corresponding allocations, would depend on the extent of the rigidity.

The two taxes, on consumption and labor income, are needed to obtain the first, instrumental, result, that the set of implementable allocations under flexible prices is also implementable under sticky prices for any degree and type of price stickiness, including staggered prices as in Calvo (1983). This result is a direct implication of the result in Corollary 1 that the set of implementable allocations under flexible prices can be implemented with constant prices. Looking more closely at the proof of Corollary 1, it is shown there that, except for the initial money supply which is not uniquely pinned down, the policy that implements each allocation with a constant price level is unique. This means that all the policy instruments are necessary to obtain that result, there are no redundant policy instruments.

Suppose, then, that one of the taxes, say the consumption tax, was set equal to zero in every date and state. Under flexible prices, as in Chari, Christiano and Kehoe (1991), it would still be possible to implement the same set of allocations. The set of implementable allocations would still

be characterized by the conditions in Proposition 1, the implementability condition, the condition that the interest rate be positive and the feasibility conditions¹⁰. Instead, under sticky prices, in particular under staggered prices, it would no longer be possible to implement that whole set.

To see this, notice that in order for the feasibility conditions to be the ones under flexible prices, producer prices would have to be constant over time. With constant prices there just wouldn't be enough policy variables to satisfy all the remaining equilibrium conditions. The interest rate would satisfy the marginal conditions between marginal rates of substitution and transformation for cash and credit goods, the labor income tax would satisfy the marginal conditions, also between marginal rates of substitution and transformation, for labor and consumption of the credit good. There would be no instruments to satisfy the intertemporal conditions (15) and the budget constraints (18), so that these would have to be added as additional conditions to the Ramsey problem.

When prices are sticky, the producer price level must be constant across time and states to eliminate the production inefficiency, but on the other hand, the price gross of consumption taxes must vary across states to satisfy the intertemporal condition and replicate real state contingent debt. Without consumption taxes, these objectives are just not compatible. The optimal allocation under sticky prices will not coincide with the optimal allocation under flexible prices and will give lower welfare. The extent of the price rigidity matters for the conduct of policy, both in terms of the allocation that can be achieved and the policies that can implement it.

Siu (2004) and Schmitt-Grohe and Uribe (2004) consider environments that are very similar to the one we consider with minor changes, but assume that consumption taxes are not available. In their numerical examples calibrated to the US post war data, they obtain that the costs of price volatility by far outweigh the gains. Production efficiency is a dominant force, so that the optimal

policy will induce a price level that is nearly constant. As a result the nominal interest rate will be greater than zero and will fluctuate, and debt will not be state contingent. The result that the government will optimally decide not to induce price level volatility in order to replicate state contingent debt is not very surprising given the results in Aiyagari, Marcet, Sargent and Seppala (2002) where it is shown that the welfare costs of debt not being state contingent are low if shocks are relatively small.

Benigno and Woodford (2003) study a very similar economy imposing that labor income taxes are set to zero¹¹. The consumption tax will be necessary to satisfy the marginal condition between labor and consumption of the credit good, so that, again, there will be no policy instruments to satisfy the intertemporal conditions and the budget constraints, respectively (15) and (18). The same exactly happens as in the other papers by Siu and Schmitt-Grohe and Uribe. The price stickiness matters for the conduct of fiscal and monetary policy.

Once the policy instruments in Benigno and Woodford and the ones in Siu and Schmitt-Grohe and Uribe are both considered, the results in those papers are fundamentally changed and we obtain the equivalence results in this paper. The optimal policy will induce producer price stability, so that the allocations will be production efficient.

An intuitive discussion on how the policy instruments implement the optimal allocation is as follows: the intratemporal conditions between cash and credit goods, and between the credit good and leisure are satisfied, respectively, by the choice of the nominal interest rate and the labor income tax, given a path of consumption taxes. The quantity of money must be chosen so as to satisfy the cash-in-advance constraint with a constant producer price level, also given a path of consumption taxes. Given that the price level is constant, the budget constraints and the intertemporal conditions determine how the consumption tax moves across states as well as

the stock of noncontingent nominal debt. The conditional volatility of the consumption tax plays the role of the ex-post volatility of the price level in replicating real state-contingent debt, as in Chari, Christiano and Kehoe (1991). The conditional average of the consumption tax allows the government to break the equality between the real and the nominal interest rates, that would otherwise be implied by perfect price stability.

The optimal policy problem can be solved in two stages, where in the first it is shown that the optimal allocation must be in the set under flexible prices and in a second stage the optimal allocation is found in the set under flexible price. This second stage has been studied extensively in the literature on optimal fiscal and monetary policy under flexible prices.

VI. Concluding remarks

A major issue of recent interest is how should we think differently about conducting monetary policy if we move from a world of flexible prices to a world with sticky prices. A contribution of this paper is to show that the answer to this question depends critically on the available set of fiscal policy instruments.

In models where fiscal policy is restricted, such as Siu (2004) and Schmitt-Grohe and Uribe (2004) that only consider labor income taxes, or Benigno and Woodford (2003) that allow for consumption taxes only, the nominal rigidity matters for the conduct of policy. In those models, both the optimal allocations and policies are affected by the degree and type of price rigidity. Instead, in our set up where we assume that there are both consumption and labor income taxes, it turns out that the optimal allocation and the policy that implements it do not depend on the nominal rigidity.

The basic intuition for our result is the following. In most models with sticky prices, the

price setting restrictions affect different firms differently. For example, in Calvo (1983) only a fraction of firms can change prices in any period. Whenever policy exploits the non neutrality resulting from these price setting restrictions there will be relative price distortions, that can be interpreted as a production inefficiency. As long as there are taxes on the final goods, this production inefficiency will be undesirable, even in a distorted, second best, environment. This result recalls a well known result in the public finance literature, due to Diamond and Mirrlees (1971) on the taxation of intermediate goods. Also in Diamond and Mirrlees (1971) as long as consumption taxes on the final goods are available it is not optimal to tax intermediate goods. We are able to establish a similar result, that in our second best world it is optimal to eliminate distortions in production. That is achieved by pursuing price stability, therefore neutralizing the effects of price setting restrictions, whatever these may be.

We make our point in a simple, and somewhat extreme, form. Given our assumptions on the available government debt instruments, just one period nominal state noncontingent debt, consumption taxes play the role of replicating real state contingent debt, and as a result both optimal consumption and labor income taxes may appear to be very volatile. This volatility can be mitigated and possibly eliminated, if instead, other government bonds were considered, such as bonds of different maturities. As shown by Angeletos (2002) and Buera and Nicolini (2004), real state contingent debt can be replicated with government debt of longer maturities. In any case, there is evidence that the welfare gains from using state contingent debt in response to small shocks are low (see Aiyagari, Marcet, Sargent and Seppala, 2002), so that, even if the optimal policy would exhibit high volatility, the costs of following an alternative policy with much lower volatility would be low.

We consider, as most of this literature, models with sticky prices where the degree of price

stickiness is exogenous. This is not a natural assumption when computing the optimal policies because the price setting restrictions will in general depend on the policy. Considering this, however, would be irrelevant, if, as we show, policy does not depend on those restrictions.

A final remark: We have considered a model with a representative household and made the standard Ramsey assumptions that are no lump sum taxes and that leisure cannot be taxed. We could have instead considered a model with heterogenous agents that differ in their unobservable skill levels, as in Costa and Werning (2002) after Mirrlees (1971). We would expect the same result in that alternative set up, that production distortions are inefficient and that price stability is optimal.

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NOTES

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¹In Khan, King and Wolman (2003), that assume staggered price setting, there is also a trade-off between the money demand distortion and the distortion associated with the price setting restrictions. In order to eliminate the latter the price level will have to be constant over time, resulting in positive and varying nominal interest rates.

²Ireland (1996) considers a special case of preferences, technology, and shocks, where the optimal allocations under flexible and sticky prices coincide.

³The profits in period t are discounted by $Q(s^{t+1}/s^{t-1})$ because they can only be used for consumption in the subsequent period. The firms maximize the value of profits net of taxes. For $\tau^d(s^t) = 1$, the production decisions are indeterminate. We consider the limiting economies as $\tau^d(s^t)$ approaches one.

⁴The notation makes explicit that the set of implementable allocations depends on the degree of price stickiness.

⁵We could prove the proposition, without imposing that the price level be constant over time. It could, for instance grow at a constant and perfectly forecastable rate. Indeed, we could have stated Corollary 1 in a weaker form, and show that there are policies under flexible prices such that the price level does not move with contemporaneous information instead of being constant. We have stated it in this form because it will be used to show that the implementable set under flexible prices can be implemented under sticky prices, also when the price setting restrictions are more restrictive, as in the case of staggered price setting.

⁶The relaxed set also contains allocations that cannot be implemented under sticky prices, since that set allows for general configurations of quantities of each good, and Lemma 1.b implies

that under sticky prices there are only two possible quantities or prices in each state.

⁷Correia, Nicolini and Teles (2002) solve the model with Calvo staggered pricing.

⁸While the price distributions can be more general when prices are staggered, it is not the case that any implementable allocation in the case of prices set in advance can be implemented with Calvo staggered pricing. Thus, in comparing $\Omega^{Stag}(\gamma)$ with $\Omega^{Sti}(\alpha)$, neither is a proper subset of the other.

⁹The relaxed set also contains allocations that cannot be implemented under sticky prices, since that set allows for general configurations of quantities of each good, and Lemma 1.b implies that under sticky prices there are only two possible quantities or prices in each state.

¹⁰The optimal allocation in this set, as shown in an example calibrated to the US post war economy in Chari, Christiano and Kehoe (1991) will be implemented with a very volatile price level. Under sticky prices, a volatile price level is inconsistent with production efficiency.

¹¹They consider a cashless economy so that there is no equilibrium condition similar to the intratemporal condition between cash and credit goods. The nominal interest rate is not uniquely pinned down.

APPENDIX

Appendix 1: Good specific taxes

When consumption taxes can discriminate across goods, they are indexed by i , $\tau_i^c(s^t)$, and the price of good i gross of consumption taxes is $p_i^c(s^t) = (1 + \tau_i^c(s^t))p_i(s^t)$. The equilibrium conditions are the ones stated in Lemma 1, except for the price setting conditions of the firms that set prices in advance, (24). The prices set by those firms can now differ across firms, depending on how they are taxed. They are $p_{si}(s^{t-1})$,

$$p_{si}(s^{t-1}) = \frac{\theta}{(\theta - 1)} E_{t-1} \left[v_i(s^t) \frac{W(s^t)}{A(s^t)} \right], \quad t \geq 1 \quad (\text{A.1})$$

where

$$v_i(s^t) = \frac{u_{C^2}(s^t) (P^c(s^t))^{\theta-1} (1 + \tau_i^c(s^t))^{-\theta} A(s^t) N(s^t)}{E_{t-1} \left[u_{C^2}(s^t) (P^c(s^t))^{\theta-1} (1 + \tau_i^c(s^t))^{-\theta} A(s^t) N(s^t) \right]}.$$

The set of implementable allocations when tax instruments are completed to include good specific taxes is described in the following proposition.

Proposition 5. The set of implementable allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$, and relative prices $\frac{p_i^c(s^t)}{P^c(s^t)}$,¹² with discriminatory consumption taxes under any degree of price rigidity $0 \leq \alpha < 1$, is characterized by the implementability conditions (32), (33), as well as the feasibility conditions

$$(C_1(s^t) + C_2(s^t) + G(s^t)) \int_0^1 \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{-\theta} di = A(s^t) N(s^t) \quad (\text{A.2})$$

where the relative prices $\frac{p_i^c(s^t)}{P^c(s^t)} \geq 0$ satisfy the restriction

$$\left[\int_0^1 \left(\frac{p_i^c(s^t)}{P^c(s^t)} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}} = 1. \quad (\text{A.3})$$

Proof:

The equilibrium conditions restricting the variables $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$, $\{R(s^t) \geq 0, \tau_i^c(s^t), \tau^n(s^t)\}_{t=0}^\infty$ and $\{W(s^t), P^c(s^t), p_i^c(s^t), p_i(s^t), p_{si}(s^t), p_f(s^t)\}_{t=0}^\infty$, are the households' marginal conditions (13), (14), (15), the cash in advance constraints (9), the budget constraints (18), the price setting equations for the flexible price and sticky price firms, respectively, (21) and (A.1), the feasibility conditions (A.2), the restriction for the price level (A.3), $p_i^c(s^t) = (1 + \tau_i^c(s^t)) p_i(s^t)$, $p_i(s^t) = p_{si}(s^t)$ for $i \in [0, \alpha]$, which are the sticky price firms with measure α , $p_i(s^t) = p_f(s^t)$ for $i \in (\alpha, 1]$, as well as the restriction that the nominal interest rate must be nonnegative, which is equivalent to (33).

The equilibrium values for $\{c_{1i}(s^t), c_{2i}(s^t)\}_{t=0}^\infty$ and $\{g_i(s^t)\}_{t=0}^\infty$ are obtained using the demand functions (11), (12) and (19). $\{\bar{B}(s^t), B(s^{t+1})\}_{t=0}^\infty$ are obtained with the market clearing conditions (28) and (29). The prices of the state-contingent debt in zero net supply, $Q(s^{t+1}|s^t)$, are given by (16).

We will now show that the only restrictions on the allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$ are the implementability conditions (32), which is condition (18) for $t = 0$, and (33), the feasibility conditions (A.2) and the conditions for the price level (A.3). The other equilibrium conditions restrict the remaining variables $\{R(s^t), P^c(s^t), \bar{B}^g(s^t), M(s^t), \tau^n(s^t), W(s^t), p_i^c(s^t), \tau_i^c(s^t), p_i(s^t), p_{si}(s^t), p_f(s^t)\}_{t=0}^\infty$.

The households marginal conditions on the choice of cash and credit goods, (13), determine uniquely the nominal interest rates, $\{R(s^t)\}_{t=0}^\infty$, which will be nonnegative because of the constraints (33). For a given $M(s_0)$, $P^c(s_0)$ is determined using the cash-in-advance constraint (9),

assuming it holds with equality. Let Φ_t be the number of states in period t , with $\Phi_0 = 1$. For any $t \geq 1$, given the values for $P^c(s^{t-1})$ and $M(s^{t-1})$, there are Φ_{t-1} intertemporal constraints, (15), and Φ_t constraints, (17), to determine $\Phi_t + \Phi_{t-1}$ variables, the price levels gross of consumption taxes and the state-noncontingent debt levels, $P^c(s^t)$ and $\overline{B}^g(s^{t-1})$. The money supply in period t and state s^t can be determined using the cash-in-advance constraint, (9). Given an initial value for the money supply $M(s_0)$, the whole sequence $\{(P^c(s^t), \overline{B}^g(s^t), M(s^t))\}_{t=0}^\infty$ is uniquely determined.

We now assume that the labor income tax rates are zero, $\tau^n(s^t) = 0$. From the households intratemporal conditions, (14), we obtain the path for the nominal wage, $\{W(s^t)\}_{t=0}^\infty$.

From the price setting conditions for the flexible price firms, (21), we obtain $\{p_f(s^t)\}_{t=0}^\infty$. Given $\{P^c(s^t)\}_{t=0}^\infty$ and $\left\{\frac{p_i^c(s^t)}{P^c(s^t)}\right\}_{t=0}^\infty$, $\{p_i^c(s^t)\}_{t=0}^\infty$ are uniquely determined. From the price setting conditions for the sticky price firms, (A.1), together with $p_i^c(s^t) = (1 + \tau_i^c(s^t))p_i(s^t)$, we obtain $\{\tau_i^c(s^t), p_i(s^t)\}_{t=0}^\infty$.

If, instead, the labor income tax, $\tau^n(s^t)$, was not restricted to be equal to zero, there would be Φ_t degrees of freedom both under flexible and sticky prices. ■