Trade, Diffusion and the Gains from Openness

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Abstract

Building on Eaton and Kortum's (2002) model of Ricardian trade, Alvarez and Lucas (2005) calculate that a small country representing 1% of the world's GDP experiences a gain of 41% as it goes from autarky to *frictionless* trade with the rest of the world. But the gains from *openness*, which includes not only trade but all the other ways through which countries interact, are arguably much higher than the gains from trade. This paper presents and then calibrates a model where countries interact through trade as well as diffusion of ideas, and then quantifies the overall gains from openness and the contribution of trade to these gains. Having the model match the trade data (i.e., the gravity equation) and the observed growth rate is critical for this quantification to be reasonable. The main result of the paper is that, compared to the model without diffusion, the gains from openness are much larger (183%) and the gains from trade are smaller (29%) when diffusion is included in the model. This last result is a consequence of a novel feature of the model, namely that trade and diffusion are substitutes, implying that trade generates smaller gains when diffusion is present.

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1 Introduction

How much does a country gain from its relationship with the rest of the world? Consider for example the recent work by Alvarez and Lucas (2005), who build on Eaton and Kortum's (2002) model of Ricardian trade. According to their quantitative model, a small country like Argentina, which represents approximately 1% of the world's GDP, experiences an income gain of 41% as it goes from autarky to *frictionless* trade with the rest of the world. But the gains from *openness*, which includes not only trade but all the other ways through which countries interact, are arguably much higher than the gains from trade. Even if a country were to shut down trade, it could still benefit from foreign ideas through foreign direct investment (FDI), migration, books, journals, the Internet, etc.

The goal of this paper is to construct and calibrate a model where countries interact through trade and diffusion of ideas, and then to quantify the overall gains from openness and the contribution of trade to these gains. The main result is that the gains from trade are smaller than those quantified by Alvarez and Lucas (29% rather than 41% for a country with 1% of the world's GDP) whereas the gains from openness are relatively large (183% for a country with 1% fo the world's GDP). An implication is that shutting down trade would generate loses that are quite small in comparison to the loses that would arise if the country were to become completely isolated by shutting down *both* trade and diffusion.

Calculating the gains from trade in a model that allows for trade and diffusion represents a significant departure from the standard practice in the literature, which is to consider trade as the only means through which countries interact. This alternative approach has at least two advantages. First, having both trade and diffusion in the model shows that the gains from trade depend on the way in which trade and diffusion interact. In the model I present here, trade and diffusion are substitutes: if a country cannot import a good then it can in principle adopt a foreign technology to produce the good domestically with a lower efficiency loss. This substitution is clearly important: think about all the trade that does not take place because technology diffusion has allowed many countries to satisfy their own demand for thousands of goods. On the other hand, if a country cannot adopt foreign technologies, it can always import the goods produced abroad with those technologies. Since trade and diffusion are substitutes, then shutting down trade in this model leads to smaller losses than in models with no diffusion such as Eaton and Kortum (2002) and Alvarez and Lucas (2005).

A second advantage from studying diffusion and trade together is that one can compare the gains from trade with the overall gains from openness, and this may provide a way to judge whether the numbers are reasonable. The usual reaction of economists to the calculated gains from trade in quantitative models is that they are "too small." Apparently, economists have a prior belief that these gains are much higher, so there has been a search for mechanisms through which trade can have a larger effect, such as scale effects, intra-industry reallocations or gains from increased variety. But the result of this search has been generally disappointing (see Tybout, 2003). This paper suggests that the reason for this may be that the gains from trade are in fact "small," while economists' priors about large gains may in fact be about the overall gains from openness. More importantly, this strategy may have relevant implications for research and policy regarding how countries integrate with the rest of the world. In particular, the result of this paper that the gains from trade appear to be quite small relative to the overall gains from openness suggests that both research and policy should at least partially redirect their attention from trade to all the other ways through which countries interact. More attention should be devoted, for example, to understanding the importance of FDI and migration in the international exchange of ideas, and to think about policies that countries can follow to speed up the adoption of foreign technologies.

In Eaton and Kortum's (2002) model of Ricardian trade with no diffusion, countries gain from openness through specialization according to comparative advantage. In the model I construct here, countries also gain from diffusion of ideas. Both the gains from trade and the gains from diffusion come from the same basic phenomenon, namely the sharing of the best ideas across countries. Consider, for example, Japan's superior technology for producing automobiles. This technology can be shared through trade by having Japan export automobiles or through diffusion by having other countries produce their own automobiles using Japan's technology. In both cases, thanks to the non-rivalry of ideas emphasized by Romer (1990), sharing ideas leads to an increase in worldwide income.

These gains from sharing the best ideas are the same ones that give rise to aggregate increasing returns to scale in models of quasi-endogenous growth such as Jones (1995) and Kortum (1997). Consider Kortum (1997). In the simplest version of this model, the arrival of new ideas is proportional to the population level and the quality of each idea is drawn from an unchanging distribution. The technology frontier at a certain point in time is the set of best ideas available to produce the given set of goods, and the average productivity of the technology frontier determines the income per capita level. A larger economy has more ideas, more ideas imply that the best available technologies are more productive, and this allows the economy to sustain a higher income level. This entails a scale effect in *levels* so that income per capita y is increasing with population L, $y = \beta L^{\eta}$, where β and η are positive constants. Jared Diamond's main argument in his book *Guns, Germs and Steel* can be interpreted as saying that this scale effect from sharing ideas is what allowed large "Eurasia" to attain a superior level of productivity (Diamond, 1997). For our purposes, the relevant implication is that a country can achieve a level of income that is much lower in isolation than sharing ideas in a world of six billion people.

A scale effect of the kind just described is the key element in quasi-endogenous growth models, as it implies that the growth rate is proportional to the growth rate of population, $g = \eta g_L$. This implication allows for a simple calibration, which reveals the magnitude of the gains from openness (in steady state levels). With $g = 1.5\%$ and $g_L = 4.8\%$,[1] the equation $g = \eta g_L$ implies that $\eta = 0.31$, which in turn implies that a country with 1% of the world's population enjoys gains from openness equal to 320% ($100^{0.31} = 4.2$). Using the quasi-endogenous growth model due to Jones (1995), Klenow and Rodríguez-Clare (2005) performed a similar exercise and also found enormous gains from openness. They finished their paper calling for "research documenting... the vehicles of knowledge diffusion," arguing that "trade, joint ventures, FDI, migration of key personnel, and imitation may all play important roles." This paper can be seen as a first step in meeting this challenge.

To explore the role of trade, it is necessary to have a model that is quantitatively consistent with both the observed growth rate and the observed trade volumes. Matching the observed growth rate is essential, since - as shown above - this is what pins down the gains from openness. I build on Eaton and Kortum's (2001) model of trade and growth, which can be seen as an extension of Kortum (1997) to incorporate trade. A key parameter in this model, θ, determines the variability of the distribution of the quality of ideas.[2] If θ is calibrated to match the

[1] The rate of growth of y is the rate of growth of income per worker after subtracting the contribution from increases in average human capital and in the capital-output ratio (see Jones, 2002, and Klenow and Rodríguez-Clare, 2005). The value for g_L comes from the rate of growth of researchers in the G5 countries (West Germany, France, the United Kingdom, the United States, and Japan) from 1950 to 1993, see Jones (2002). Note that this is significantly higher than the 1.1% rate of growth of population observed in the OECD in the last decades because of an increasing share of the population devoted to research. Doing this exercise with a lower g_L would lead to even larger gains from openness.

[2] In Eaton and Kortum (2001) the quality of ideas is distributed Pareto with parameter θ. Thus, the variance of this distribution *increases* as θ *falls*. I instead follow Alvarez and Lucas (2005), who flip this parameter

gravity equation, as is done in Eaton and Kortum (2002), then a puzzle emerges in that the implied growth rate is almost an order of magnitude lower than the one we observe for the OECD countries in the last decades. Alternatively, if θ is calibrated to match the observed rate of growth in the OECD, then the model generates too much trade, since the pattern of comparative advantage is too strong and dominates the estimated trade costs.

There are (at least) two ways to deal with this puzzle: first, by allowing for diffusion of ideas across countries, and second, by allowing for knowledge spillovers.[3] In this paper I focus on diffusion because my main goal is to explore the implications of this phenomenon for the gains from openness. To understand why diffusion may allow the model to match both the gravity equation and the growth rate, note that the excessive volume of trade generated by the high θ needed to match growth of 1.5% per year is dampened when countries can share ideas through diffusion rather than trade. Introducing diffusion into the model leads to a gravity equation with a discontinuous border effect (i.e., trade falls discontinously as trade costs increase from zero) that is not present in Eaton and Kortum (2001, 2002). Estimating θ from this equation leads to $\theta = 0.22$ rather than Eaton and Kortum's $\theta = 0.12$, and this helps to increase the model's implied growth rate from $g = 0.29\%$ to $g = 0.53\%$. But this is still significantly below the observed $g = 1.5\%$.

To increase the model's implied growth rate without affecting its trade implications, I allow for progress and diffusion in ideas that are relevant for non-tradable goods. I refer to these ideas as "NT ideas" to differentiate them from the ideas associated with tradable goods, which I will call "T ideas." Analogously to the role played by θ for T ideas, parameter γ determines the variability of the distribution of the quality of NT ideas. One can then use $\theta = 0.22$ to match the gravity equation, and $\gamma = 0.2$ so that the model generates $g = 1.5\%$.[4] Having this model that is quantitatively consistent with observed growth and trade volumes, I can then calculate the gains from openness and the contribution of trade to these gains. The main result is as stated above: the gains from openness are large (183%), while the gains from trade are in fact smaller than in the model without diffusion (29% rather than 41%).[5]

around and have a higher θ increase the variability of the quality of ideas.

[3] I thank Sam Kortum for suggesting knowledge spillovers as a way to deal with this puzzle.

[4] In the calibrated model the growth rate is $g = (\theta/2 + \gamma)g_L$. Thus, $\theta = 0.22$ and $\gamma = 0.2$ together with $g_L = 4.8\%$ imply $g = 1.5\%$.

[5] These gains from openness of 180% differ from the ones stated above (320%). The reason for this is that the model developed in the paper and its calibration incorporate frictions in the diffusion process that are not present in the simple numerical exercise considered above.

This paper is related to the literature on trade and endogenous growth associated with Grossman and Helpman (1991) and Rivera-Batiz and Romer (1991), among others. This group of papers showed that under some conditions trade or international knowledge spillovers would lead to a higher growth rate thanks to the exploitation of scale economies in R&D at the global level. This is essentially what Jones (1995) called a "strong scale effect," whereby larger markets exhibit higher growth rates. Jones' empirical analysis showed that such a strong scale effect is not consistent with the data, however, so there has been a shift towards quasi-endogenous growth models, where the growth rate is not affected by scale variables. In this paper I focus on this class of models and explore the *quantitative* implications of openness on steady state income levels.

Another related literature is the one that focuses on international technology diffusion.[6] The closest paper is by Eaton and Kortum (1999), who develop and calibrate a model of technology diffusion and growth among the five leading research economies. These authors then perform a counterfactual analysis to see the implications for the U.S. of detaching itself from sharing ideas with the rest of the world. My contribution in this paper is to build a model where one can explore the gains from *both* trade and diffusion.

The rest of the paper is organized as follows. In the next section I lay out the basic model with T ideas and no diffusion to introduce the basic notation and assumptions, and to establish a benchmark agains which to compare the results of the full model. In this section I also show that if θ is calibrated to match trade volumes then the implied growth rate is too low. In Section 3 I present the full model, which entails both technological progress in the production of non-tradables through the introduction of NT ideas, and diffusion for both T ideas and NT ideas. In this Section I calculate the gains from openness and the contribution of trade to these gains. I establish the result discussed above that trade and diffusion are subsitutes, and show that this implies that the gains from trade are lower than in a model with no diffusion. In section 4 I calibrate the model to match trade volumes and the observed growth rate, and in Section 5 I use the calibrated model to quantify the gains from openness and the role of trade. The final section offers some concluding comments and topics for future research.

[6] See Keller (2004) for recent a survey.

2 Trade and growth without diffusion

In this section I first present a model of trade and growth without diffusion based on Eaton and Kortum (2001).[7] I then calibrate an enriched version of the model to compute the gains from trade and the implied growth rate.

2.1 A model of trade and growth

There is a single factor of production, labor, I countries indexed by i, and a continuum of tradable intermediate goods indexed by $u \in [0, 1]$. The intermediate goods are used to produce a final consumption good via a CES production function with an elasticity of substitution $\sigma > 0$. The productivity with which individual intermediate goods are produced (i.e., output per unit of the labor) varies across intermediate goods u and across countries, and this gives rise to trade. Let us focus on a single country for now so that we can momentarily leave aside the use of country subscripts. It is convenient to work with the inverse of productivity. To do so, let $x(u)$ be a parameter that determines the cost of producing intermediate good u at time t. In particular, let the cost of producing such a good be given by $x(u)^{\theta} w$, where w is the wage level. Note that the parameter θ, which will be constant across goods and countries, magnifies the variability of the cost parameter x on the actual cost structure across goods and countries. This parameter will be crucial in the analysis that follows.

At any point in time the cost parameters $x(u)$ are the result of previous research efforts in each country. Following Kortum (1997) and Eaton and Kortum (2001), research is modeled as the creation of ideas, although for simplicity here I assume that this is exogenous. In particular, I assume that there is an instantaneous (and constant) rate of arrival ϕ of new ideas per person. In the concluding section I argue that the main results of the paper would not change significantly if research efforts were endogenous.

Ideas are specific to goods, and the good to which an idea applies is drawn from a uniform distribution in $u \in [0, 1]$. Since this interval has unitary mass, then at time t there is a probability $R(t) \equiv \phi L(t)$ of drawing an idea for any particular good, where $L(t)$ is the population level at time t. This implies that the arrival of ideas is a Poisson process with rate function $\phi L(t)$, so the number of ideas that have arrived for a particular good by time t is distributed

[7] One difference with Eaton and Kortum (2001) is that I assume perfect competition and an exogenous process for the arrival of ideas, whereas they have Bertrand competition and endogenous innovation. As I explain below, this is just to simplify the presentation: the main results would not be affected by endogeneizing innovation.

Poisson with rate $\lambda(t) \equiv \int_0^t R(s)ds$. Again, since the set of goods has unitary mass, then $\lambda(t)$ also represents the total stock of ideas (applying to all goods) at time t. (From here onwards, I will suppress the time index as long as it does not cause confusion.) Assuming that L grows at the constant rate g_L (assumed to be common across countries) then in steady state we must have $\lambda = R/g_L$, so λ also grows at rate g_L.

Ideas for producing a particular intermediate good differ only in terms of a "quality" parameter, and the economy's productivity for intermediate good u is determined by the best idea available for the production of this good. The quality of ideas is *independently* drawn from a distribution of quality which is assumed to be Pareto with support in $[1, \infty]$ and parameter one.[8,9] Letting $x(u)$ be the inverse of the quality of the best idea that has arrived up to time t for good u, then it is easy to show that $x(u)$ is distributed exponentially with parameter λ.[10]

Transportation costs are of the iceberg type, with one unit of a good shipped from country j resulting in $k_{ij} \leq 1$ units arriving in country i. I assume that $k_{ii} = 1$, that $k_{ij} = k_{ji}$, and that the triangular inequality holds (i.e., $k_{ij} \leq k_{ik}k_{kj}$ for all i, j, k).

2.1.1 Equilibrium

Following Alvarez and Lucas (2005), I relabel goods by $x \equiv (x_1, x_2 ... x_I)$ rather than u. The price of good x in country i is then

$$p_i(x) = \min_j \left\{ \left(\frac{w_j}{k_{ij}} \right) x_j^\theta \right\}$$

Letting $s_i(x) \equiv \min_j \left\{ (w_j/k_{ij})^{1/\theta} x_j \right\}$, then $p_i(x) = s_i(x)^\theta$. From the properties of the exponential distribution it follows that $s_i(x)$ is distributed exponentially with parameter ψ_i,[11]

[8] Kortum (1997) shows that the Pareto assumption for the distribution of quality is necessary for there to be a steady state growth path.

[9] Eaton and Kortum (2001) assume that the distribution of quality is Pareto with parameter θ, whereas here I assume instead a Pareto distribution with parameter 1, with θ being a parameter that expands the differences in cost across ideas, as in Alvarez and Lucas (2005). The two approaches are equivalent except that the θ here is the inverse of Eaton and Kortum's θ.

[10] Letting q represent the quality of ideas, then $\Pr(Q \leq q) = H(q) = 1 - 1/q$. Letting v be the quality of the best idea that has arrived up to time t, then using $e^x \equiv \sum_{k=0}^{\infty} x^k/k!$ we get $\Pr(V \leq v) = \sum_{k=0}^{\infty} \left(e^{-\lambda}(\lambda)^k/k!\right) H(v)^k = e^{-\lambda/v}$, and hence $x \equiv 1/v \sim \exp(\lambda)$. There is a discrepancy in that here $v \geq 1$ (because $q \geq 1$) whereas the exponential distribution has range in $[0, \infty[$. As shown by Kortum (1997), this can be safely ignored because quality levels below one become irrelevant as λ gets large.

[11] These properties are: (1) if $x \sim \exp(\lambda)$ and $k > 0$ then $kx \sim \exp(\lambda/k)$; and (2) if x and y are independent, $x \sim \exp(\lambda)$ and $y \sim \exp(\mu)$, then $\min\{x, y\} \sim \exp(\lambda + \mu)$.

where

$$\psi_i \equiv \sum_j \psi_{ij} \text{ and } \psi_{ij} \equiv (w_j/k_{ij})^{-1/\theta}\,\lambda_j \tag{1}$$

Letting p_{mi} be the price index of the final good, then $p_{mi}^{1-\sigma} = \int p_i(x)^{1-\sigma} dF(x)$ and assuming $1+\theta(1-\sigma) > 0$,[12] we get

$$p_{mi} = C_T \psi_i^{-\theta} \tag{2}$$

where $C_T = \Gamma[1+\theta(1-\sigma)]^{1/(1-\sigma)}$, with $\Gamma()$ being the Gamma function.

To determine wages we introduce the trade-balance conditions. As shown by Eaton and Kortum (2002), the average price charged by any country j in any country i is the same, and hence the share of total income in country i spent on imports from country j, D_{ij}, is equal to the share of goods for which country j is the lowest cost supplier in country i. In turn, this share is equal to the probability that $(w_j/k_{ij})x_j^\theta = \min_l\{(w_l/k_{il})x_l^\theta\}$. From the properties of the exponential distribution, this probability is $D_{ij} \equiv \psi_{ij}/\psi_i$. Given that total income in country i is $L_i w_i$, then the trade balance conditions are simply

$$L_i w_i = \sum_j L_j w_j D_{ji} \tag{3}$$

The previous conditions determine a competitive equilibrium. In particular, a competitive equilibrium is a couple of vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mI})$ and $w = (w_1, w_2, ..., w_I)$ such that, together with the vector $(\psi_1, \psi_2, ..., \psi_I)$ that satisfies equations (1) and (2) and $D_{ij} \equiv \psi_{ij}/\psi_i$, the trade balance conditions (3) are satisfied.

2.1.2 Growth and the gains from trade

I now turn to the implications of the model for growth and the gains from trade. Once we choose a numeraire, wages are constant in steady state since all λ_i are growing at the same rate g_L. The growth rate in real wages is then given by the rate of decline in p_{mi}. But from (2) it is clear that p_{mi} falls at rate θg_L, so the growth rate of real wages or consumption is

$$g = \theta g_L \tag{4}$$

[12] The assumption that $1+\theta(1-\sigma) > 0$ entails $\sigma < 1 + 1/\theta$. In principle, I could explore whether this inequality holds given estimates of σ and given the values of θ that I will discuss in the text below. In practice, however, the empirical value of σ depends on the level of aggregation that we use for inputs, which in turn should be determined by the level at which technologies differ in the way specified in the model. Thus, the restriction $1+\theta(1-\sigma) > 0$ must be taken as an assumption for now.

This is a simple version of Kortum (1997). Note in particular that growth of income per capita depends on the growth rate of population (the hallmark of quasi-endogenous growth models) and that a higher θ implies a higher growth rate. The reason for this positive role of θ is that a high θ increases the probability that high-quality ideas will be generated, and such good realizations are what fuel growth in this model.

The gains from trade are determined by the increase in the real wage, w_i/p_{mi}, as a country goes from autarky to trade. In calculating these gains here and in the following sections I focus on the case of frictionless trade because this allows for simpler derivations and because this establishes an upper bound for the gains from trade. The result I derive in the following sections that the gains from trade are small compared to the gains from openness would only be strengthened if instead I calculated the gains from trade as those generated by moving from autarky to trade with realistic trade costs. In autarky $\psi_i = w_i^{1/\theta}\lambda_i$, and plugging into (2) and using $\lambda_i = R_i/g_L$ yields $p_{mi}/w_i = C_T(R_i/g_L)^{-\theta}$. Similarly, with frictionless trade we have $p_m = C_T\left(\sum_j w_j^{-1/\theta}R_j/g_L\right)^{-\theta}$. With $\phi_i = \phi$ then it is easy to show that there is factor price equalization (i.e., $w_i = w_j$ for all i, j), and hence the gains from trade are simply

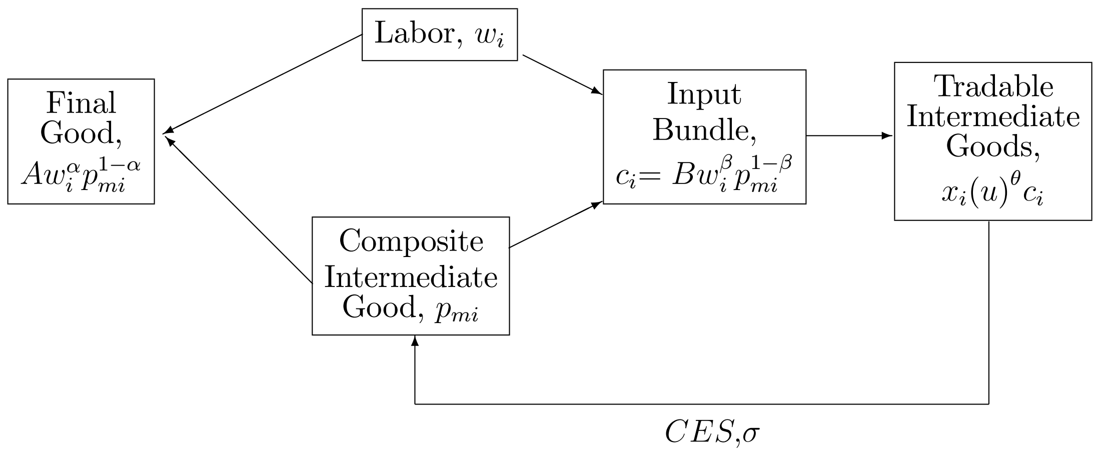
$$GT_i = \left(\frac{\sum L_i}{L_i}\right)^{\theta}$$

Since they generate a smaller share of the world's best ideas, smaller economies have more to gain from integrating with the rest of the world. Moreover, a high θ leads to higher gains from trade. As explained in the Introduction, the reason for this is that a high θ increases the variability of cost differences across countries and hence leads to a stronger pattern of comparative advantage.

2.2 Towards a quantitative model

I now enrich and calibrate the model to explore its quantitative implications. There are two modifications. First, as in Eaton and Kortum (2002) and Alvarez and Lucas (2005) it is assumed that intermediate goods are used in the production of intermediate goods, thus generating a "multiplier" effect that expands the gains from trade and the growth rate. Second, as in Alvarez and Lucas, it is assumed that production of the consumption good uses labor directly and not only through intermediate goods. Alvarez and Lucas introduce this feature to capture the existence of non-tradables that dampen the gains from trade. In the model of this section (but

Figure 1: The Production Structure



not in the one of Section 3) this will also reduce the growth rate because technological progress is confined to tradable intermediates.

These two modifications are illustrated in Figure 1 and captured formally as follows. The intermediate goods are used to produce a "composite intermediate good" with a CES production function with elasticity σ, so that p_{mi} - which above was the price index of the consumption good - is now the price index of this composite good. In turn, the composite good together with labor are used to produce intermediate goods with a Cobb-Douglas production function with labor share β. One can think of an "input bundle" produced from labor and the composite intermediate good that is in turn used to produce all the intermediate goods. The cost of the input bundle in country i is then $c_i \equiv Bw_i^{\beta}p_{mi}^{1-\beta}$ where $B \equiv \beta^{-\beta}(1-\beta)^{\beta-1}$, while the cost of intermediate good u in country i is now $x(u)^{\theta}c_i$. Finally, the consumption good is produced from the composite intermediate good and labor with a Cobb-Douglas technology with labor share α. Thus, the price of the consumption good is $p_i = Aw_i^{\alpha}p_{mi}^{1-\alpha}$, where $A \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. Note that if $\beta = 1$ and $\alpha = 0$ then we are back to the model above. The individual intermediate goods are the only tradeable goods.

These modifications do not substantially affect the qualitative results above; the only difference is that now the wage w_i must be substituted by the unit cost of the input bundle, c_i, in the definition of ψ_{ij} in equation (1).[13] But there are important quantitative implications. In

[13] As shown by Alvarez and Lucas (2005), the trade balance conditions are not affected by the values of α or β (at least for the case in which there are no tariffs, as here).

particular, the growth rate is now

$$g = \theta \left(\frac{1-\alpha}{\beta} \right) g_L \tag{5}$$

If intermediate goods have a high share in the production of intermediate goods (i.e., high $1-\beta$), so that there is a large multiplier $1/\beta$, then the growth rate will be higher. Similarly, the growth rate increases with the share of intermediate goods in the production of the consumption good (i.e., $1-\alpha$). The term $\left(\frac{1-\alpha}{\beta}\right)$ also affects the gains from trade, which in the case of $\phi_i = \phi$ considered above are

$$GT_i = \left(\frac{\sum L_i}{L_i} \right)^{\theta(1-\alpha)/\beta} \tag{6}$$

The key parameters of the model are θ, α, β, and g_L. Eaton and Kortum (2002) estimate θ from the gravity equation generated by the model together with bilateral import and price data for the OECD countries. They focus on the way in which θ determines the impact of trade costs on trade volumes. To isolate this aspect of the gravity equation, Eaton and Kortum focus on "normalized trade flows." Let the normalized bilateral imports of country i from country j be D_{ij}/D_{jj}. If there are no trade costs then $\psi_i = \psi_j$, $p_{mi} = p_{mj}$ and hence $D_{ij}/D_{jj} = 1$ for all i, j. With trade costs we have

$$D_{ij}/D_{jj} = \left(\frac{p_{mj}}{p_{mi} k_{ij}} \right)^{-1/\theta}$$

Taking logs, and letting $m_{ij} \equiv \ln(D_{ij}/D_{jj})$ and $\kappa_{ij} = \ln(p_{mj}/p_{mi}k_{ij})$, then

$$m_{ij} = -(1/\theta)\kappa_{ij} \tag{7}$$

Eaton and Kortum (2002) construct m_{ij} from 1990 data on trade and production of manufactures for 19 OECD countries and κ_{ij} from data on prices from the UN ICP 1990 benchmark study, which gives retail prices for 50 manufactured products in these countries.[14] A simple method of moments estimation of $1/\theta$ in (7) or an OLS regression with no intercept (as required by theory) yields $\theta = 0.12$.

Alvarez and Lucas (2005) calibrate the parameters α and β to match the fraction of U.S. employment in the non-tradables sector and the share of labor in the total value of tradables produced, respectively. They find $\alpha = 0.75$ and $\beta = 0.5$. For g_L I could use the growth rate of

[14] The 19 countries included in the sample are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States.

population in the OECD over the last decades, which is $g_L = 1.1\%$. But as Jones (2002) has emphasized, there has been an upward trend in the share of people devoted to R&D in rich countries over the last decades. According to Jones, the rate of growth of researchers has been 4.8% over the period 1950-1993 in the G-5 countries.[15] Plugging these values in equation (5) together with Eaton and Kortum's $\theta = 0.12$ yields $g = 0.29\%$, which is significantly lower than the observed rate of growth of productivity in the OECD countries, which is close to $g = 1.5\%$. One could, of course, calibrate θ to match the observed growth rate, but this would lead to inconsistent implications for the role of gravity in trade. In particular, bilateral trade volumes would decline too slowly as trade costs increase.

Turning to the gains from trade, these parameters ($\theta = 0.12$, $\alpha = 0.75$, and $\beta = 0.5$) imply from (6) that the gains from frictionless trade for a country with 1% of the world's total population are $100^{0.06} = 1.3$, or 30%.[16] If instead we use the "central value" of θ in Alvarez and Lucas (2005), namely $\theta = 0.15$, then the gains from trade are 41%, as mentioned in the Introduction.

3 Diffusion, trade and growth

In this section I extend the previous model to introduce international diffusion of ideas and make it quantitatively consistent with the observed growth rate and trade volumes. First, I allow for technological progress in the production of non-tradables. In particular, I assume that just as there are ideas that increase the productivity of tradable intermediate goods, there are ideas that increase the productivity of non-tradable consumption goods. I will refer to the first type of ideas as "T ideas" and to the second type of ideas as "NT ideas." (I will supress the T and NT labels except when necessary to avoid confusion.) Second, I allow for international diffusion of both types of ideas. The introduction of NT ideas into the model is necessary to have the model match the observed growth rate, whereas diffusion of both T and NT ideas is a key mechanism for the gains from openness that I want to explore. Moreover, as will be shown, diffusion of T ideas helps to make the model better match the trade data.

[15] The G5 countries are France, West Germany, the United Kingdom, the United States, and Japan. Clearly, an increasing share of people engaged in research implies that the system is not in steady state, but the system can still attain a constant growth rate where these formulas are valid (see Jones, 2002).

[16] Note that if $\phi_i = \phi_j$ for all i, j then with frictionless trade wages are equal across countries, so a country with 1% of the population also has 1% of the world's GDP. For convenience, I used this case to refer to the gains from trade in the Introduction.

To model the role of NT ideas, I assume that there is a continuum of non-tradeable consumption goods indexed by $v \in [0,1]$. These goods enter the representative consumer's instantaneous utility through CES preferences with elasticity of substitution σ.[17] They are produced from labor and the composite intermediate good with a Cobb-Douglas production function at cost $Az(v)^{\gamma} w^{\alpha} p_m^{1-\alpha}$, where $z(v)$ is a cost parameter associated with good v. Analogously to the way in which T ideas determine the cost parameters $x(u)$ for intermediate goods, $z(v)$ is the inverse of the quality of the best NT idea that has arrived for good v. Note that the parameter γ plays the same role in affecting the cost of non-tradeable consumption goods as the parameter θ plays in affecting the cost of the tradeable intermediate goods.

The generation and diffusion of T and NT ideas is assumed to be identical, so I suppress the T and NT labels for now. I assume that the world is composed of two regions: the North and the South. To simplify, I take the South to be a single economy, whereas the North contains n countries. I denote the set of north countries by Ω_N and similarly use Ω_S to denote the (unitary) set of South countries. I use index i for north countries and the indexes j and l for all countries (i.e., $i \in \Omega_N$ and $j, l \in \Omega_N \cup \Omega_S$).

Only countries in the North generate ideas. Ideas at first are "national" (as in the previous section), but then diffuse to other north countries, from which they finally diffuse to the South. Thus, there are $n+2$ pools of ideas: one pool for each north country, a pool of ideas that have diffused among the north countries (the "north ideas"), and the pool of ideas that have diffused to the South. Ideas in the latter pool are available in all countries, so I refer to these ideas as "global ideas."

Following Eaton and Kortum (2006), I assume that diffusion is probabilistic, with each idea having a constant probability of diffusing. Letting δ be the rate of diffusion among countries in the North, and δ' be the rate of diffusion from the pool of north ideas to the pool of global ideas, and letting λ_N and λ_G be the stocks of north and global ideas, respectively, then $\dot{\lambda}_i = \phi_i L_i - \delta \lambda_i$, $\dot{\lambda}_N = \delta \sum \lambda_i - \delta' \lambda_N$, $\dot{\lambda}_G = \delta' \lambda_N$. In steady state the stock of national ideas in north country i is

$$\lambda_i = (\phi_i/(g_L + \delta))L_i \tag{8}$$

while the stock of north ideas is

$$\lambda_N = \widetilde{\delta} \sum \lambda_i \tag{9}$$

[17] The assumption that the elasticity of substitution is the same here as in the production of the composite good is made to minimize notation and plays absolutely no role in the results.

where $\widetilde{\delta} \equiv \delta/(\delta' + g_L)$. Finally, the stock of global ideas in steady state is

$$\lambda_G = \delta' \lambda_N / g_L \tag{10}$$

All these stocks of ideas grow at rate g_L in steady state.

For each good there are n best national ideas (one for each north country), a best north idea, and a best global idea. Letting $x_i(u)$, $x_N(u)$ and $x_G(u)$ be the cost parameters associated with the best idea in north country i, the best north idea, and the best global idea for intermediate good u, then $x_i(u)$, $x_N(u)$ and $x_G(u)$ are exponentially distributed with parameters λ_i, λ_N and λ_G, respectively.

In the case of consumption goods, the fact that these goods are non-tradable implies that - for north countries - we care only about the best idea available in each country, irrespective of whether they are national ideas or not (see below). The cost parameter associated with the best idea available in north country i for good v, $z_i(v)$, is distributed exponentially with parameter be $\lambda_i + \lambda_N + \lambda_G$, which grows at rate g_L in steady state. Similarly, the cost parameter for good v in the South, $z_G(v)$, is determined by the best global idea and is distributed exponentially with parameter λ_G.

3.1 Equilibrium

In equilibrium, consumption goods are sold at cost, hence the price of consumption good v is $Az(v)^{\gamma} w^{\alpha} p_m^{1-\alpha}$. The price index for the consumption bundle is then

$$p = Aw^{\alpha} p_m^{1-\alpha} \left(\int_0^1 z(v)^{\gamma(1-\sigma)} dv \right)^{1/(1-\sigma)}$$

Since $z(v)$ in north country i is distributed exponentially with parameter $\lambda_i + \lambda_N + \lambda_G$, then (assuming that $1 + \gamma(1-\sigma) > 0$ so that the integral above is well defined) we have that the price index for the consumption bundle in i is

$$p_i = Aw_i^{\alpha} p_{mi}^{1-\alpha} C_{NT} \left(\lambda_i + \lambda_N + \lambda_G \right)^{-\gamma} \tag{11}$$

where $C_{NT} \equiv \Gamma(1 + \gamma(1-\sigma))^{1/(1-\sigma)}$. The price index for the South is given by a similar expression with $\lambda_i + \lambda_N + \lambda_G$ replaced by λ_G.

Turning to intermediate goods and trade, each country has $3n + 1$ ways of procuring a good: country l can buy the good from each of the n north countries (including itself if $l \in \Omega_N$)

produced with the exporting country's best national idea or with the best north or global idea, or it can buy the good from the South produced with the best global idea. Labeling intermediate goods by $\widetilde{x} = (x_1, x_2, ..., x_n, x_N, x_G)$, and letting $\widetilde{c}_l \equiv \min_j \{c_j/k_{lj}\}$ and $\widetilde{c}_l^N \equiv \min_i \{c_i/k_{li}\}$ (recall that i necessarily belongs Ω_N), then the price of good $\widetilde{x}$ in country l is

$$p_l(\widetilde{x}) = \min\left\{\min_i \left\{\left(\frac{c_i}{k_{li}}\right) x_i^\theta\right\}, \widetilde{c}_l^N x_N^\theta, \widetilde{c}_l x_G^\theta\right\} \equiv \xi_l(\widetilde{x})^\theta$$

Given the properties of the exponential distribution, ξ_l is distributed exponentially with parameter

$$\hat{\psi}_l \equiv \sum\nolimits_i (c_i/k_{li})^{-1/\theta} \lambda_j + \left(\widetilde{c}_l^N\right)^{-1/\theta} \lambda_N + (\widetilde{c}_l)^{-1/\theta} \lambda_G \tag{12}$$

This parameter determines the price index for intermediate goods in country l. In particular, and analogous to (2), we now have

$$p_{ml} = C_T \hat{\psi}_l^{-\theta} \tag{13}$$

Wages are determined by the trade-balance conditions, as in (3), but the trade shares are now different. To determine these shares, note that $(c_i/k_{li})^{-1/\theta} \lambda_i / \hat{\psi}_l$ is the share of goods which country l can procure most cheaply from i produced with $i's$ best national ideas. The fact that l may also buy north and global goods from i establishes that

$$D_{li} \geq (c_i/k_{li})^{-1/\theta} \lambda_i / \hat{\psi}_l \text{ for } i \in \Omega_N \tag{14}$$

To proceed, let $M_l^N \equiv \arg\min_i \{c_i/k_{li}\}$ denote the set of countries from which country l would buy all goods produced with north ideas (i.e. if country l buys a good produced with a north idea, it must be buying this good from $i \in M_l^N$). The share of goods that country l will actually buy from countries $i \in M_l^N$ produced with north ideas is given by $\left(\widetilde{c}_l^N\right)^{-1/\theta} \lambda_N/\hat{\psi}_l$. The fact that l may also buy global goods from countries $i \in M_l^N$ establishes that

$$\sum_{i \in M_l^N} D_{li} \geq \left(\left(\widetilde{c}_l^N\right)^{-1/\theta} / \hat{\psi}_l\right) \left(\sum_{i \in M_l^N} \lambda_i + \lambda_N\right) \tag{15}$$

Finally, let $M_l \equiv \arg\min_j \{c_j/k_{lj}\}$ denote the set of countries from which l would buy all goods produced with global ideas. If the South were the unique member of M_l then country l would buy all goods produced with global ideas from the South, and then

$$D_{lS} = (c_S/k_{lS})^{-1/\theta} \lambda_G / \hat{\psi}_l$$

In this case (15) would have to be satisfied with equality. If there are north countries in M_l, however, then country l will import from these countries goods produced with national, north and global ideas, and hence

$$\sum_{j\in M_l} D_{lj} = \left(\left(\widetilde{c}_l\right)^{-1/\theta}/\hat{\psi}_l\right)\left(\sum_{i\in M_l\cap\Omega_N}\lambda_i + \chi\left(M_l\cap\Omega_N\right)\lambda_N + \lambda_G\right) \tag{16}$$

where $\chi\left(M_l\cap\Omega_N\right) = 1$ if $M_l\cap\Omega_N \neq \varnothing$ and $\chi\left(M_l\cap\Omega_N\right) = 0$ otherwise.

The competitive equilibrium is determined by the vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mn}, p_{mS})$ and $w = (w_1, w_2, ..., w_n, w_S)$ such that together with the vector $(\hat{\psi}_1, \hat{\psi}_2, ..., \hat{\psi}_n, \hat{\psi}_S)$ that satisfies equations (12) and (13) and the matrix $\{D_{jl}; j, l = 1, 2..., n, S\}$ that satisfies (14) – (16), the trade-balance conditions (3) are satisfied.

In steady state wages are constant, so the common growth rate is given by $g = -\dot{p}_l/p_l$. But equations (12) and (13) imply that p_{ml} declines at rate $\theta g_L/\beta$ (using $c_l = Bw_l^{\beta}p_{ml}^{1-\beta}$), so from (11) we find

$$g = \left[\theta\left(\frac{1-\alpha}{\beta}\right) + \gamma\right]g_L \tag{17}$$

The growth rate is composed of two terms: the first term, $\theta(\frac{1-\alpha}{\beta})g_L$, is associated with technological progress in tradeable (intermediate) goods, whereas the second term, γg_L, is associated with technological progress in non-tradeable (consumption) goods. It is worth noting that the first term is exactly the same as in the model with no diffusion of the previous section (see equation (5)). This reveals that diffusion has no effect on steady state growth in this model; as will become clear below, there is only a level effect.

3.2 Gains from trade and diffusion

I now turn to the derivation of the gains from trade and diffusion of both T and NT ideas. As in the previous section, I consider the gains from *frictionless* trade. To do so, I first derive the real wage for the case of no trade (with and without diffusion), and then for the case of frictionless trade with diffusion. I then compare these wages to establish the gains from trade and diffusion, and discuss several implications from these results.

3.2.1 No trade

When there is no trade, the price index of intermediate goods in country l, p_{ml}, is given by (13) but with

$$\hat{\psi}_l = c_l^{-1/\theta} \eta_l$$

where η_l is the stock of ideas in country l. For each north country i, this stock is composed of ideas originated in i and foreign ideas that have diffused (i.e., foreign ideas that have become north or global ideas), while for the South this stock is composed entirely of global ideas. Thus,

$$\eta_l = \begin{cases} (1 + \delta/g_L)\lambda_l & \text{if no diffusion and } l \in \Omega_N \\ 0 & \text{if no diffusion and } l \in \Omega_S \\ \lambda_l + \lambda_N + \lambda_G & \text{if there is diffusion and } l \in \Omega_N \\ \lambda_G & \text{if there is diffusion and } l \in \Omega_S \end{cases} \tag{18}$$

From (13) we then get

$$p_{mi}/w_i = (BC_T)_i^{1/\beta} \, \eta_l^{-\theta/\beta}$$

Note that if there is no diffusion, then this expression is not well defined for the South since in that case $\eta_l = 0$. From (11) we finally get the real wage for country l, namely

$$w_l/p_l = (AC_{NT})^{-1} \, (BC_T)^{-(1-\alpha)/\beta} \, \eta_l^{\theta(1-\alpha)/\beta+\gamma} \tag{19}$$

3.2.2 Frictionless trade

Turning to the characterization of the equilibrium under frictionless trade, it is convenient to introduce the notions of "national i goods," "north goods," and "global goods" (this is relevant only for intermediate goods). National i goods are those for which the best idea is a national idea in country i (i.e., $x_i = \arg\min\{x\}$); north goods are those for which the best idea is a north idea (i.e., $x_N = \arg\min\{x\}$); and global goods are those for which the best idea is a global idea (i.e., $x_G = \arg\min\{x\}$).

There are three different equilibrium configurations. First, the equilibrium could entail wage equalization across all countries, in which case all countries produce global goods and all north countries produce north goods. Second, the equilibrium could entail wage equalization among north countries and an inferior wage in the South. All north countries would produce north goods, and only the South would produce global goods. Finally, the equilibrium could exhibit three wage levels: a high wage in the north countries with the highest research intensities, a middle wage in the north countries with the lowest research intensities, and a low wage in the

South. North goods would be produced in the north countries with low research intensities, and global goods would be produced in the South. To simplify the exposition, I will focus on the equilibrium with two wage levels: a low wage in the South and a common wage for north countries. This equilibrium is possible even if the research intensity differs among north countries: thanks to diffusion, north countries with low research intensities can specialize in north goods and attain trade balance in spite of the fact that their stock of national ideas per person is relatively low. The key for this equilibrium configuration is that all north countries produce north goods. Under frictionless trade, this requires that the unit cost of the input bundle be equal across countries, i.e. $c_i = Bw_i^{\beta} p_{mi}^{1-\beta}$ for all $i \in \Omega_N$, so that there is indifference about where to buy north goods. I refer to this condition as the ECN condition. Since under frictionless trade we have $p_{ml} = p_m$ for all l, then this condition entails $w_i = w_N$ for all i.

In equilibrium, country i will at least supply the whole world of national i goods. Using (8), (9), and (10), and letting $R_i \equiv \phi_i L_i$ and $R_N \equiv \sum_i R_i$, the share of national i goods among all goods is

$$\lambda_i / (\sum \lambda_i + \lambda_N + \lambda_G) = \frac{R_i / R_N}{1 + \widetilde{\delta} + \delta' / g_L}$$

Given the absence of trade costs and the ECN condition, this is also the share of each country's total spending that will be allocated to buying national i goods from country i. Letting $\phi_N \equiv R_N / L_N$ (with $L_N \equiv \sum_i L_i$) be the average research intensity in the North, then a condition necessary for an equilibrium with wage equalization among north countries is

$$\phi_i / \phi_N \leq 1 + \widetilde{\delta} \tag{20}$$

This inequality ensures that - given the ECN condition - every north country has some resources left over for producing north goods. This requires that ϕ_i / ϕ_N be not too high, for otherwise there would be a country that would have so many national goods that it would not be able to satisfy the world demand for its national goods given the ECN condition, and the equilibrium could not take the form that I have postulated here. Note that the condition is relaxed as $\widetilde{\delta}$ increases. This is because a higher δ implies that a lower share of goods are national goods.

The wage in the South relative to the North can be obtained from the conditions $\frac{L_S w_S}{L_N w_N} = \frac{D_S}{1 - D_S}$ and $D_S = \left(c_S^{-1/\theta} / \hat{\psi} \right) \lambda_G$. This yields

$$w_S / w_N = \left(\frac{\widetilde{\delta} \delta'}{\left(1 + \widetilde{\delta} \right) g_L} \frac{L_N}{L_S} \right)^{1/(1+\beta/\theta)} \tag{21}$$

For the conjectured equilibrium with $w_S < w_N$ we then need the following condition:

$$\delta/g_L \leq L_S/L_N \quad \text{or} \quad \delta/g_L > L_S/L_N \;\; \text{and} \; \delta' < \frac{L_S\,(g_L+\delta)\,g_L}{\delta L_N - g_L L_S} \tag{22}$$

The first inequality or the second and third inequalities together imply that the stock of ideas in the South is too low, so $w_S/w_N < 1$.

If conditions (20) and (22) are satisfied, then there is an equilibrium of the form that I have conjected (i.e., wage equalization in the North and $w_S < w_N$). For future reference, note that as diffusion increases then wages in South and North become equalized (i.e. $w_S = w_N$). Formally, if $\delta > g_L L_S/L_N$ then there exists a $\bar{\delta}'$ such that wages are equalized if $\delta' \geq \bar{\delta}'$. Similarly, if $\delta' > g_L L_S/L_N$ then there exists a $\bar{\delta}$ such that wages are equalized if $\delta \geq \bar{\delta}$. Similarly, wages become equalized for any δ and δ' if South is sufficiently small.

Now, from (12) and (13) we get

$$p_m = (BC_T)^{1/\beta}\, w_N \left(\sum\nolimits_j \lambda_j + \lambda_N + (w_S/w_N)^{-\beta/\theta}\,\lambda_G\right)^{-\theta/\beta} \tag{23}$$

and from From (11) and (23) we get the inverse of the real wage in north country i,

$$w_N/p_i = (AC_{NT})^{-1}\,(BC_T)^{-(1-\alpha)/\beta}\left(\sum\nolimits_j \lambda_j + \lambda_N + (w_S/w_N)^{-\beta/\theta}\,\lambda_G\right)^{\theta(1-\alpha)/\beta}(\lambda_i+\lambda_N+\lambda_G)^{\gamma} \tag{24}$$

The corresponding result for the South is

$$w_S/p_S = (AC_{NT})^{-1}\,(BC_T)^{-(1-\alpha)/\beta}\left((w_N/w_S)^{-\beta/\theta}\left(\sum\nolimits_j \lambda_j + \lambda_N\right) + \lambda_G\right)^{\theta(1-\alpha)/\beta}\lambda_G^{\gamma} \tag{25}$$

3.2.3 Gains from diffusion and frictionless trade

The overall gains from openness for north country i can be seen as the increase in the real wage from the case with no trade and no diffusion to the case with diffusion under frictionless trade. From (19) with $\eta_i = (1+\delta/g_L)\lambda_i$ and (24), and using the expressions for λ_i, λ_N and λ_G in equations (8) – (10), the gains from openness for north country i are

$$GO_i = r_i^{-\theta(1-\alpha)/\beta}\left(1+\frac{(w_N/w_S)^{\beta/\theta}-1}{(g_L+\delta)\,/\delta'\widetilde{\delta}}\right)^{\theta(1-\alpha)/\beta}\left(\frac{\delta/r_i+g_L}{\delta+g_L}\right)^{\gamma} \tag{26}$$

where $r_i \equiv R_i/R_N$ is the share of worldwide research done by i. The first term capturesare the gains associated with North-North trade of intermediate goods and diffusion of T ideas; the second term captures the gains from trading with the South; and the final term captures the gains from diffusion of NT ideas. Clearly, countries that account for a smaller share of worldwide research have more to gain from openness. Moreover, as long as δ' is not too low, then $\delta \to \infty$ implies that $w_N = w_S$ and hence $GO_i \to r_i^{-\theta(1-\alpha)/\beta-\gamma}$. Since $\theta(1-\alpha)/\beta + \gamma = g/g_L$, this coincides with the simple logic pursued in the Introduction to compute the gains from openness as a pure scale effect in a quasi-endogenous growth model. Of course, if diffusion is finite, then the gains from openness would be smaller than what this simple calculation suggests.

What is the contribution of trade to these gains from openness? The problem in decomposing the overal gains from openness into the contributions of trade and diffusion (of T ideas) is that these two channels are substitutes, in the sense that if one is present then the other one is less important. To see this, it is best to start with an extreme case in which trade and diffusion of T ideas among north countries are *perfect substitutes*. Consider the case in which $\phi_i = \phi$ for all i (symmetry) and again let $\delta \to \infty$ (with δ' not too low). Assume also that $\gamma = 0$ (no NT ideas) to simplify the exposition. Recall that the stock of ideas originated in country i is R_i/g_L. Thus, if all countries shut down diffusion it is easy to show that the real wage under frictionless trade in country i is given by

$$w_i/p_i = A^{-1} \left(BC_T\right)^{-(1-\alpha)/\beta} \left(R_N/g_L\right)^{\theta(1-\alpha)/\beta} \tag{27}$$

On the other hand, $\delta \to \infty$ implies that for all i we have $\lambda_i + \lambda_N + \lambda_G \to R_N/g_L$, so using (19) we see that under autarky but with diffusion the real wage is also given by (27). Thus, in this extreme case, the contribution of diffusion is zero when there is trade, and the contribution of trade is zero when there is diffusion. The general result under normal conditions (i.e., trade is costly and diffusion is finite) is that trade and diffusion are substitutes in the sense that if there is diffusion (trade) then the gains from trade (diffusion) are lower than if diffusion (trade) is not present. Intuitively, imports allow a country to benefit from foreign ideas that have not yet diffused, and diffusion allows a country to benefit from foreign ideas even without trade; diffusion acts as a substitute for trade in the international exchange of ideas among north countries.[18] Another way to state this is that shutting down trade leads a country to rely more on diffusion and this attenuates the resulting losses.

This result implies that it is to some extent arbitrary to decompose the overall gains from openness into separate contributions of trade and diffusion. But it is still meaningful to ask how a country would lose by shutting down trade. Equivalently, we can ask how a country

[18] This substitutability between trade and diffusion is similar to the substitutability between trade and factor flows in Mundell (1957).

gains by going from a case with diffusion and autarky to a case with diffusion and frictionless trade. The result can then be compared to the calculated gains from trade in a model without diffusion (as in Section 2) and to the overall gains from openness. This entails comparing (19) with $\eta_i = \lambda_i + \lambda_N + \lambda_G$ to (24), which yields (again, using (8) – (10))

$$GT_i = \left(\frac{1 + \delta/g_L + \left[(w_N/w_S)^{\beta/\theta} - 1 \right] \delta' \widetilde{\delta}/g_L}{r_i + \delta/g_L} \right)^{\theta(1-\alpha)/\beta} \tag{28}$$

Countries with a lower share of worldwide research gain more from trade. Moreover, the gains from trade for north countries are increasing in w_N/w_S, as this reduces their relative cost of procuring global goods. This implies that an increase in the size of the South enlarges the gains from trade for the North. This is a standard terms of trade effect. The result in (28) also shows that, holding w_N/w_S constant, an increase in the rate of North-South diffusion δ' increases the gains from trade for the North. Of course, as shown in (21), an increase in δ' increases the South's relative wage. Thus, an increase in δ' has two opposite effects on the North's gains from trade: on the one hand, it allows more goods to be produced cheaply in the South, which is beneficial to the North, but on the other hand this generates a terms of trade loss for north countries. It is easy to show that GT_i behaves like an inverted U with respect to δ': when diffusion is low then higher diffusion benefits the North, and the opposite occurs when diffusion is high. Thus, the gains from trade for north countries may either increase or decrease as North-South diffusion increases. A similar result obtains (and the same reasoning applies) regarding the impact of North-North diffusion on the North's real wage: w_N/p_i behaves like an inverted U with respect to δ.

We can now explore further the implications of diffusion for the gains from trade. Consider first the case in which wages are equalized between North and South. The result that trade and diffusion are substitutes can be appreciated clearly in this case by noting from (28) (with $w_N/w_S = 1$) that GT_i decreases with δ. Another way to see this is to note from the first term of (26) that the joint gains from trade and diffusion of T ideas among north countries are $r_i^{-\theta(1-\alpha)/\beta}$, which does not depend on δ. Since the real wage in the North increases under autarky with δ, then necessarily GT_i will decrease with δ.

Consider now the case in which there is no wage equalization between North and South. The result discussed above that GT_i behaves like an inverted U with respect to δ' implies that, if δ' is sufficiently small, then an increase in δ' actually increases the gains from trade in north countries. Thus, North-South diffusion and trade can be complements. The impact of North-North diffusion on the gains from trade in north countries is less straightforward: on the one hand, as mentioned above, an increase in δ implies lower gains from North-North trade, but on the other hand, if δ is sufficiently small, then an increase in δ increases the North's gains from trading with the South. Thus, with no wage equalization between North and South, it is conceivable that higher North-North diffusion leads to higher gains from trade in north countries.

Turning now to the gains from trade for the South (the gains from openness are not well defined because under isolation the stock of ideas in the South is zero, implying zero income). This is given by

$$GT_S = \left(\frac{(w_N/w_S)^{-\beta/\theta} (\delta' + g_L + \delta) + \delta\delta'/g_L}{\delta\delta'/g_L} \right)^{\theta(1-\alpha)/\beta} \tag{29}$$

If $w_S = w_N$ then GT_S is decreasing in both δ and δ', implying that trade and diffusion are substitutes for the South. In the general case, we again have conflicting effects: on the one hand, diffusion implies that the South has less to gain from the North because in a sense it already has many of the North's technologies, but on the other hand higher diffusion implies an improvement in the terms of trade for the South, leading to higher gains from trade with north countries. It is easy to show that GT_S behaves like an inverted U witn respect to either δ and δ': when diffusion is low, the terms of trade effect dominates, whereas for high diffusion the substitution effect dominates.

4 Calibration

To explore the quantitative implications of the model, I need to choose values for the parameters α, β, θ, γ, δ and δ'. I also need to choose a reasonable value for L_N/L_S, as this determines the relative wage w_N/w_S, which is crucial in determining the gains from trade. Again, I follow Alvarez and Lucas (2005) and set $\alpha = 0.75$, and $\beta = 0.5$. To assign a value to θ I use a procedure that is similar to Eaton and Kortum (2002), but amended to account for the effect of diffusion. In particular, I assume that trade among north countries can be seen as an equilibrium outcome of the model presented in the previous section (with trade costs) for the particular case in which there is no trade among north countries in north goods. That is, I think of the trade data as coming from the equilibrium of the model for a set of research intensities and trade costs so that each north country satisfies its own demand for north goods with domestic production. I refer to this as the NTNG condition.

Recalling the definition $\widetilde{c}_l^N = \min_i \{c_i/k_{li}\}$, the NTNG condition implies that $\widetilde{c}_i^N = c_i$ for all i. One case in which the NTNG condition is satisfied entails a common research intensity across north countries. Formally, if $\phi_i = \phi_j$ for all $i, j \in \Omega_N$ and if $x_N(u) = \min_i\{x_i(u)\}$ then there is an equilibrium in which there is no North-North trade in intermediate good u (see the Appendix). To gain some intuition for this result, consider the case of no trade costs. With a common research intensity in the North, the absence of trade costs implies that in equilibrium all north countries have the same wage and the same unit cost for the input bundle, i.e. $c_i = c_j$ for all $i, j \in \Omega_N$. In turn, this implies that in equilibrium one can have every north country satisfy its own demand for north goods. If trade costs are positive, then *a fortiori* the NTNG condition will be satisfied.

The assumption of a common research intensity across north countries is only necessary for the NTNG condition to be satisfied for *any* structure of trade costs k_{ij}. Alternatively, one could assume a simple structure of trade costs with $k = k_{ij}$ for all i, j, and then find the maximum k (i.e., minimum trade costs) necessary for the NTNG condition to be satisfied given any vector of research intensities $(\phi_1, \phi_2, \ldots, \phi_n)$. Clearly such a maximum k exists since the NTNG condition is satisfied for k close to zero.

I now assume that research intensities are sufficiently similar that given the presence of trade costs the NTNG condition is satisfied. Although clearly this is not a reasonable characterization for the whole world, it is a reasonable assumption to characterize trade among the richest countries.[19] I also assume that $\widetilde{c}_l \equiv \min_j \{c_j/k_{lj}\} = c_S/k_{lS}$ for all l, so that the South produces all global goods. Applying the equilibrium characterization derived in the previous section to the case with $\widetilde{c}_i^N = c_i$ for all i and $\widetilde{c}_l = c_S/k_{lS}$ for all l we see that now the price index of intermediate goods in north country i is given by

$$p_{mi} = C_T \widetilde{\psi}_i^{-\theta} \tag{30}$$

where

$$\widetilde{\psi}_i = \sum_j \widetilde{\psi}_{ij} \text{ and } \widetilde{\psi}_{ij} = \begin{cases} (c_j/k_{ij})^{-1/\theta} \lambda_j & if \quad i \neq j \text{ and } j \in \Omega_N \\ c_i^{-1/\theta}(\lambda_i + \lambda_N) & if \qquad i = j \\ (c_S/k_{iS})^{-1/\theta} \lambda_G & if \qquad j = S \end{cases} \tag{31}$$

[19]Readers interested in the characterization of trade in global goods among rich and poor countries should consult Eaton and Kortum (2006).

Trade shares are given by $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$. The relationship between normalized import shares (D_{ij}/D_{jj}) and trade costs $(p_{mj}/p_{mi}k_{ij})$ can now be shown to be (in logs)

$$m_{ij} = -\ln\left(1 + \widetilde{\delta}R_N/R_j\right) - (1/\theta)\kappa_{ij} \tag{32}$$

This is similar to the (normalized) gravity equation in (7) except that now there are source-country fixed effects. These effects are more important for small countries (i.e., m_{ij} is more negative when R_N/R_j is higher) because in such countries an important part of domestic production is related to north goods, which are not exported to other north countries. This decreases the imports by any country from small countries in relation to (or normalized by) their domestic purchases.

Using the same data on trade volumes and trade costs as Eaton and Kortum (2002), and R&D employment as a proxy for aggregate research levels R_j and R_N in (32), I estimated $\widetilde{\delta}$ and θ from this equation using non-linear least squares among 19 OECD countries and $g_L = 0.048$. Both parameters are precisely estimated. The estimate of $1/\theta$ is 4.57 with a s.e. of 0.32 while the estimate of $\widetilde{\delta}$ is 0.13 with a s.e. of 0.03.[20] Note that this implies $\theta = 0.22$, significantly higher than Eaton and Kortum's $\theta = 0.12$.

The higher value of θ helps the model better match the observed growth rate even with no technological progress for consumption goods: imposing $\gamma = 0$ in (17) and using $\alpha = 0.75$, $\beta = 0.5$ and $g_L = 4.8\%$, the implied growth rate would now be $g = 0.53\%$ rather than $g = 0.29\%$ obtained in the model without diffusion. But this is still significantly below the observed $g = 1.5\%$. I can now calibrate γ to match this growth rate in (17). This yields $\gamma = 0.2$.

To calibrate δ and δ' given $\widetilde{\delta} = 0.13$ I use (21) to get

$$\delta' = (w_S/w_N)^{\beta/\theta}\left(\frac{w_S L_S}{w_N L_N}\right)\left(1 + 1/\widetilde{\delta}\right) g_L \tag{33}$$

Imposing the condition that north countries constitute a 75% of world GDP, then $\frac{w_S L_S}{w_N L_N} = 1/4$.[21] I could also require the model to be consistent with some realistic relative wage for "the South," but the problem is that wage differences could also be coming from TFP differences outside the model, which would manifest themselves here through differences in efficiency units per worker.

[20] These are robust standard errors. There are 342 observations, and the R-squared is 0.93.

[21] This is the share of worldwide GDP of the 19 OECD countries used by Eaton and Kortum (2002) in their estimation of θ. The GDP data comes from the WDI cdRom, average 1994-2000.

Instead of this, I follow Alvarez and Lucas (2005) and require the model's implied p_i/p_{mi} to match observed relative prices of consumption to machinery. As an approximation, I consider the model's implications under frictionless trade. In this case, since intermediates are tradable then $p_{ml} = p_m$ for all l, and hence I focus on differences in consumption prices. The model implies

$$p_i/p_S = (w_N/w_S)^{\alpha} \left(\frac{(1/\widetilde{\delta})R_i/R_N + 1 + \delta'/g_L}{\delta'/g_L} \right)^{-\gamma} \tag{34}$$

I follow Alvarez and Lucas (2005) in using the benchmark PWT year 1996 to compute p_i/p_{mi}. The average for developing countries is 0.7, whereas for the U.S. it is 1.37. This implies $p_{US}/p_S = 1.96$. For the U.S. we also have $R_i/R_N = 41\%$ (the share of scientists and engineers engaged in research that reside in the U.S.). Thus from (33) and (34) we then get $\delta' = 0.00019$ and $\delta = 0.0063$. This also implies $w_N/w_S = 16$. To check robustness later on, I also consider the implications of imposing $w_N/w_S = 1$. From (33) this would result in $\delta' = 0.1$ and $\delta = 0.02$.

4.1 Discussion

It is interesting to explore further the way in which $\widetilde{\delta}$ was estimated from (32) above. This equation implies that smaller countries should have lower normalized exports to any other country. This relationship between size as measured by R_j/R_N and normalized exports is what should be pinning down $\widetilde{\delta}$ in the estimation. To see whether this is indeed the case, I ran a linear regression of m_{ij} on κ_{ij} with source-country dummies and compared the exponential of the estimated coefficients for the dummies (z_j) with R_N/R_j. As shown in Figure 2, there is clear positive relationship between these two variables.[22] This suggests that smaller countries do have lower normalized exports, and that the estimated $\widetilde{\delta}$ in the NLS procedure above is capturing this relationship.

One concern with this estimate is that it implies a very slow diffusion process: in fact, $\delta = 0.0063$ implies a mean diffusion lag of $1/\delta = 156$ years, which may seem too high. Comin, Hobijn and Rovito (2006) estimate diffusion lags for several technologies. They find median diffusion lags that range from 8 years for the Internet to 74 years for cars, but of course such lags are smaller among rich countries. Eaton and Kortum (1999) calculate diffusion lags from international patent data and find an average mean diffusion lag of ten years among advanced countries. For any reasonable value of δ', if I imposed $\delta = 0.1$ in the model above the implied

[22] A linear regression of z_j on R_N/R_j yields an estimated slope coefficient of 0.26 with a s.e. of 0.09.

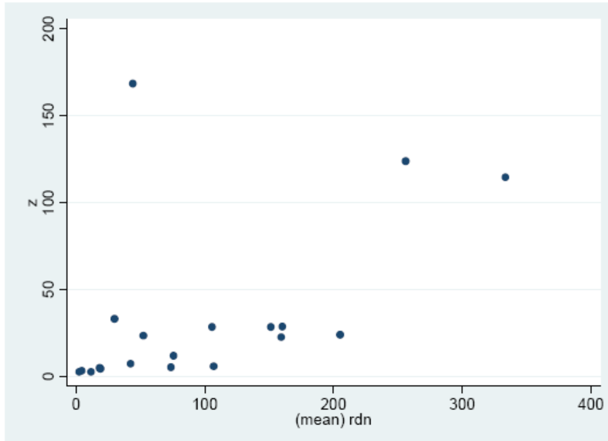


Figure 2: Source country dummies versus model (size)

trade volumes would be way too low.

There are several reasons for the discrepancy between the diffusion lag estimated from the trade data and the diffusion lags estimated from data on the usage of technologies or from international patenting. First, only a small subset of technologies are patented, and it is reasonable to expect that it is precisely those technologies that are likely to rapidly diffuse that are patented. Thus, the mean international diffusion lag found by Eaton and Kortum (1999) may significantly understate the diffusion lag for all ideas conducive to trade. Second, the estimated rate of diffusion may be low because I am implicitly assuming that all observed trade is Ricardian trade, whereas in reality a significant part of this trade (especially among OECD countries) is explained by increasing returns and product differentiation. Finally, note also that diffusion as modeled here occurs simultaneously to *all other* countries, whereas what we observe in practice is sequential diffusion to just one or a few countries. A more realistic model would entail ideas diffusing to one country at a time, and in the mean time there would be trade between the countries that have and the ones that do not have the technology. To see the importance of this for trade volumes, imagine the extreme case where ideas diffuse only to one country. With ten countries and a mean diffusion lag of ten years, the import share with $\theta = 0.22$, $k = 0.7$, and $g_L = 0.048$ would be 41% whereas with complete diffusion (as modeled above) and the same parameters but $\delta = 0.0063$ the import share would be 44%. Pursuing this line of research to explore the implications of a more realistic process of diffusion on trade flows is an important topic for future research, but does not seem essential for the calculation of the gains from trade and diffusion that I pursue in this paper.

This is a good place to discuss the implication of the NTNG condition for the estimation of θ above. Imagine that ideas diffused from country j to country i (in the North) at an instantaneous rate of δ^{ij}. Evidence suggests that countries that are closer together, or have a common language or a common border, have stronger knowledge flows (see Keller, 2004). Thus, δ^{ij} is likely to be negatively correlated with trade costs $1/k_{ij}$ or κ_{ij}. Since a high δ^{ij} implies lower trade flows from j to i then omitting this from the regression above is likely to *underestimate* the impact of trade costs on trade flows, and hence overestimate θ. Intuitively, high trade costs go together with trade flows that are not too low *not* because of a high θ but rather because of little diffusion between such countries. A lower θ would imply that the gains from openness are lower, with the gains from trade falling even more (see below).

5 Gains from trade and diffusion: quantitative results

I now use the parameters calibrated in the previous section to compute values for the gains from openness and the gains from trade for a country that does 1% of the world's total research. I then generalize to countries with different shares of world research.

The overall gains from openness for a country with $r_i = 1\%$ are $GO_i = 2.83$, or gains of 183% relative to the income level under isolation. These gains are large relative to the gains from openness in the model without diffusion: with $\theta = 0.12$ (as estimated by Eaton and Kortum, 2002) the gains from openness are 30%, whereas for the central value of θ used in Alvarez and Lucas (2005), namely $\theta = 0.15$, these gains are 41% (see Section 2). Partly, this is a result of the gains from diffusion of NT ideas, which from (26) are

$$\left(\frac{\delta/r_i + g_L}{\delta + g_L}\right)^{\gamma} = 1.67$$

But this also comes from the increase in the estimated value of θ to 0.22 that arises when diffusion is allowed into the model. To see this at an informal level, note that the gains from trade and diffusion among north countries, which is captured by the first term on the RHS of (26), are $100^{0.11} = 1.66$ for $\theta = 0.22$ rather than 1.41 for $\theta = 0.15$. Thus, the increase in the estimated θ that arises from the gravity equation in the model with diffusion explains part of the increase in the implied gains from openness.

What is the role of trade in generating these gains? Applying (28) we get $GT_i = 1.29$, or

29% gains of going from autarky (but with diffusion) to frictionless trade. These gains seem small compared to the large overall gains from openness calculated above. It is also interesting to compare this result to the gains from trade if there is no diffusion and all countries have the same research intensity (i.e., $\phi_i = \phi$ for all i), so that wage equalization holds. In this case $GT_i = r_i^{-\theta(1-\alpha)/\beta} = 1.66$, or 66% gains. There are two main differences between this result and the result of $GT_i = 1.29$ derived above: diffusion and wage differences between North and South. To understand the role of these two differences, consider an intermediate situation with diffusion but no wage differences between North and South. Imposing $w_N = w_S$ in (28) we get $GT_i = 1.26$. This shows that the gains from trade drop from 66% to 26% as we introduce diffusion but retain wage equalization; allowing for wage differences between North and South increases the gains from trade only slightly, from 26% to 29%. An important implication is that diffusion decreases the gains from trade in spite of the fact that it leads to "new" gains from trading with the South. The implication is that the complementarity of diffusion and trade with the South is dominated by the substitutability between North-North trade and diffusion; in other words, although in theory trade and diffusion could be complements via trade with the South, this is not the case for the set of parameters considered here.

The previous calculations have been made for the case of $R_i/R_N = 1\%$. Figure 3 shows how these results generalize to different levels of r. The curve GO represents the gains from openness whereas the curve $GO1$ represents the gains from North-North trade and diffusion of T ideas (first term on the RHS of 26) and the curve $GO3$ represents the gains from North-North diffusion of NT ideas (third term on the RHS of 26). The second term on the RHS of (26) – which corresponds to the gains from North-South trade – is not plotted because its value is constant and equal to 1.025 for any R_i/R_N. Figure 4 shows the relationship between the gains from trade in this model and the gains from trade in the Alvarez and Lucas model (i.e., $\theta = 0.15$ and no diffusion). The result presented above for $R_i/R_N = 1\%$ that the gains from trade in the model with diffusion are lower than the AL model remains valid for high $1/r$ values (small countries); for large countries, the larger θ in the model with diffusion dominates, leading to higher gains from trade than in the AL model.

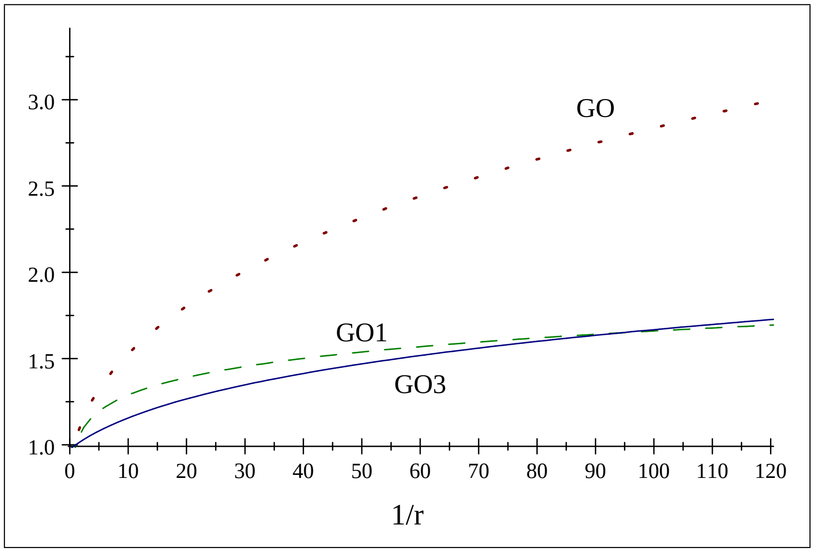


Figure 3: Gains from Openness (GO = Gains from Openness, GO1 = Gains from North-North Trade and Diffusion of T ideas, GO3 = Gains from North-North Diffusion of NT ideas)

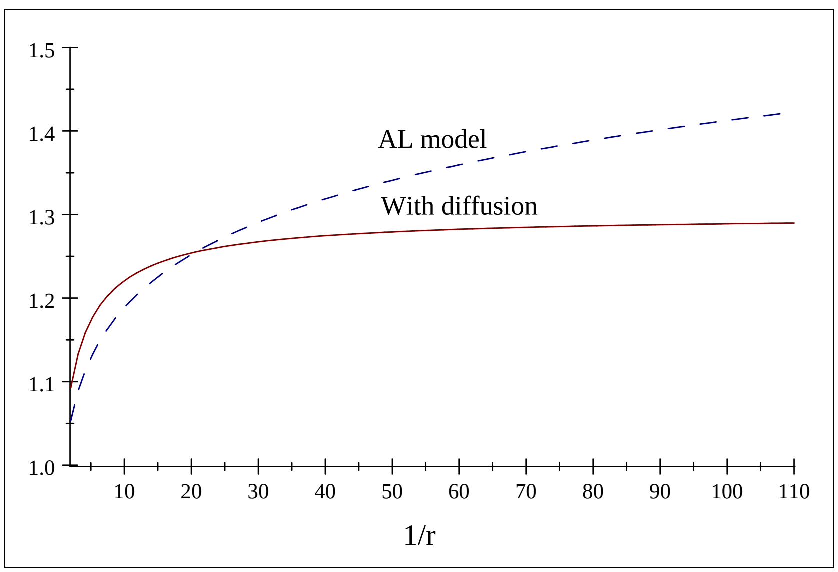


Figure 4: Gains from Trade in the model with diffusion versus the AL model

It is interesting to consider the implied gains from diffusion and trade if instead we calibrated δ and δ' by imposing $w_S/w_N = 1$, as discussed in the previous section. Then $GO_i = 3.29$ and $GT_i = 1.14$. The implication is that in this case the gains from openness are even higher, while the gains from trade are lower than under the main calibration above. The reason is that imposing $w_S/w_N = 1$ requires higher rates of diffusion, and this generates higher overall gains from trade and diffusion but lower gains from trade.

Finally, I turn to the gains from trade for the South. Applying (29) yields $GT_S = 1.19$, or 19% gains. The main gains for the South come not from trade but from diffusion of ideas from the North.

6 Conclusion

Countries benefit from openness to the rest of the world in many different ways. Trade and diffusion are surely two of the most important channels for the realization of these benefits. Building on Eaton and Kortum (2001, 2002) and Alvarez and Lucas (2005) I developed a model of trade and diffusion where growth is caused by technological progress in tradable and non-tradable goods. Calibrating the model to match trade data and the observed growth rate I then computed the model's gains from openness and the gains from trade. The main result is that the gains from openness are quite large, whereas the gains from trade are even smaller than in the model without diffusion.

One concern is that the analysis implicitly assumes that the flow of ideas is independent of the volume of trade. Perhaps trade has a much larger role precisely through a positive effect on diffusion. This could happen through several channels, such as flows of ideas arising from the interaction between nationals and foreigners via trade, the competitive pressure from imports inducing domestic firms to engage in faster technology adoption, or complementarities between foreign technologies and foreign inputs.[23] There is a large empirical literature exploring the significance of these and similar mechanisms through which trade may induce productivity growth in domestic firms.[24] This seems like an important topic for future research.

A limitation of the model concerns the way in which diffusion is captured. There are at least

[23] We tend to see countries that are closed to trade also being closed to foreign ideas; think of North Korea. But this empirical association between trade and diffusion does not imply any causal role for trade in accelerating diffusion.

[24] See for instance Bernard and Jensen (1999), and Hallward-Driemeier et. al. (2002), and Tybout (2003).

three specific tasks ahead. First, as mentioned above, assuming that diffusion entails national ideas becoming available in all rich countries and then globally is clearly unrealistic. It seems important to model diffusion according to the qualitative and hopefully quantitative features found in the data (see for example Jaffe and Trajtenberg, 1999, and Comin et. al. 2006) and explore the implications of this for growth and trade.

Second, FDI is surely one mechanism through which diffusion takes place. Ramondo (2006) has already shown a way to model FDI within the Eaton and Kortum framework when there is no trade. The next step is to have a model with trade, FDI and (pure) diffusion and quantify the gains from each of these channels separately. Including migration into such a model would be another worthwhile step.

Finally, this paper has ignored the importance of differences across countries in technology adoption. One conjecture that would be interesting to explore is that countries with lower rates of technology adoption have less to gain from diffusion and more to gain from trade, with lower overall gains from openness.

A final remark concerns the assumption that research efforts are exogenous. How would the results change if this assumption were relaxed? In the simplest model with no diffusion of T ideas, as in Eaton and Kortum (2001), trade does not affect countries' research intensity (i.e., the share of the labor force devoted to research). The reason for this is the standard one that although trade expands the market for ideas, it also increases competition, and these two effects exactly balance out. This implies that, to a first approximation, the gains from trade would not be affected by having endogenous research efforts. Something similar happens with the gains from diffusion. To see this, consider the extreme case in which diffusion is instantaneous (no frictions to the international diffusion of ideas, i.e. $\delta, \delta\prime \rightarrow \infty$) and research productivities are equal across north countries (i.e., $\phi_i = \phi$ all i). Shutting down diffusion would imply larger returns to ideas in the home market, but would prevent the exploitation of ideas in foreign markets. As shown in Eaton and Kortum (2006), these two effects exactly cancel out, so diffusion has no effect on innovation. This discussion suggests that the results obtained here with exogenous research efforts would not be significantly affected by the extension to endogenous research. Still, a thorough analysis of this issue seems worthwile, and is left for future research.

Appendix (needs updating)

This Appendix proves that if $\phi_i = \phi_j$ for all i, j then the equilibrium characterized by (30) and (3) with $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$ and (31) is consistent with the NTG condition. The following condition is both necessary and sufficient for the NTG condition to hold:

$$c_i \leq c_j/k_{ij} \text{ for all } i, j \tag{35}$$

The following Lemma is sufficient to prove the claim:

Lemma 1 *Assume that* $\phi_i = \phi$ *for all* i. *If* $p_m = (p_{m1}, p_{m2}, ..., p_{mn})$ *and* $w = (w_1, w_2, ..., w_n)$ *satisfy* (30) *and* (3) *with* $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$ *and* (31), *then* (35) *is satisfied.*

The rest of the Appendix proves this Lemma. Letting $\mu \equiv (BC_S)^{-1/\theta}$, then it is easy to show that

$$D_{li} = \begin{cases} \mu c_i^{-1/\theta} p_{ml}^{1/\theta} k_{li}^{1/\theta} \lambda_i & \text{for } l \neq i \\ \mu c_i^{-1/\theta} p_{mi}^{1/\theta} (\lambda_i + \lambda_T) & \text{otherwise} \end{cases}$$

Since ϕ is the same across countries, I can use $\phi = g_L$ without no loss of generality, so $\lambda_i = L_i$ for all i. Then the trade balance conditions are $\sum \lambda_l w_l D_{li} = \lambda_i w_i$, which can be expressed as

$$\sum_{l \neq i} \mu c_i^{-1/\theta} p_{ml}^{1/\theta} k_{li}^{1/\theta} \lambda_i \lambda_l w_l + \mu c_i^{-1/\theta} p_{mi}^{1/\theta} (\lambda_i + \lambda_T) \lambda_i w_i = \lambda_i w_i$$

Dividing by λ_i and multiplying by $c_i^{1/\theta}/\mu$ this becomes

$$c_i^{1/\theta} w_i/\mu = \sum_{l \neq i} p_{ml}^{1/\theta} \lambda_l w_l k_{li}^{1/\theta} + p_{mi}^{1/\theta} (\lambda_i + \lambda_T) w_i$$

Moreover, given (30) then

$$p_{mi}^{-1/\theta}/\mu = \sum_{l \neq i}^{n} c_l^{-1/\theta} k_{il}^{1/\theta} \lambda_l + c_i^{-1/\theta} (\lambda_i + \lambda_T)$$

Letting $b \equiv 1/\theta$, $a_{il} \equiv k_{il}^b$, and $z_i \equiv (p_{mi}/c_i)^b \lambda_i$, and then using $a_{il} = a_{li}$, these two equations can be transformed into:

$$\begin{aligned} 1/\mu &= \sum_{l \neq i} a_{li} z_l (w_l/w_i)(c_l/c_i)^b + z_i(1 + \lambda_T/\lambda_i) \\ 1/\mu &= \sum_{l \neq i}^{n} a_{li} z_l (p_{mi}/p_{ml})^b + z_i(1 + \lambda_T/\lambda_i) \end{aligned}$$

Both of these equations have the same structure, namely

$$1/\sigma = \sum_{l \neq i} a_{li} z_l x_l + z_i(1 + \lambda_T/\lambda_i)$$

This can be seen as a system of n linear equations in n unknowns, x_i for $i = 1, ..., n$, with z_i for $i = 1, ..., n$ as constants. In matrix notation, this is $Ax' = d'$, where x' is the transposed $x = (x_1, x_2, ..., x_n)$ vector and d' is the transposed $d = (d_1, d_2, ..., d_n)$, where $d_i = 1/\mu - z_i(1+\lambda_S/\lambda_i)$, and where $A = A_1 * A_2$, and

$$A_1 = \begin{bmatrix} 0 & a_{21} & \dots & a_{n1} \\ a_{12} & 0 & \dots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & 0 \end{bmatrix} \text{ and } A_2 = \begin{bmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n \end{bmatrix}$$

This system has a unique solution iff $\det(A) = \det(A_1)\det(A_2) \neq 0$. Given that the $z's$ are non-zero, then $\det(A_2) \neq 0$. On the other hand, the set of values of a_{ij} such that $\det(A_1) = 0$ has dimension $n - 1$. Thus, without loss of generality we can say that A is invertible. This implies that the system $Ax' = d'$ has a unique solution and hence

$$(w_j/w_i)(c_j/c_i)^b = (p_{mi}/p_{mj})^b$$

Recall that $c_i = Bw_i^{\beta} p_{mi}^{1-\beta}$. Plugging this into the previous equation we get

$$\begin{aligned} (w_j/w_i)\left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{w_i^{\beta} p_{mi}^{1-\beta}}\right)^b &= (p_{mi}/p_{mj})^b \\ (w_j/w_i)^{1+b\beta} &= (p_{mi}/p_{mj})^{b(2-\beta)} \\ w_j/w_i &= (p_{mi}/p_{mj})^{b(2-\beta)/(1+b\beta)} \end{aligned}$$

Plugging this back above we get

$$c_j/c_i = (w_j/w_i)^{\beta} (p_{mj}/p_{mi})^{1-\beta}$$

and hence

$$c_j/c_i = (p_{mi}/p_{mj})^{\gamma} \quad (*)$$

where

$$\gamma \equiv \beta - 1 + b\beta(2 - \beta)/(1 + b\beta)$$

(Note that this is not the same γ used for NT-ideas in the main text.) For future reference, note that

$$\begin{aligned}\gamma &= \frac{(\beta-1)(1+b\beta)+b\beta(2-\beta)}{1+b\beta} \\ &= \frac{\beta-1+b\beta^2-b\beta+2b\beta-b\beta^2}{1+b\beta} \\ &= \frac{\beta-1+b\beta}{1+b\beta}\end{aligned}$$

so $\gamma \in]1,1[$.

Now, note that $p_{mi} < p_{mj}/k_{ij}$ is equivalent to $a_{ij}p_{mj}^{-b} < p_{mi}^{-b}$, and

$$\begin{aligned}a_{ij}p_{mj}^{-b}/\mu &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + a_{ij}a_{ji}c_i^{-b}\lambda_i + a_{ij}c_j^{-b}(\lambda_j+\lambda_T) \\ &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i + c_i^{-b}\lambda_i + a_{ij}c_j^{-b}(\lambda_j+\lambda_T)\end{aligned}$$

But from

$$p_{mi}^{-b}/\mu = \sum_{l\neq i}^{n} a_{il}c_l^{-b}\lambda_l + c_i^{-b}(\lambda_i+\lambda_T)$$

we see that

$$c_i^{-b}\lambda_i = p_{mi}^{-b}/\mu - \sum_{l\neq i}^{n} a_{il}c_l^{-b}\lambda_l - c_i^{-b}\lambda_T$$

Plugging this above we get

$$\begin{aligned}a_{ij}p_{mj}^{-b}/\mu &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i + p_{mi}^{-b}/\mu - \sum_{l\neq i}^{n} a_{il}c_l^{-b}\lambda_l - c_i^{-b}\lambda_c + a_{ij}c_j^{-b}(\lambda_j+\lambda_T) \\ &= a_{ij}\sum_{l\neq i,j}^{n} a_{jl}c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i + p_{mi}^{-b}/\mu - \sum_{l\neq i,j}^{n} a_{il}c_l^{-b}\lambda_l - c_i^{-b}\lambda_c + a_{ij}c_j^{-b}\lambda_T\end{aligned}$$

and hence

$$a_{ij}p_{mj}^{-b}/\mu = p_{mi}^{-b}/\mu + \sum_{l\neq i,j}^{n}\left[a_{ij}a_{jl}-a_{il}\right]c_l^{-b}\lambda_l + (a_{ij}^2-1)c_i^{-b}\lambda_i - c_i^{-b}\lambda_c + a_{ij}c_j^{-b}\lambda_T \qquad (36)$$

From the triangular inequality, $k_{il} \geq k_{ij}k_{jl}$ for all i,j,l, we have $a_{il} \geq a_{ij}a_{jl}$. Hence the second term on the RHS of the last line is non-positive. Since $k_{ij} \leq 1$ for all i,j, then this is also the case for the third term. Note also that if $k_{ij} < 1$ then the third term is strictly negative. Thus $a_{ij}p_{mj}^{-b} < p_{mi}^{-b}$ if $\lambda_T = 0$ or

$$c_i^{-b} > a_{ij}c_j^{-b}$$

This implies that

$$\begin{aligned} c_j/k_{ij} &> c_i \Longrightarrow p_{mi} < p_{mj}/k_{ij} \qquad (+) \\ p_{mi} &> p_{mj}/k_{ij} \Longrightarrow c_j/k_{ij} < c_i \end{aligned}$$

Imagine first that $\gamma = 0$. Then c_i/c_j for all i, j, and hence clearly $c_i \leq c_j/k_{ij}$ for all i, j.

Now consider the case $\gamma > 0$. There are two possibilities: $p_{mj} > p_{mi}$ or $p_{mj} < p_{mi}$. In the first case then from (*) we see that

$$c_i/c_j = (p_{mj}/p_{mi})^{\gamma} > 1$$

(thanks to $\gamma > 0$) which implies

$$c_i/k_{ji} > c_i > c_j$$

In the second case, $p_{mj} < p_{mi}$, then from (*) we see that $c_i < c_j < c_j/k_{ij}$. From (+) this implies that $p_{mi} < p_{mj}/k_{ij}$. But from (*) we see that

$$c_j/c_i = (p_{mi}/p_{mj})^{\gamma} < (1/k_{ij})^{\gamma} < 1/k_{ij}$$

where the second inequality follows from $\gamma > 0$ and the last inequality follows from $\gamma < 1$. Thus, we also get $c_i/k_{ij} > c_j$.

Finally, consider the case $\gamma < 0$. The following lemma will prove useful:

Lemma 2 *If $\gamma < 0$ and $p_{mi} < p_{mj}/k_{ij}$ for any i, j then $c_i < c_j/k_{ij}$ for any i, j*

Proof. $p_{mi} < p_{mj}/k_{ij}$ for any i, j is equivalent to $p_{mj} < p_{mi}/k_{ji}$ for any i, j. This implies

$$(p_{mj}/p_{mi})^{-\gamma} < (1/k_{ji})^{-\gamma}$$

Thus,

$$\begin{aligned} c_j/c_i &= (p_{mi}/p_{mj})^{\gamma} \\ &= (p_{mj}/p_{mi})^{-\gamma} < (1/k_{ji})^{-\gamma} < 1/k_{ji} \end{aligned}$$

where the last step follows from $1 > -\gamma > 0$. ■

We now show that if $\gamma < 0$ then $c_i \leq c_j/k_{ij}$ for any i, j. From inequality 36 or $i \neq j$ we have

$$\mu^{-1}\left(\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b\right) < \lambda_T\left(\left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b\right) \tag{37}$$

Assume that $\lambda_T = 0$. Then,

$$\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b < 0 \iff p_{mi} < p_{mj}/k_{ij} \quad \text{for any } i, j$$

From Lemma 2 we obtain that $c_i < c_j/k_{ij}$, which is equivalent to $\left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b < 0$. Hence, we proved that for $\lambda_T = 0$: $\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b < 0$ and $\left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b < 0$ for any i, j. From continuity there exists ε such that for $\lambda_T \in [0, \varepsilon)$: $\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b < 0$ and $\left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b < 0$ for any $i \neq j$.

Suppose there exists $\lambda_T > 0$ such that $\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b > 0$ or $\left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b > 0$ for some i and j. From continuity and (37) we know that there must be a value of λ_T, λ_T^*, such that $\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b < 0$ and $\left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b = 0$ for some i and j. Thus, we have for some i and j

$$\begin{aligned} p_{mi} &< p_{mj}/k_{ij} \\ c_j/k_{ij} &= c_i \end{aligned}$$

This implies that

$$\begin{aligned} \left(p_{mi}/p_{mj}\right)^\gamma &= \frac{c_j}{c_i} = k_{ij} < p_{mj}/p_{mi} \implies \\ \left(p_{mi}\right)^{\gamma+1} &< \left(p_{mj}\right)^{\gamma+1} \implies p_{mi} < p_{mj} \end{aligned}$$

where the last step follows from the fact that $\gamma > -1$. Since γ is negative then $p_{mi} < p_{mj}$ implies that $c_j/c_i > 1$, which then implies $k_{ij} > 1$, a contradiction. This establishes that λ_T^* does not exist, and hence for any $\lambda_T \geq 0$ we have

$$\left(k_{ij}/p_{mj}\right)^b - \left(1/p_{mi}\right)^b < 0, \quad \left(k_{ij}/c_j\right)^b - \left(1/c_i\right)^b < 0 \quad \text{for any } i \neq j.$$

Q.E.D.

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