Heterogeneity and Competition in School Markets
WORK IN PROGRESS

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June 3, 2008

1 Introduction

The use of education vouchers to enhance competition and efficiency in the school market has been a controversial issue since Friedman's original proposal of the policy in the 1950s (Friedman, 1958). The debate has become stronger in the last decade, as countries as Britain, The Netherlands, and Sweden have partially adopted the system. Chile, among developing countries, has had a nation-wide voucher system for over 20 years, with private schools,both non-profit and for-profit, accounting for nearly half of total student enrollment.

In many dimensions, the discussion on the overall effects of vouchers is far from settled, and the issue still remains a divisive one (for an overview of the recent discussion, see Neal, 2002 and 2008, and Ladd, 2002, for two sides of the debate).

Several studies in recent years have studied the theoretical implications of education vouchers for the distribution of school quality and the allocations of students across different types of schools (Epple and Romano, 1998 and 2002; Nechyba, 2000; McMillan, 2005). While most of the studies have allowed for heterogeneity across students (both in income and ability), less attention has been given to heterogeneity across schools, and to other aspects such as intertemporal dynamics or the role of non-academic factors.

In its current incarnation, this paper tries to contibute to the literature on school vouchers on two relevant dimensions. First, it presents a framework for the behavior of competitive education markets with heterogenous schools and students. Second, it applies the framework in a general equilibrium setup to analyze the transition across different types of institutional regimes.

[0]I appreciate the comments and suggestions of Gary Becker, Francisco Gallego, Casey Mulligan, Derek Neal, and Jose Tessada.

This paper does not incorporate peer effects, a central issue in the sorting outcomes observed in theoretical studies such as Epple and Romano (2002)[1]. In those models, school quality is simply a by-product of the class composition chosen by the school. In here, schools choose their quality levels, and there are no cost externalities between students.

The closest paper to the approach presented here is Urquiola and Verhoogen (2007), who explicitly allow for differences in productivity across schools when trying to analyze classroom size in a partial equilibrium framework. However, they leave out differences in ability across students, and model a market with no threat of entry.

The main results of the paper can be summarized as follows:

1. With vouchers with no payment on top, there need not be assortative matching between more able students and more productive profit-maximizing schools. Education quality differences between schools may only reflect differences in student ability. Qualities are determined by threat of entry of the marginal school.

2. Even if there is no assortative maching with profit-maximizing schools., assortative matching will occur between more able students and more productive non-profit schools that choose to maximize quality. Education quality differences between schools reflect differences in student ability and school productivity. Qualities do not depend on threat of entry. Assortative matching is not related to peer effects.

3. When profit-maximizing schools are allowed to charge freely, there will be assortative matching between more productive schools and students with higher willingness to pay for education. Education quality differences between schools reflect differences in student ability and school productivity. Qualities depend on threat of entry and competition within existing schools. Assortative matching is not related to peer effects.

4. For a given level of public spending for education, a voucher system never reduces education quality for publicly-funded schools in a context of schools with fixed scale and homogenous composition. Quality will always increase if public schools are rent-seekers, as competitive pressures upon them become stronger. If public schools are not rent seekers, quality will increase if they are replaced by more productive private schools.

5. Changes in the composition of productivities of schools outside and inside the market affect qualities in both the privately-funded and the publicly-funded sector. In fact, quality in the fully-paid private schools could decrease after the adoption of a voucher, leading to smaller differences in quality between income groups.

The organization of the paper is as follows. Section 2 presents the basic results on education qualities and equilibrium allocations in a competitive market with heterogenous private schools and students. Section 3 presents some extensions of the basic framework: intertemporal considerations, the role of preferences for non-academic aspects of education, and the choice of scale. Sec-

[1] In any case, the empirical relevance of peer effects is far from clear.

tion 4 applies the results of the basic framework to a general equilibrium setup. Section 5 discusses future extensions of this line of research.

2 A basic framework on competition between private schools

This section provides a general overview on the operation of competitive education markets with private schools in a static setup. Section 2.1 describes the general framework, and the characteristics of households and schools. Section 2.2 describes competition and equilibrium allocations when vouchers with no payments on top are in place. Section 2.3 considers the case of fully-paid private schools.

2.1 General Setup

2.1.1 Households

In the spirit of Becker (1964), households have the general utility function:

$$U = U(C, h) \tag{1}$$

, where c is total household consumption and h is the human capital of the household's only child.

Households differ on two aspects: their income level, y; and the ability of their only child, a.

Households are assumed to live in a specific location, which defines the relevant market, and will choose among the schools operating in it. Implicitly, there are no transportation costs of attending different schools within a given geographic location, but those costs approach infinity for schools outside the location. M households (and thus, M potential students) live in a given location.

The funding of school vouchers is assumed to be exogenous: there are no explicit taxes, so at this point there are no general equilibrium considerations on how the central government finances the voucher. Section 4 discusses taxation for school-funding on a general equilibrium setup.

The technology for the accumulation of human capital is a function of the quality of education received by student i at school j, q_{ij},and of the student's exogenous ability, a_i:

$$h_i = f(a_i, q_{ij}) \tag{2}$$

, where $f_a, f_q, f_{aq} > 0$, $f_{aa}, f_{qq} \leq 0$, where a_i is the child's ability, q_s is the quality of education provided by school s. a_i is a random draw from the distribution of talents across the population, $F(a_{\max}, a_{\min})$.

The household's budget constraint is:

$$y_i = c_i + p_i q_i \tag{3}$$

, where c_i is the total value of consumption (the price of consumption is normalized to 1) and p_i is the unit price of school quality faced by household i.

2.1.2 Schools

All schools have the same technology to produce quality, and have capacity to enroll only one student. The production function of quality for school j depends on the school's productivity, γ_j, the student's ability, a_i, and variable inputs, e (teachers, materials, books, etc.).

$$q = g(a_i, \gamma_j, e) \tag{4}$$

, where $g_{a_i}, g_{\gamma_j}, g_e > 0, g_{a_i a_i}, g_{\gamma_j \gamma_j}, g_{ee} \leq 0$, and $g_{a\gamma}.g_{\gamma e}, g_{ae} \geq 0$. There are no fixed costs of setting up a school. Schools can always select which student they enroll if they have more than one applicant. A share of schools are assumed to be profit-maximizers while the rest are non-profit. Nonprofit schools maximize the quality they provide, subject to a balanced budget. There are N potential schools, where $N > M$. Schools can be ranked in terms of their productivity, γ_1 to γ_N,

2.2 Flat voucher with no payments on top

2.2.1 A general result

In the case of flat vouchers without payments on top, all operating schools get the same revenue, v. As the voucher does not depend on the characteristics of the household, all students are worth the same in terms of revenue.

The equilibrium is characterized as a set of M matches $m(a_i,\gamma_j) = m_{ij}$, where a_i identifies a student and γ_j a school. Associated payoffs are $U_{ai}(m_{ij})$, for the student, and $U_{\gamma j}(m_{ij})$, for the school, and must be such that there are no incentives to deviate and create new matches. This is, for any i, j, k, l,if $m(a_i,\gamma_j)$ and $m(a_k,\gamma_{l'})$ are two equilibrium matches, there are no deviations which make both participants of a new match strictly better off.

This is, $U_{ai}(m_{il}) > U_{ai}(m_{ij}) \Rightarrow U_{\gamma_l}(m_{il'}) < U_{\gamma_l}(m_{kl})$ and $U_{\gamma_j}(m_{kj}) > U_{\gamma_j}(m_{ij}) \Rightarrow U_{a_k}(m_{kj})) < U_{a_k}(m_{kl})$.

Claim 1 *In equilibrium, more productive schools and more able students are strictly better off.*

Claim 2 *As the most productive schools must be strictly better off, and all operating schools must be earning non-negative profits and offering a positive level of quality, only the M more productive schools will be able to operate.*

2.2.2 Relative costs and ability

Focus for a moment on a market where only profit-maximizing schools exist. It is easy to see that, for any given student with ability a_i, it must be true that the equilibrium quality he receives, q_i satisfies the following no-entry condition:

$$q_i \geq q^{\max}(m_{iM+1})$$

, where $q^{\max}(a_i, \gamma_{M+1})$ is such that:

$$v = c(q^{\max}(m_{iM+1}), a_i, \gamma_{M+1})$$

$c(q^{\max}(m_{iM+1}), a_i, \gamma_{M+1})$ is the cost function associated to the quality production function of the marginal school outside the market, with productivity γ_{M+1}, when matched with the student with ability a_i. Equilibrium quality must always be such that the marginal competitor cannot enter profitably.

Whether actual qualities exceed $q^{\max}(m_{iM+1})$ will depend on how relative costs between schools change with the ability of the student with which they match.

If, for any given level of quality, and $\gamma_j > \gamma_k$

$$\frac{\partial \left[\frac{c(q,\gamma_j,a_i)}{c(q,\gamma_k,a_i}\right]}{\partial a_i} = 0 \text{ for all } a_i, q$$

, relative costs are not affected by ability, but only by relative productivity. In that case, schools have no incentives to provide more than $q^{\max}(m_{iM+1})$ (this is discussed in more detail below). Figure 1 shows the cost of providing $q^{\max}(m_{iM+1})$ for schools of different productivity as a function of a_i. Even if $g_{a\gamma} > 0$, the relative cost of more productive schools does not decrease as ability grows. The cost advantage of higher productivity is constant across the ability spectrum. As discussed in more detail below,this implies that there will be no assortative matching between school productivity and student ability.

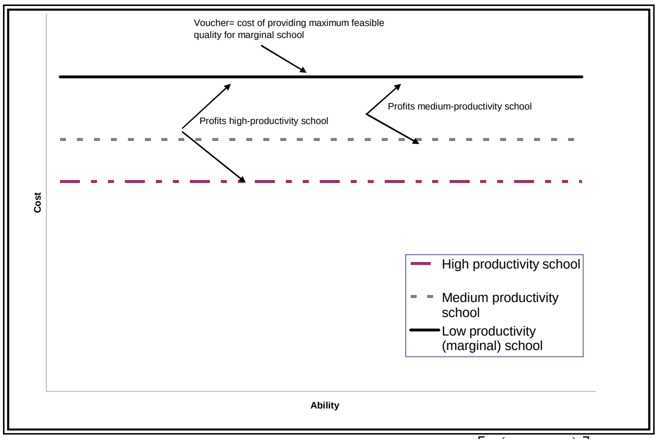

Figure 1: Costs of $q^{\max}(m_{iM+1})$ when $\dfrac{\partial\left[\frac{c(q,\gamma_j,a_i)}{c(q,\gamma_k,a_i}\right]}{\partial a_i} = 0$

If, however, for $\gamma_j > \gamma_k$:

$$\frac{\partial\left[\frac{c(q,\gamma_j,a_i)}{c(q,\gamma_k,a_i}\right]}{\partial a_i} < 0 \text{ for all } a_i, q$$

, the cost advantage of more productive schools becomes larger as ability increases. This implies that qualities for abler students must exceed $q^{\max}(m_{iM+1})$, as otherwise more productive students would be more profitable (as illustrated in Figure 2). It can be shown that this will imply assortative matching between more productive schools and more able students, with equilibrium qualities strictly larger than $q^{\max}(m_{iM+1})$ for all students except the least able.

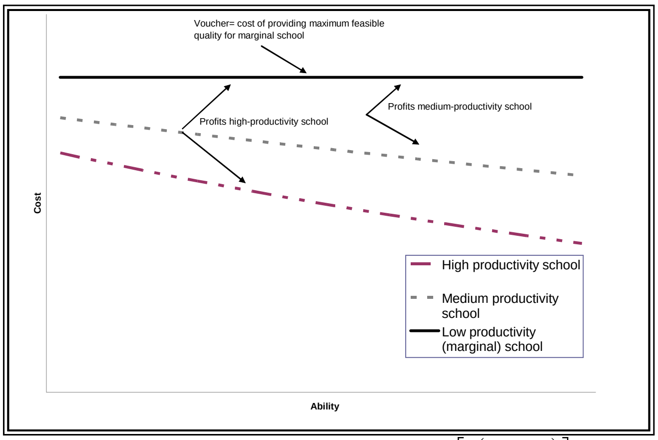

Figure 2: Costs of $q^{\max}(m_{iM+1})$ if $\dfrac{\partial\left[\frac{c(q,\gamma_j,a_i)}{c(q,\gamma_k,a_i}\right]}{\partial a_i} < 0$

The analysis below assumes a particular functional form for $g(a_i, \gamma_j, e)$, such that $\frac{\partial\left[\frac{c(q,\gamma_j,a_i)}{c(q,\gamma_k,a_i}\right]}{\partial a_i} = 0$. However, the main results of the paper do not depend crucially on this assumption, and would still hold in a more general setup.

2.2.3 Equilibrium with relative costs independent of ability

Assume that the production of quality can be described as:

$$q = a_i \gamma_j e \tag{5}$$

This is, the school has constant returns to scale in producing quality with the variable input e.Normalizing the unit cost of inputs to 1, the cost of producing quality q is therefore

$$c(q) = e = \frac{q}{a_i \gamma_j} \tag{6}$$

To understand the equilibrium allocations, it is simpler to look first at markets where only one of the two types of private schools exist, and then to apply the features of that solution to a market where both types operate.

Claim 3 *In a market where schooling is financed by vouchers that do not allow payments on top, and where only nonprofit schools operate, there will be positive assortative matching between productive schools and able students, with each school providing its maximum feasible quality $q^{\max}(m_{ii}) = va_i\gamma_i$. Threat of entry is irrelevant in this market.*

Proof. The second part of the statement simply comes from the objective function of non-profit schools, and would be true in any allocation.

For any school with productivity γ_j and students with ability a_i and a_k, $a_i > a_k$, $q^{\max}(m_{ij}) > q^{\max}(m_{kj}) \Rightarrow U_{\gamma i}(m_{ij}) > U_{\gamma i}(m_{kj})$, so all schools strictly prefer to match with more able students. This implies that, for the more productive school, γ_1, matching with the most productive student, a_i, provides higher utility than any other potential match: $U_{\gamma 1}(m_{11}) > U_{\gamma 1}(m_{i1})$ for all i.

For any student with ability a_i and schools with productivity γ_j and γ_k, $\gamma_j > \gamma_k$, $q^{\max}(m_{ij}) > q^{\max}(m_{ik}) \Rightarrow U_{ai}(m_{ij}) > U_{ai}(m_{ik})$, so all students strictly prefer to match with the more productive schools, as they provide more quality. This implies that, for the more able child, a_1, matching with the most productive school, γ_1, provides higher utility than any other potential match: $U_{a1}(m_{11}) > U_{a1}(m_{1i})$ for all $i \neq 1$.

As both the most productive school and the most able student maximize their utilities when matching, they will do so in equilibrium. No other school or student could make an offer than makes them better off. For the second school and student, γ_2 and $a_2, U_{a2}(m_{22}) > U_{a2}(m_{2i})$ and $U_{\gamma 2}(m_{22}) > U_{\gamma 2}(m_{i2})$ all

$i > 2$.Only matching with γ_1 or $a_1$would make them better off, but neither γ_1 or a_1 would gain from that. Thus, the best feasible match is m_{22}. In general, for school γ_i, student a_i, the payoff of m_{ii} exceeds the payoff of any other match in set of feasible candidates, m_{ij} and m_{ji} , $j > i$. ■

In this setup, better nonprofit schools (those who provide higher quality) are not only better because they enroll the more able students, but also because they are intrinsically more productive. The gap in quality between more able and less able students is enhanced by the differences in the productivity of the school in which they are enrolled. This is, in a world where nonprofit schools maximize the quality they provide in equilibrium, differences in equilibrium school quality between students with ability a_i and $a_{i+1}, a_i > a_{i+1}$, are more than proportional to their ability differences, reflecting also the productivity gap between the schools in which they enroll.

$$\frac{q_{ii}}{q_{i+1,i+1}} = \frac{a_i\gamma_i}{a_{i+1}\gamma_{i+1}} = \left(\frac{a_i}{a_{i+1}}\right)\left(\frac{\gamma_i}{\gamma_{i+1}}\right), \text{ where } a_i > a_{i+1} \Rightarrow \gamma_i > \gamma_{i+1} \tag{7}$$

Consider now the case where only profit-maximizing schools exist.

Claim 4 *In a market where schooling is financed by vouchers that do not allow payments on top, and where only profit-maximizing schools operate, quality does not depend on each school's productivity, but only on the productivity of the fringe competitor and the ability of each student. Schools are indifferent among all potential students, so there need not be assortative matching.*

Claim 5 *Equilibrium qualities are uniquely determined by threat of entry. In particular, by the productivity of the marginal school.*

Proof. For any student with ability a_i , its equilibrium quality is bounded by below by $q^{\max}(m_{i(M+1)}) = va_i\gamma_{M+1}$. This is the maximum quality that the school with productivity γ_{M+1} can supply, and get non-negative profits, if matched with student a_i. Thus, $q_i \geq q^{\max}(m_{i(M+1)})$, as otherwise school γ_{M+1}, the fringe competitor, could profitably enter the market. This holds true for every student, as the quality level at which the fringe competitor would make non-negative profits depends on the ability of the student with which it would potentially match. This is, $q_i \geq q^{\max}(m_{i(M+1)})$ for all i.

Take now any given allocation of schools and students, with quality for any given student given by $q^{\max}(m_{i(M+1)})$.Given that any given school γ_j must supply $q^{\max}(m_{i(M+1)})$ to match with student i, its profits are $\pi(m_{ij}) = v' - \frac{q^{\max}(m_{i(M+1)})}{a_i\gamma_j} = v' - \frac{v'a_i\gamma_{M+1}}{a_i\gamma_j} = v'\left(1 - \frac{\gamma_{M+1}}{\gamma_j}\right)$. This is an equilibrium allocation, as there are no incentives to increase quality. As all students provide the same revenue, given by the fixed voucher, profits do not depend on the characteristics of the students, but only on relative school productivities. Profits for any given school do not depend on the ability of its match, but only on the school's productivity relative to the productivity of the fringe competitor. There are no incentives to increase quality to attract a more able student, as costs would increase and revenue, v, would be the same. Thus, any allocation where qualities are described by $q^{\max}(m_{i(M+1)})$ for all i is an equilibrium. ■

Profits are strictly increasing in productivity, and more able students get strictly higher quality. However, both student and schools are indifferent across matches, as their payoffs are identical across all potential partners. This is, for all j, $U_{\gamma j}(m_{kj}) = U_{\gamma j}$ for all k and, for all i, $U_{aij}(m_{il}) = U_{ai}$. Thus, any matching allocation between the students and the M more productive schools will be an equilibrium allocation. As schools cannot enroll more than one student or charge a price different than the voucher, quality is not determined by competition between operating schools, but only by the threat posed by the fringe competitor.

As in the non-profit schools case, more able students are strictly better off. However , the quality gap (and thus, the human capital gap) is now smaller, as it is only caused by the ability gap.

$$\frac{q_{ii}}{q_{i+1,i+1}} = \frac{a_i \gamma_{M+1}}{a_{i+1}\gamma_{M+1}} = \left(\frac{a_i}{a_{i+1}}\right) \tag{8}$$

Schools that provide higher quality need not to be intrinsically more productive: their higher quality only reflects their enrollment of more able students. In this setup, quality gaps between schools in reveal no information about the schools' underlying characteristics, which can only be inferred by looking at profits.

While there is a complementarity between more productive schools and more able students, the fact that there is no price premium associated to higher quality prevents the more productive schools from supplying relatively more quality (compared to the less productive schools) to the more able students.

The insights provided by analyzing separately the markets for profit-maximizing and non-profit schools prove useful to understand how the allocation and provision of quality would look like in a market where both types of school exist.

Claim 6 *In equilibrium, profit-maximizing schools will choose not to compete with the non-profit schools, leaving the more able students in the non-profit sector.*

Proof. As shown, if private schools compete between themselves, profits for the school with productivity γ_j are $\pi_j = v'(1 - \frac{\gamma_{M+1}}{\gamma_j})$. If the same school wanted to compete for a given student with ability a_i with a nonprofit school of productivity γ_k, it would have to offer quality $q_{ij} = v' a_i \gamma_k$ [2]. Profits of any match of that type are $\pi_j = v'(1-\frac{\gamma_k}{\gamma_j})$[3]. But, for all operating nonprofit schools,

[2] Strictly, $q_{ij} = v' a_i \gamma_k + \epsilon$.

[3] Strictly speaking, even smaller:

$\gamma_k > \gamma_{M+1}$, so profits are strictly inferior when competing with the non-profit school. Thus, any profit-maximizing school is better off avoiding competing with a non-profit school. As non-profit schools unambiguously prefer to match with more able students (they only care about quality), profit-maximizing schools will optimally avoid competition for those students. ■

Thus, the market equilibrium allocation will imply positive assortative matching between the set of N_n^* most productive non-profit schools and the N_n^* more able students, with the remaining $N_p^* = M - N_n^*$ students enrolling in the profit-maximizing sector. The more able student will receive the maximum quality that can be offered by the most productive non-profit school (which, of course, need not to be the most productive school of the whole set, given the behavior of profit-maximizing schools). This is, $q^{\max}(m_{11(N_n^*)}) = va_1\gamma_{1(N_n^*)}$. In general, quality for the N_n^* more able students is of the form $q^{\max}(m_{ii(N_n^*)}) = va_i\gamma_{i(N_n^*)}, i = 1$ to N_n^*. For the remaining students, who enroll in the profit-maximizing sector, the quality is $q^{\max}(m^*_{N_n^*+i(M+1)}) = va_{N_n^*+i}\gamma_{M+1}, i = 1$ to N_p^*.[4]

For constant ability and productivity differences across all schools and students, the quality gap is higher between more able students (enrolled in the non-profit sector) than between less able students (enrolled in the profit-maximizing sector). There is (potentially) large drop in quality between the least able student in the non-profit sector and the most able in the profit-maximizing school, as the input provided by the school drops to reflect only the competitive pressure provided by the fringe competitor. Figure 3 illustrates the equilibrium allocation.

[4]When $n_{\max} > 1$, it was irrelevant whether the fringe competitor was a non-profit or a profit-maximizing school. Now, the condition that would make a profit-maximizing school indifferent (that he would get zero profits) implies that the non-profit school would strictly prefer to be inside the market. Thus, the quality level supplied by profit-maximizing schools in that case should be marginally above the one discussed here.

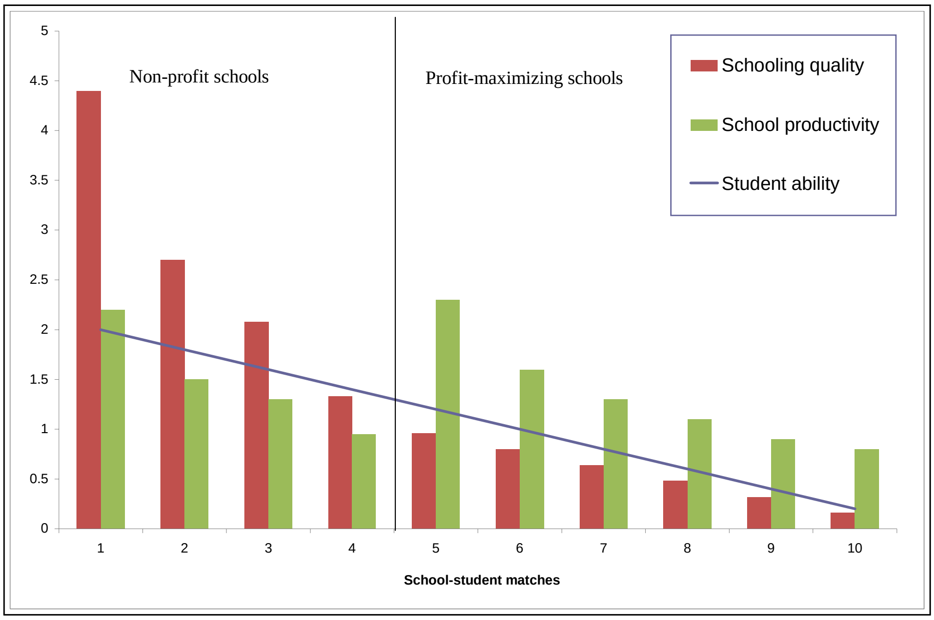

Figure 3: Education quality, student ability, and school productivity

2.3 No limits on charging

Taking the same static setup, assume now that schools are free to charge as they wish. Assumptions about the production function and capacity restrictions (one school student per school) still hold. For the moment, leave aside nonprofit schools, so our attention will be focused on the behavior of profit-maximizing schools.

As in the case with pure vouchers, only the M most productive schools will be able to operate, and the productivity of the fringe competitor will be relevant for setting equilibrium qualities. However, unlike the previous case, equilibrium qualities will not be solely determined by the threat of entry and the productivity of the marginal school. When schools are allowed to charge a higher price for higher qualities, more productive schools can exploit their competitive advantage to satisfy the demand of consumers with a higher willingness to pay. Equilibrium qualities of prices will not only reflect productivity from the school outside the market (competitive pressure from outside), but also the productivities of schools within the market.

When only pure vouchers were considered, demand and preferences were largely irrelevant, as long as higher quality was strictly preferred to less. Here, however, the exact form of preferences, and the resulting demand for quality, will be central for the behavior of schools and the equilibrium outcomes.

While education can be reasonably considered a normal good, and thus will increase with parental income, the effects of ability are ambiguous. On the one hand, higher ability increases the marginal return of investment, making investing in more education more attractive. On the other hand, as for any given investment the human capital of the descendant is higher (so the marginal utility associated is smaller), substituting education investments for parental consumption becomes more attractive. For the moment, assume that both effects cancel out (the substitution effect is of the same size as the scale effect), so that the demand for quality does not depend on the kid's characteristics, but only on parental income. In particular, assume that education expenditure is a fixed share of the households income:

$$p_i q_i = \beta Y_i \tag{9}$$

where β is the same across all households and p_i is the price per unit of quality faced by household i (the price will depend on household's characteristics).

To solve for the equilibrium, we rank all households in terms of income, Y_1 to Y_M, with Y_1 being the highest. a_i is the ability of the kid coming from the household with income Y_i.This implies, of course, that abilities are not ranked, unless there was perfect positive correlation between parental income and descendant ability, which need not be. Thus, in general, while for $j > i, Y_i \geq Y_j, a_i \geq a_j$ or $a_i \leq a_j$.

Claim 7 *When schools are allowed to set prices freely and education expenditures are a constant share of income, there is strict assortative matching between more productive schools and higher income households. Differences in equilibrium qualities reflect the characteristics of both the school and the student.*

Claim 8 *Equilibrium qualities and prices are only partially determined by threat of entry (the productivity of the marginal school). Except for the student with the lowest income, students face lower prices and receive higher quality than what would be implied by threat of entry of the marginal school.*

To see this, consider the marginal cost of providing quality for the fringe competitor, the school with productivity γ_{M+1} :

$$c'(q, a_i) = \frac{1}{a_i \gamma_{M+1}} \tag{10}$$

If, as in the pure voucher case, only the productivity of the marginal school was relevant, then the ability-adjusted price per unit of quality would be the one that keeps γ_{M+1} out of the market:

$$p_i = \frac{1}{a_i \gamma_{M+1}} \tag{11}$$

It is clear that this cannot be an equilibrium, as it implies that parents with a given ability would be indifferent across schools (they all charge the same per unit of quality), but schools would not be indifferent across students, as profits would be strictly higher when enrolling richer parents. Take, for example, the profits for school γ_i of matching with students Y_j and Y_k at $p_i = \dfrac{1}{a_i \gamma_{M+1}}$.

$$\pi_i(j) = \beta Y_j \left(1 - \frac{\gamma_{M+1}}{\gamma_i}\right) > \beta Y_k \left(1 - \frac{\gamma_{M+1}}{\gamma_i}\right), Y_j > Y_k \tag{12}$$

As the markup per unit of quality is constant at $1 - \frac{\gamma_{M+1}}{\gamma_i}$, selling more quality is always more profitable.

Thus, as students from richer households are more attractive, competition between schools implies that they must face lower ability-adjusted prices (of course, actual prices for a richer household might be larger than the price of a household with lower income if the relative ability of the student from a poor background is high enough). Moreover, as more productive schools are more efficient at producing at a lower cost, they will be able to match with the households with higher willingness to pay.

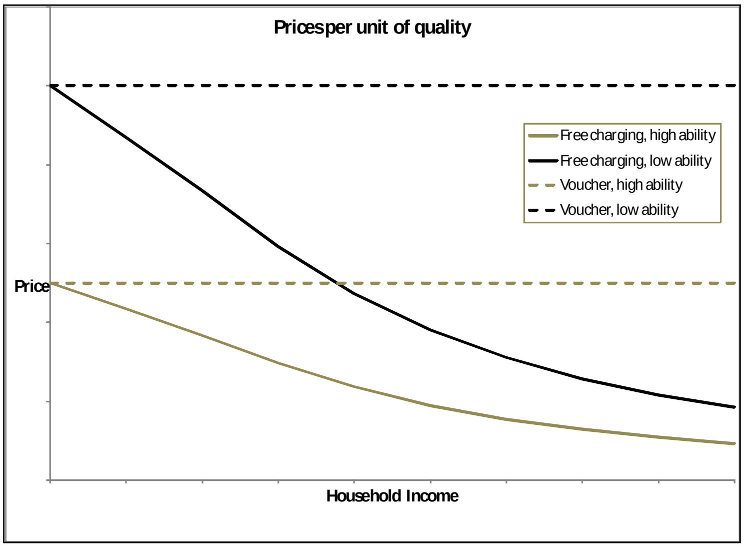

Figure 4: Pricing for students with different ability and income

To illustrate this, take the case of the least productive school inside the market, γ_M. For the student with the lowest income, Y_M, the ability-adjusted equilibrium price, p_M, is given solely by the productivity of the fringe competitor, $p_M = \dfrac{1}{a_M \gamma_{M+1}}$, and the associated equilibrium quality would simply be $q_M = \beta Y_M a_M \gamma_{M+1}$. The ability-adjusted quality (ability-adjusted price) for the student with the second-lowest income,Y_{M-1}, must be high (low) enough that school γ_M is indifferent between matching with any of them:

$$\pi_M(Y_M) = \pi_M(Y_{M-1}) \rightarrow \beta Y_M \left(1 - \frac{\gamma_{M+1}}{\gamma_M}\right) = \beta Y_{M-1} - \frac{q_{M-1}}{a_{M-1\gamma_M}} \qquad (13)$$

, which implies that the quality and price for student Y_{M-1} must be:

$$\begin{aligned} q_{M-1} &= \beta(a_{M-1})\left[Y_{M-1}(\gamma_M) + Y_M(\gamma_{M+1} - \gamma_M)\right] \\ p_{M-1} &= \frac{\beta Y_{M-1}}{\beta(a_{M-1})\left[Y_{M-1}(\gamma_M) + Y_M(\gamma_{M+1} - \gamma_M)\right]} \end{aligned}$$

It is clear that this quality is higher (and thus, the unit price lower) than the one that the one associated only with the productivity of the fringe competitor:

$$\begin{aligned} &\beta(a_{M-1})\left[Y_{M-1}(\gamma_M) + Y_M(\gamma_{M+1} - \gamma_{M+1})\right] - \beta(a_{M-1})(Y_{M-1})(\gamma_{M+1}) \\ &= (Y_{M-1} - Y_M)\left(\gamma_M - \gamma_{M+1}\right) > 0 \end{aligned}$$

Also notice how school γ_{M-1} is better off, at ability-adjusted qualities q_M and q_{M-1}, matching with the student with income Y_{M-1}. To see this, calculate the profits for each match:

$$\begin{aligned} \pi_{M-1}(Y_M) &= \beta Y_M - \frac{\beta Y_M(\gamma_{M+1})}{\gamma_{M-1}} \\ \pi_{M-1}(Y_{M-1}) &= \beta Y_{M-1} - \frac{\beta Y_{M-1}(\gamma_M)}{\gamma_{M-1}} - \frac{\beta Y_M(\gamma_{M+1})}{\gamma_{M-1}} + \frac{\beta Y_M(\gamma_{M+1})}{\gamma_{M-1}} \end{aligned}$$

The difference in profits between matching with Y_{M-1} and Y_M can be written as:

$$\begin{aligned} &\pi_{M-1}(Y_{M-1}) - \pi_{M-1}(Y_M) \\ =\ &\beta Y_{M-1} - \frac{\beta Y_{M-1}(\gamma_M)}{\gamma_{M-1}} - \frac{\beta Y_M(\gamma_{M+1})}{\gamma_{M-1}} + \frac{\beta Y_M(\gamma_M)}{\gamma_{M-1}} - \beta Y_M + \frac{\beta Y_M(\gamma_{M+1})}{\gamma_{M-1}} \\ =\ &\beta Y_{M-1} - \frac{\beta Y_{M-1}(\gamma_M)}{\gamma_{M-1}} + \frac{\beta Y_M(\gamma_M)}{\gamma_{M-1}} - \beta Y_M \\ =\ &\beta(Y_{M-1} - Y_M)\left[1 - \frac{\gamma_M}{\gamma_{M-1}}\right] > 0 \text{ as } Y_{M-1} > Y_M \text{ and } \gamma_{M-1} > \gamma_M \end{aligned}$$

Thus, under the proposed equilibrium prices, the least productive school is indifferent across matches, while the school with a higher productivity prefers to match with the student with higher income. Thus, this can hold as an equilibrium with assortative matching.

By extension, and making each school indifferent between matching with the student that holds the same ranking in the income distribution as the school holds in the productivity distribution or with the student immediately above, one can derive that the equilibrium ability-adjusted quality and price for student with income Y_{M-j} will be:

$$\begin{aligned} q_{M-j} &= \beta(a_{M-j}) \left[Y_{M-j}(\gamma_{M+1-j}) - \sum_{i=0}^{j-1} Y_{M-i} \left(\gamma_{M-i} - \gamma_{M+1-i} \right) \right] \\ &= \beta(a_{M-j}) \left[Y_M \gamma_{M+1} + \sum_{i=0}^{j-1} \gamma_{M-i} \left(Y_{M-1-i} - Y_{M-i} \right) \right] \end{aligned} \tag{14}$$

$$p_{M-j} = \frac{Y_{M-j}}{(a_{M-j}) \left[Y_{M-j}(\gamma_{M+1-j}) - \sum_{i=0}^{j-1} Y_{M-i} \left(\gamma_{M-i} - \gamma_{M+1-i} \right) \right]} \tag{15}$$

Ability-adjusted equilibrium prices (qualities) are strictly lower(higher) than the ones that would be determined solely by threat of entry of school γ_{M+1}. Differences in quality across students not only reflect differences in ability, as in the vouchers case, but also differences in income and in school productivity. Controlling for ability, richer students are better off because they can purchase more quality at any given price, and because they face lower prices, reflecting the fact that they attend schools that are intrinsically more productive.

Notice, however, that the most productive school need not be the one that provides the highest quality. Its quality may well be relatively low, compared to other less productive schools, if the ability of the student from the richest household is sufficiently low.

3 Some extensions of the basic setup

This section presents some simple extensions of the basic framework of competitive markets with vouchers with no payments on top.

3.1 Intertemporal model and the role of entry

So far, the analysis has been restricted to a static setup with only one period, which could interpreted as an intertemporal model with independent periods. To allow for periods to be connected in time, consider now a setup in which schools must make a fixed investment to operate. An entrepreneur lives for two periods, and can open his school in the first or second period of his existence. Opening a school implies paying a cost f, which is paid only once, even if the school operates for two periods. The school can be closed at no cost, and, if operational, always closes after the entrepreneur's "death " at the end of his second period. Again, we focus only on profit-maximizing schools. There is no discount rate.

Begin at period 1 with a new cohort of entrepreneurs. Assume that there are no "old " entrepreneurs from the previous period, and that no new entrepreneurs are expected in period 2. Thus, the same set of potential schools operates both periods.

In general, and in particular in a general equilibrium setup where the voucher is endogenous, nothing prevents the voucher from being different across periods. For the moment, we impose a constant voucher in both periods.

Claim 9 *If entry is feasible in both periods, and the fringe competitor could successfully deter entry on the second period, equilibrium qualities in the first period are larger than in the second period for all schools that could be successfully deterred. Within those schools, first period quality is the largest for the least productive school*

(Formal proof is omitted for the moment)

Feasibility in every period implies that $v > f$, so a school can get positive profits even if it operates for only one period. Thus, threat of entry is present in period 1 and period 2. The second part of the statement refers to the competitive advantage of being already in the market in the second period. For a new entrant in that period, γ_j,the zero-profit quality to student with ability a_i is $(v-f)a_i\gamma_j$. For any school with the same productivity that is already in the market, the zero-profit quality for the same student is strictly larger, $va_i\gamma_j$.

This implies that, conditional on first period entry, the school with productivity γ_{M+1}, which is always outside the market in a static setup, could leave a more productive school out of the market, and get non-negative profits, as long as

$$va_i\gamma_{M+1} > (v-f)a_i\gamma_j \rightarrow \frac{\gamma_{M+1}}{\gamma_j} > \frac{(v-f)}{v} \tag{16}$$

If the above condition holds, school γ_{M+1} would be willing to incur a loss in period 1 in order to enjoy the competitive advantage of being a successful incumbent and earn profits on period 2.

Thus, the static equilibrium qualities in period 1, which imply zero-profits for school γ_{M+1}, will be too low for schools for which $\gamma_{M+1}\frac{(v)}{v-f} > \gamma_j$, as the fringe competitor will profitably provide more quality.Notice that this is not true for the most productive schools, those for which $\gamma_{M+1}\frac{(v)}{v-f} < \gamma$,as school γ_{M+1} could never profitably deter their entry in the second period, and thus will not be willing to get negative profits in period 1.

Thus, first-period quality for the least productive entry must be such that the potential losses for school γ_{M+1} offset the potential profits associated to being an incumbent in period 2. This is, for school with productivity γ_j, its first period quality per unit of ability, q_{1j}, must imply that intertemporal profits of entry for school γ_{M+1} are zero:

$$\pi_{M+1}(a_i, t=1) + \pi_{M+1}(a_i, t=2) \leq 0 \rightarrow \tag{17}$$

$$v - f - \frac{q_{ti}}{a_i \gamma_{M+1}} + v - \frac{(v-f)\gamma_j}{\gamma_{M+1}} \leq 0 \rightarrow \tag{18}$$

$$q_{1j}(a_i) = (v-f)a_i\gamma_{M+1} + a_i\left[v\gamma_{M+1} - (v-f)\gamma_j\right]$$

$$q_{1j} = (v-f)\gamma_{M+1} + \left[v\gamma_{M+1} - (v-f)\gamma_j\right]$$

$\frac{(v-f)\gamma_j}{\gamma_{M+1}}$ is the cost for school γ_{M+1} of providing the quality that would leave γ_j out in the second period if γ_{M+1} is able to enter in period 1.The first term in the equilibrium quality is the same as the ability-adjusted quality provided in the static case. The second term, positive if $\gamma_{M+1}\frac{(v)}{v-f} > \gamma_j$, reflects the profit that school γ_{M+1} could make on period 2 conditional on entry on period 1. As less productive schools are more vulnerable, they must provide higher qualities in period 1 to keep school γ_{M+1} out. Notice that the least productive schools potentially earn negative profits on the first period.

Thus, on the first period, equilibrium qualities per unit of ability are:

$$q_{1j} = (v-f)\gamma_{M+1} + \left[v\gamma_{M+1} - (v-f)\gamma_j\right] \text{ if } \gamma_j < \gamma_{M+1}\frac{(v)}{v-f} \tag{19}$$

$$q_{1j} = (v-f)\gamma_{M+1} \text{if } \gamma_j \geq \gamma_{M+1}\frac{(v)}{v-f} \tag{20}$$

Notice how this implies that the least productive school would be strictly preferred by all students, as it provides strictly more quality. However, as before, schools are indifferent between students.

On the second period, the setup reverts to a static case with fixed costs, and qualities are identical across all schools, and high enough to prevent entry from school γ_{M+1} :

$$q_{2j} = (v-f)\gamma_{M+1} \text{ for all } j. \tag{21}$$

Claim 10 *If entry is feasible in both periods, but productivity of the fringe competitor is sufficiently low, equilibrium qualities are the same as in the static case*

This is simply the case in which $\gamma_M \geq \gamma_{M+1}\frac{(v)}{v-f}$, so that no school can be left out of the market if γ_{M+1} became an incumbent for period 2. Thus, quality per unit of ability that makes school γ_{M+1} stay out in each isolated period is enough to prevent entry:

$$q_{1j} = q_{2j} = (v-f)\gamma_{M+1} \text{ for all } j.$$

Claim 11 *If one-period entry is not feasible, second-period qualities are as small as feasible, as there is no threat of entry. First-period qualities are strictly larger than in the static case for all schools.*

In this case, $v < f$ [5].Thus, on the second period, no school presents a threat, and quality will collapse as there are no competitive pressures. Threat of entry only exists on the first period.

Assume that there is some exogenous minimum level of quality, $q_{\min}$, that must be supplied (otherwise, any student is better off staying at home and the school does not get the voucher). It is easy to see that $q_{\min}$ will be the quality supplied to the least able student on the second period, and that competition implies that quality with ability a_i will simply be $\frac{a_i}{a_M}q_{\min.}$

If school γ_{M+1} is able to enter in period 1, then, it could make profits in period 2, as it will not face any competition. As before, quality in period 1 must exceed the zero-profit level for γ_{M+1}.Unlike the previous case, however, all schools must supply this higher quality in the first period, as an incumbent can always stay in the market in period 2 given that income for one period does not cover costs.

Thus, first period quality must imply that intertemporal profits for school γ_{M+1} are zero:

$$v - f - \frac{q_{ti}}{a_i\gamma_{M+1}} + v - \frac{q_{\min.}}{a_M\gamma_{M+1}} \leq 0 \rightarrow \tag{22}$$
$$q_{1j}(a_i) = (v-f)a_i\gamma_{M+1} + a_i\left[v\gamma_{M+1} - \frac{q_{\min.}}{a_M}\right]$$
$$q_{1j} = (v-f)\gamma_{M+1} + \left[v\gamma_{M+1} - \frac{q_{\min.}}{a_M}\right] \text{ for all j}$$

Obviously, the smaller $q_{\min}$,the larger the equilibrium quality in the first period, and the larger the decrease in quality between both periods.

Now, consider a case in which new entrepreneurs can appear in the second period For simplicity, assume that only one new entrepreneur is expected to appear, with expected productivity γ'.

Before a more generalized analysis of new potential entrants is made, a simple result can .be written as direct extensions of what was discussed before.

[5]But,of course, $2v > f$, as otherwise no school could open in period 1 either.

Claim 12 *Expectations of new entry on the second period may lower quality in the first period, as the fringe competitor might lose incentives to enter on the first period*

This is a simple extension from the case where $\dfrac{\gamma_{M+1}}{\gamma_M} < \dfrac{(v-f)}{v}$. If the fringe competitor in period 1 knows that the potential newcomer in period 2 could not be deterred ($\gamma' > \gamma_{M+1}\frac{(v)}{v-f}$), the potential profits on period 2 vanish, and entering at a loss in period 1 is no longer attractive. Thus, as expected profits in period 2 collapse to zero, operating schools in period 1 just need to supply the quality that makes profits for school γ_{M+1} zero in that period. First-period quality is the same as in the static case.

3.2 The choice of scale

With the same technology as in Section 2, allow schools to choose the number of students they enroll. For each student it enrolls, the school must pay a fixed cost f, plus the marginal cost of providing quality to each student. Within a school, all students must receive the same quality (if it enrolls students with heterogeneous ability, the school cannot offer them different qualities). Schools are financed with vouchers with no payments on top.

Take the simplest case, with 2 profit-maximizing schools, with productivities $\gamma_1 > \gamma_2$, which compete to serve two students, with abilities $a_1 > a_2$. From the previous discussion, it is easy that there are two potential allocations. In the first one, the most productive school serves both students. Thus, there is a unique "large " school with a heterogeneous student body. There is only level of quality provided in equilibrium. In the other allocation, there are two small homogeneous schools, and students with different abilities receive different qualities.

Claim 13 *If the heterogeneity in abilities is relatively larger than the heterogeneity in productivity, both schools operate. If the reverse holds, the most productive school will hold both types of students.*

To prove this, consider the profits of the most productive school if it serves both students. To be able to do that, and keep γ_2 out of the market, the quality provided must be $q_1(a_1, a_2) = (v-f)a_1\gamma_2$. At every other quality below that, the least productive school can provide a higher quality to the ablest student and get a positive profit, breaking the allocation. Thus, enrolling both students implies providing the highest feasible quality that γ_2 can provide to the ablest student. Profits for school γ_1 are then:

$$\pi_1(a_1, a_2, q_1(a_1, a_2)) = (v-f) - \frac{q_1(a_1,a_2)}{a_1\gamma_1} + v - f - \frac{q_1(a_1,a_2)}{a_2\gamma_1} \quad (23)$$
$$= (v-f)\left(1-\frac{\gamma_2}{\gamma_1}\right) + (v-f)\left(1-\frac{a_1\gamma_2}{a_2\gamma_1}\right)$$

The marginal profit for enrolling the most able student conditional on enrolling both students, $\pi_1(a_1/a_1, a_2) = (v-f)\left(1-\frac{\gamma_2}{\gamma_1}\right)$, is always positive as $\frac{\gamma_2}{\gamma_1} < 1$. The marginal profitability of the less able student, $\pi_1(a_2/a_1, a_2)$, however, will depend on whether $\frac{a_1\gamma_2}{a_2\gamma_1} < 1$. In particular, marginal profits are only positive if $a_1\gamma_2 < a_2\gamma_1 \Leftrightarrow \frac{a_1}{a_2} < \frac{\gamma_1}{\gamma_2}, \pi_1(a_2/a_1, a_2) > 0$. This is, if the ability gap between students is smaller than the productivity gap between schools, the most productive school can make a profit by accepting the second student.

In allocation where each school gets one student, the equilibrium qualities must satisfy three conditions:

(a) No arbitrage across students: $\pi_i(a_1, q_1(a_1)) = \pi_i(a_2, q_2(a_2)), i = 1, 2$

(b) The most productive school has no incentive to attract a second student: $\pi_1(a_2/q_1) = 0$

(c) Profits for the least productive school are non-negative: $\pi_2(a_1, q_1) = \pi_2(a_2, q_2) \geq 0$

The first condition simply states that, in equilibrium, there cannot be a profitable deviation for any of the schools if they shift to serve the other student. As before, that implies that they will be indifferent between them, so we cannot say which school matches what student. From this condition, it is easy to pin down the quality ratio in equilibrium as $\frac{q_1}{q_2} = \frac{a_1}{a_2}$.

The quality level is determined by (b), which must hold so that the most productive school has no incentive to expand and hold two students. While in the case with fixed capacity schools quality levels were determined by the threat of entry of the fringe school, here they are determined by the threat of expansion of the most productive school. Competitive pressure occurs from within the market. From (b), $\pi_1(a_2/q_1) = 0 \Rightarrow (v-F) = \frac{q_1(a_1)}{a_2\gamma_1}$, so that:

$$q_1(a_1) = (v-f)a_2\gamma_1 \quad (24)$$
$$q_2(a_2) = (v-f)\frac{a_2^2}{a_1}\gamma_1$$

Finally, it must be true that the least productive school is able to make non-negative profits.

$$\pi_2(a_1, q_1) = (v-f) - (v-f)\frac{a_2\gamma_1}{a_1\gamma_2} \Rightarrow \pi_2(a_1,q_1) \geq 0 \Leftrightarrow a_2\gamma_1 \leq a_1\gamma_2$$
$$\pi_2(a_1, q_1) \geq 0 \Leftrightarrow \frac{\gamma_1}{\gamma_2} \leq \frac{a_1}{a_2}$$

, which implies that the second school is viable, and the allocation with two schools sustainable, if differences in ability are larger than differences in productivity. This is just the reverse of the condition on whether the productive school made profits by serving both students. In fact, it can be shown that $-\pi_1(a_1/a_1, a_2) = \pi_2(a_1, q_1)$, and that $\pi_1(a_1/a_1, a_2) > 0 \Leftrightarrow \pi_1(a_1, a_2, q_1(a_1, a_2)) > \pi_1(a_1, q_1(a_1)) = \pi_2(a_2, q_2(a_2))$.

Thus, which equilibrium holds depends on how the ability gap measures against the productivity gap. If $\frac{\gamma_1}{\gamma_2} < \frac{a_1}{a_2}$, and ability differences are relatively larger, supplying the same (high) quality to both types of students is never attractive, and schools specialize. Schools are small, homogeneous, and quality varies reflecting differences in ability. If If $\frac{\gamma_1}{\gamma_2} > \frac{a_1}{a_2}$, and the technological advantage of the most productive school is relatively large, that school will find it attractive to expand and have an heterogeneous school body. There is a unique level of quality across the market, so that abler students are no longer better off relative to the less able.

3.3 Preferences on non-academic orientation

Assume that the households' utility function can be written now as:

$$U = U(C, h, R)$$

R measures the distance between the non-academic aspects provided by the school in which the child is enrolled and the ones that best reflect the preferences of the households (such as moral values, religion, discipline, social status, etc.)[6].

More specifically, define the utility function as:

$$U = v(c_i) + \beta u(h_i) - \delta R \tag{25}$$

where b is a measure of parental altruism and $v\prime, u\prime > 0, v\prime\prime, u\prime\prime \leq 0, \delta \geq 0$.

The distance from the school's orientation is defined as:

$$R = (r_s - r_i)^2 \tag{26}$$

, where r_s are the observable "values" provided by school s and r_i is the optimal set of values the household would like its child to receive. Think, for

[6] Of course, it could be also interpreted as a measure of transportation costs.

example, as the $r_s = 0$ representing non-religious education and $r_s = 1$ religious education. For simplicity, assume that there only the two extreme locations are feasible, so that schools cannot be only partially religious[7]. Also assume that all nonprofit schools are religious (this is, they will "locate" at $r_s = 1$). This assumption is by no means crucial, but helps to obtain a clearer analysis. Private profit-maximizing schools, who do not derive direct utility from their orientation, must decide whether to provide non-religious ($r_s = 0$) or religious education ($r_s = 1$). The choice of orientation is costless.

A household would be indifferent between sending its kid with ability a_i to schools at either location if:

$$\beta u(f(q_{i0}, a_i)) - \delta(r_i - 0)^2 = \beta u(f(q_{i1}, a_i)) - \delta(r_i - 1)^2$$

$$u(f(q_{i0}, a_i)) - u(f(q_{i1}, a_i)) = \frac{\delta\left[2r - 1\right]}{\beta} \tag{27}$$

The result is pretty straightforward. Agents will demand increasingly more relative quality to the school that is farther from their religious preferences. A non-religious parent will be willing to send his kid to a religious school only if the quality-premium for doing so is sufficiently high.

Is this affected by ability? A larger distance from the household's religious preferences can be interpreted as the price of having access to higher quality. Then, whether differences in the schools' orientation become less relevant relative to differences in quality as ability increases will depend on the effect of ability on the demand for quality. If the demand for quality does not depend on ability, such that the substitution and scale effects associated to it cancel out, the indifference condition between quality and school orientation will not depend on the student's ability level. If the demand for quality is increasing in the student's ability, differences in orientation will become less relevant as we move towards the top of the ability distribution.

Take, for example, the simple case where $u(h_i) = h_i$ and $f(q_{ij}, a_i) = a_i^{\alpha} q_{ij.}$The indifference condition simply becomes:

$$\begin{aligned} \beta a_i^{\alpha} q_0 - \delta(r_i - 0)^2 &= \beta a_i^{\alpha} q_1 - \delta(r_i - 1)^2 \\ \beta a_i^{\alpha}\left[q_0 - q_1\right] &= \delta\left[2r - 1\right] \\ \left[q_0 - q_1\right] &= \frac{\delta\left[2r - 1\right]}{\beta a_i^{\alpha}} \end{aligned} \tag{28}$$

As ability grows, differences in orientation become relatively less important than differences in quality, as the substitution effect associated to higher ability

[7] So this setup is more restrictive than usual location decisions.

is strong enough. Parents with more able children are less willing to give up a high quality school for a worse school that better suits their orientation. Without any kind of specific selection from the school, in a high-quality religious school we would expect to see, among the children from non-religious households, a higher share of more able students than on the set of children from religious households.

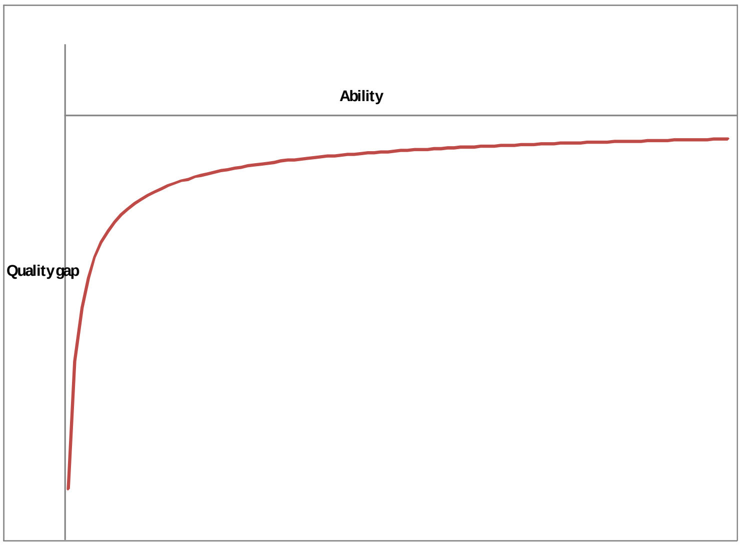

Figure 5: Ability and the quality gap

To simplify the analysis further, also assume that the distribution of preferences is such that parents are either completely religious ($r_i = 1$) or completely non-religious ($r_i = 0$) . Unlike the discussion in Section 2.2, now it will not be always true that the best students go to the nonprofit schools. When preferences for orientation are included, profit-maximizing schools have the competitive advantage of being able to tailor their orientation to the preferences of non-religious students, who would be willing to take less quality if they are able to attend a non-religious school. Under certain circumstances, this will allow profit-maximizing schools to leave out of the market religious schools who are more productive than them, or to provide lower quality than they would otherwise require to. While this outcome will imply that average quality can be smaller, students are compensated with an education that better suits their preferences, so that their welfare is at least the same (otherwise, there would be no room for "arbitrage" by the profit-maximizing schools).

Claim 14 *For any given distribution of preferences for orientation, the equilibrium allocation in setups where there are only nonprofit or only profit-maximizing* *schools will be the same as if parents only cared about quality*

Proof. *If only nonprofit schools exist, they all provide religious education (they are at* $r_s = 0$*). Under the assumption that schools only care about quality, it is still true that they all share the same ordering when ranking their potential matches. As non-religious students face the same loss when attending any potential school (*δ*), potential schools only differ in the quality they offer. The same is true for religious students, who get their preferred orientation at any school. Thus, quality is the only distinction among all potential matches, and the allocation would be the same as in Section 2.2, although religious students are strictly better off, for a given ability, than non-religious ones. If only profit-maximizing schools exist, for any given student they must match the best feasible offer from the fringe school. That offer includes the zero-profit quality for the fringe school - given the student's ability - as well as an orientation that matches the student's . This is, in equilibrium no religious student will be offered non-religious education, and viceversa. Quality levels are the same as when no preferences for orientation were considered.* ■

Thus, the introduction of preferences for orientation does not change the main results for markets where exclusively one class of school exists. However, when both types of schools exist, the fact that profit-maximizing schools enjoy an advantage in their ability to supply the type of education most preferred by non-religious students might alter some of the results discussed previously.

The parent of a non-religious student would be indifferent between quality offer q_R made from a religious school and the quality offer q_{NR} from profit-maximizing school providing non-religious education if $q_{NR} = q_R - \frac{\delta}{\beta a_i^{\alpha}}$. Parents of children with smaller ability or who care relatively more about the school's orientation are willing to accept a higher drop in quality in order to get enrolled in a non-religious school. This implies that, if it finds it profitable to do so, a profit-maximizing school can outbid a non-profit school even if the religious school is more productive. Of course, this effect does not exist when dealing with religious students.

When will profit-maximizing schools exploit their advantage when dealing with non-religious students? A simple example will illustrate the point.

Assume a market with two students, identified by abilities a_1 and a_2. There are three potential schools, identified by productivities γ_1 to γ_3.

Claim 15 *In the equilibrium allocation, the schools inside the market need not to be the most productive ones.*

Proof. *Assume the two most productive schools are nonprofit and that both students are non-religious. If those schools remain in the market, they will match assortatively. The school with productivity* γ_2 *would match with the student with ability* a_2 *at* $q^{\max}(\gamma_2, a_2) = va_2\gamma_2$*. The profit-maximizing school would get negative profits at that quality. However, given that the student is non-religious,* *he would strictly prefer to leave the religious school if* $q_{NR} > q_{22} - \frac{\delta}{a_2^\alpha}$. *Assume that* $va_2\gamma_3 > q_{22} - \frac{\delta}{a_2^\alpha} + \varepsilon$, *where* $\varepsilon \to 0$. *Thus, the profit-maximizing school can get into the market and earn positive profits, driving out a more productive religious school.* ■

Profit-maximizing schools can, in fact, exploit their advantage and be able to get students even if the supply a smaller quality. While this reduces the aggregate level of quality, on the margin students are not worse off (they would not accept the offer otherwise) as the reduction in quality is compensated by an orientation that better suits their preferences. Notice, also, that even if they profit-maximizing school could arbitrage against the higher ability student a_1, it would never do so, as profits would be strictly smaller (while it is true that more able students allow to produce cheaper quality, they are matched with more productive schools, and are not willing to sacrifice as much quality in order to get their preferred orientation). In that sense, it is still true that, in general, profit-maximizing schools prefer to avoid competition with non-profit schools. If they decide to compete, they will do it for the less able students.

However, the gains to a profit-maximizing school of its advantage to freely choose orientation are limited by the fact that potentially all profit-maximizing schools can do the same. Assume that γ_1 is non-profit and that γ_2, γ_3 are profit-maximizing schools. a_1 is avnon-religious student, a_2 religious. If γ_2 matches with a_2, it must provide $q_{22} = q^{\max}(a_2, \gamma_3) = va_2\gamma_3$. If so, it earns profits $\pi_2(a_2) = v\left(1 - \dfrac{\gamma_3}{\gamma_2}\right)$. Alternatively, it could exploit the opportunity to get the non-religious student a_1 at $q_{12} = va_1\gamma_1 - \frac{\delta}{a_1^\alpha} + \varepsilon, \varepsilon \to 0$, and get $\pi_2(a_1) = v\left(1 - \dfrac{\gamma_1}{\gamma_2}\right) + \dfrac{\delta}{\gamma_2 a_1^{\alpha+1}}$. However, it is easy to show that if $\pi_2(a_1) > \pi_2(a_2)$, so the arbitrage is profitable, then $\pi_3(a_1) > 0$, so the arbitrage can also be made by the fringe profit-maximizing school[8]. This implies that, in order to get student a_1, school γ_2 must not only deal with the religious school, but also against the threat of entry of the profit-maximizing school γ_3.Then, $q_{12} = q^{\max}(a_1, \gamma_3) = va_1\gamma_3 > va_1\gamma_1 - \frac{\delta}{a_1^\alpha}$, such that on equilibrium $\pi_2(a_1) = \pi_2(a_2)$.This implies that the potential gains of exploiting the orientation handicap of non-profit schools are washed away by competition among profit maximizing schools[9].

Thus, the introduction an additional dimension in preferences, one that is not related to quality, opens the space for competitive outcomes that are characterized by smaller average quality, as students are willing to receive less quality

[8] Note that neither profit-maximizing school could compete with the nonprofit school for the religious student a_2.

[9] This also leads to the somehow uncomfortable result that we have two equilibria, with the two most productive schools potentially matching any of the two students. While both equilibria are the same in terms of utility for the profit-maximizing schools, they are not the same in terms of utility for any of the other actors. The nonprofit school is better off in the equlibrium where it matches with a_1, while both students are better off in the other equilibrium.

in order to get the non-academic characteristics that they prefer. While this can go against the spirit of the voucher (to get higher quality), households cannot be worse if they are willing to enroll in those schools. Moreover, as long as potential competitors exist, and those non-academic characteristics can be replicated at a small cost, the negative effect on quality will be smaller.

4 An Application: Schooling regimes in a general equilibrium setup

4.1 Households

The economy is populated by N households. Each household has exactly one child in schooling age. Households maximize separable utility over current consumption and on the human capital of the child,

$$U(c,h) = u(c) + \beta v(h_1) \tag{29}$$

,where the utility function satisfies the usual assumptions and β is a measure of parental altruism.

As before, the production of human capital is a function of the quality of education received by student i at school j, q_i,and of the student's given ability, a_i :

$$h_{i1} = f(q_{ij}, a_i)$$

, where $f_a, f_q, f_{aq} > 0$, $f_{aa}, f_{qq} < 0$.

Notice that there are no direct parental inputs on the production of schooling, and that parents only care about the aspects of education that directly affect the student's human capital.

Parents are endowed with their own human capital, h_{i0}.Human capital is the sole input in the production of the economy's unique non-storable good, which can be used for consumption or as an input the production of human capital at the school level. There is no physical capital, and agents can only invest in the human capital of their child. There are no capital markets, so agents cannot lend or borrow.

As agents have no utility on leisure, and human capital production does not require parental time, agents inelastically supply all their human capital to production. There is no explicit labor market: agents are self-employed, and have access to a common production technology, where production equals the monetary income of the parents:

$$y_i = h_{it0}^{\varphi} \tag{30}$$

In general, the relevant budget constraint for household i can be written as:

$$c_i + p_i q_i = (1-\tau)y_1 = (1-\tau_t)h_{i0}^{\varphi} \tag{31}$$

where τ_t reflects an income tax levied to finance public education (either direct or through vouchers) and p_i is the market price per unit of school quality faced by the household.

4.2 Schools

Besides their human capital endowments, households are endowed with productivity as educational entrepreneurs. Schools produce education quality in exchange for tuition fees. More productive entrepreneurs can produce a given level of quality at a smaller cost. As households have no altruistic preferences on the utility of other households, all schools will be profit-maximizers (or, as discussed below, rent-seekers).

There are no fixed costs of opening a school. Education quality is produced by a standard technology, $q = g(.)$, which can be written as:

$$q_{ij} = g(y, a_i, \gamma_j)$$

, where $g_y, g_a, g_\gamma > 0$, $g_{yy} \leq 0$. For a given input, quality increasing in school productivity and student ability. The only input in the production of education quality is the economy's consumption good, y [10][11]. All schools have a fixed (exogenous) scale.

4.3 Solving the model

To solve explicitly for equilibrium, assume that the household's utility function can be written as:

$$U(c,h) = \ln(c) + \beta \ln(h_1) \tag{32}$$

, with the human capital production being described by:

$$h_{i1} = q_i^{\alpha} a_i^{1-\alpha} \tag{33}$$

[10] This is, an entrepreneur does not use his time endowment if it decides to open a school, so he can still work in the production of consumption goods. We could also state a model in which households had to choose between working in the production of goods or as educational entrepreneurs. While that is certainly an interesting extension, it is not crucial for the main topics analyzed here, and it inclusion would not make a significant difference for the results presented below.

[11] There are no explicit teachers in the model. In fact, as everybody is self-employed, there is no labor market, either. Explicitly allowing for a labor market with agents that can work as teachers or in goods production is certainly an relevant part of a more realistic model, which is left aside for the moment. As discussed in the previous note, the main qualitative results of the analysis should not be affected by this simplification.

Thus, the household's demands for consumption and school quality can be written as:

$$c_i = (1-\eta)y_i \tag{34}$$

$$q_i = \frac{\eta y_i}{p_i} \tag{35}$$

, where $\eta = \frac{\beta\alpha}{1+\beta\alpha}$. Ability plays no role in the demand for education, as the substitution and scale effects associated to it being an input in human capital production exactly cancel out.

Assume that the initial distribution of human capital is such that there are 3 income groups: high-income households, with income Y_1 , medium-income households, with income Y_2,and low-income households with Y_3[12]. The groups are equally sized, so each income category has $N/3$ households. Also assume that there is no heterogeneity in ability, and that the ability level per student is 1.

The quality production function for each school exhibits constant return to scale on y :

$$q_j = a_i \gamma_j y$$

, from where the cost function of quality provision simply becomes:

$$c(q_j) = \frac{q_j}{\gamma_j} \tag{36}$$

Assume, conveniently, that the scale of operation is fixed, and that the technology is such that each school can enroll $N/3$ students. Thus,there will be 3 schools in equilibrium. The set of productivities for all potential entrepreneurs can be ranked from γ_1 to γ_M,with γ_1 being the most productive.

Thus, heterogeneity in the model comes both from parental income - and the associated demand for education - and the productivity of school entrepreneurs. Differences in ability are not considered at this moment. As discussed in the previous sections, competitive pressures should imply that more able students face lower prices, and most of the analysis that will be presented below would still go through. Leaving ability aside, however, simplifies the analysis as it allows to bypass the choice of scale and, as a related issue, the composition and quality choice of schools with heterogeneous students. While this is clearly a relevant issue, I omit at this point, and focus on the competitive outcomes in this simplified market, which still provides powerful conclusions.

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[12] For simplicity, it is assumed that the number of households is sufficiently large, so that profits from school operation are irrelevant in the aggregate, and the three income groups described here adequately represent all the population.

4.3.1 Private Equilibrium

Consider first the case of fully private schooling market. All schools are profit-maximizers. Given the assumptions on capacity and population size, only the three most productive educational entrepreneurs will be able to operate in equilibrium.

Given preferences, total educational spending for each household is of the form:

$$p_i q_i = \eta Y_i$$

, where i=1,2,3.

From Claim 7, we know that there will be positive assortative matching between school productivity and household income. High-income households will attend the school with the highest productivity, γ_1, middle-income students will enroll in the school with productivity γ_2, and low-income students will attend the school with productivity γ_3.

Equilibrium prices (and the associated qualities) must be such that:

(a) The price and quality supplied by the least productive school in the market, γ_3 , to the low-income households prevent entry from the fringe competitor, the entrepreneur with productivity γ_4.

(b) The price and quality supplied by school γ_2 to middle-income households prevent school γ_3 from profitably supplying them quality at a smaller price.

(c) The price and quality supplied by school γ_1 to high-income households prevent school γ_2 from profitably supplying them quality at a smaller price.

In particular, the set of equilibrium prices can be written as:

$$\begin{aligned} p_1^{private} &= \frac{Y_1}{[Y_1 - Y_2]\,\gamma_2 + [Y_2 - Y_3]\,\gamma_3 + Y_3\gamma_4} < p_2^{private} \\ p_2^{private} &= \frac{Y_2}{[Y_2 - Y_3]\,\gamma_3 + Y_3\gamma_4} < p_3^{private} \\ p_3^{private} &= \frac{1}{\gamma_4} \end{aligned}$$

, with associated qualities

$$\begin{aligned} q_1^{private} &= \eta\,[Y_1 - Y_2]\,\gamma_2 + \eta\,[Y_2 - Y_3]\,\gamma_3 + \eta Y_3\gamma_4 > q_2^{private} \\ q_2^{private} &= \eta\,[Y_2 - Y_3]\,\gamma_3 + \eta Y_3\gamma_4 > q_3^{private} \\ q_3^{private} &= \eta Y_3\gamma_4 \end{aligned} \tag{37}$$

For a given level of ability, households with higher overall education spending face lower prices, as they are more attractive for all schools. The quality gap between students, therefore, reflects not only that richer households spend more in education - in absolute terms-, but also that they can buy more quality out of every unit invested. Also notice that, for a given dispersion in income, the dispersion in equilibrium qualities will increase as differences in productivity between schools become larger. In particular, quality gaps between low and middle income students will depend on the productivity gap between the fringe competitor and the school that serves the low-income sector, while the quality gap at the top of the income distribution will also depend on the productivity of the middle-income school. The productivity of the most efficient school, γ_1, plays no explicit role in the determination of equilibrium qualities.

4.3.2 Public education

Now, introduce taxation to allow public-funding of schools. Among the set of education entrepreneurs, some given number ($P \geq 3$) is eligible to receive public funds from education, while the remaining competitors must only rely on public funding. A "public" school is not allowed to charge tuition to the parents. As public schools have the same preferences as the rest of the entrepreneurs, they are rent-seekers, and will only provide higher quality if forced to do so by competitive pressures and threat of entry[13]. Public schools have the same fixed scale as the rest of the schools.

Public education is financed through a flat income tax, τ[14]. Households attending a public school make no education expenditures, and can fully consume their after-tax income, $(1-\tau)Y$. Agents who decide to stay in the private sector, paying full tuition, spend η of their after-tax income on education, and thus can only consume $(1-\tau)(1-\eta)Y$. The fact that staying in the private sector implies a lower consumption means that, of course, quality in an operating private school must be strictly larger. In particular, an agent will strictly prefer private education only if:

$$(1-\eta)^{1/\eta} q^{priv} > q^{pub} \tag{38}$$

This is the competitive advantage enjoyed by a rent-seeking public school against a potential private competitor. As discussed later, it still is an advantage for altruistic public schools who want to provide as much quality as possible but are relatively less productive than their profit-seeking private competitors. This is exactly the same kind of competitive advantage discussed in Section 3.4, when a school that matched the student's non-academic preferences could profitably supply less quality than a (potentially more productive) school with a different non-academic orientation. Unlike that case, however, the public school's competitive advantage is not eroded by the fact that others schools can

[13] The case in which public schools do not behave as rent-seekers is discussed in Section 4.

[14] Again, we assume that school profits are small relative to aggregate income, so that tax revenue does not explicitly account for them. Alternatively, schools are not taxed.

do the same. In Section 3.4, any profit-maximizing school could become non-religious if that proved profitable. Here, private schools cannot become public to receive taxation revenue.

The maximum quality the public school can feasibly produce is a function of its productivity and the public funds it receives. Total taxation revenue, and thus total resources available for the public sector, are simply $T = \tau \frac{N}{3}(Y_1 + Y_2 + Y_3)$. This revenue is distributed equally between the public schools that are actually able to operate in equilibrium.

The minimum public quality demanded by an income group to shift from the private sector to the public sector is increasing in the income level:

$$q^{pub} \geq (1-\eta)^{1/\eta}(1-\tau)\eta Y_i \gamma_{priv}, \tag{39}$$

, where , $(1-\tau)\eta Y_1 \gamma_{priv}$ is the maximum feasible quality provided by the private school outside the market.

If the public sector is able to serve two income groups in equilibrium (for example, Y_2 and Y_3), the quality it will provide to Y_2 is $q_2^{pub} = (1-\eta)^{1/\eta}(1-\tau)\eta Y_2 \gamma$. If it is a rent-seeker, the public school that actually serves Y_2 has no incentives to provide more.

What happens to the quality provided to Y_3? The minimum quality that would make the low income group choose the public sector is just $q_3^{pub} = (1-\eta)^{1/\eta}(1-\tau)\eta Y_3 \gamma$, smaller than q_2^{pub} as $Y_3 < Y_2$. However, that cannot be an equilibrium quality, as the public school that enrolls the low-income groups would enjoy higher rents, as it receives the same funds per students but has a smaller cost by providing lower quality. Competition for rents between public schools implies that the quality of the low income group must increase until it matches that of the middle-income sector, $q_2^{pub} = q_3^{pub} = (1-\eta)^{1/\eta}(1-\tau)\eta Y_2 \gamma$. Threat of entry from the private sector and competition for rents between public schools implies that having a higher income group on the public sector increases the quality for lower income agents. This is similar to the argument made by McMillan (2004) in the case of a unique public school that can choose its scale. However, our results will in general differ from his when we introduce a voucher.

For a given distribution of income, the size of the public sector, then, will depend on the size of the tax rate - as it determines the sector's funding- and on the productivity of public schools relative to private competition. If the tax rate is too small, or the productivity disadvantage too large, no public school will be viable. If tax rates are sufficiently large, and public schools productive enough, the private sector might end up being not competitive and disappearing entirely. In intermediate cases, households below a certain income threshold will be choose to attend the public sector, while richer households will end up in private schools.

For instance, if school productivities are homogenous at γ, the highest-income group will choose to remain in the private sector if and only if:

$$q^{pub} \leq (1-\eta)^{1/\eta}(1-\tau)\eta Y_1 \gamma$$

, where $(1-\tau)\eta Y_1\gamma$ is the maximum feasible quality provided by the private school. The maximum quality provided by the public school is a function of the revenue it receives, and the number of students it enrolls. If all students were enrolled in the public sector, maximum feasible public quality would then be:

$$q^{pub}_{\max} = \tau \frac{1}{3}(Y_1 + Y_2 + Y_3)\gamma \tag{40}$$

Thus, the high-income group student will choose the private sector if and only if:

$$\tau \frac{1}{3}(Y_1 + Y_2 + Y_3)\gamma \leq (1-\eta)^{1/\eta}(1-\tau)\eta Y_1 \gamma \tag{41}$$

, which sets an upper bound for the tax rate:

$$\tau < \frac{3\eta(1-\eta)^{1/\eta} Y_1}{(Y_1 + Y_2 + Y_3) + 3\eta(1-\eta)^{1/\eta} Y_1} \tag{42}$$

If the tax rate is above that bound, the high-income group will choose to participate in the public sector, as a new public school can now operate and feasibly provide enough quality for the agent to be better off than in the private sector. The higher the income of the high-income groups relative to the economy's average income, the higher the maximum tax rate at which the private sector can still operate.

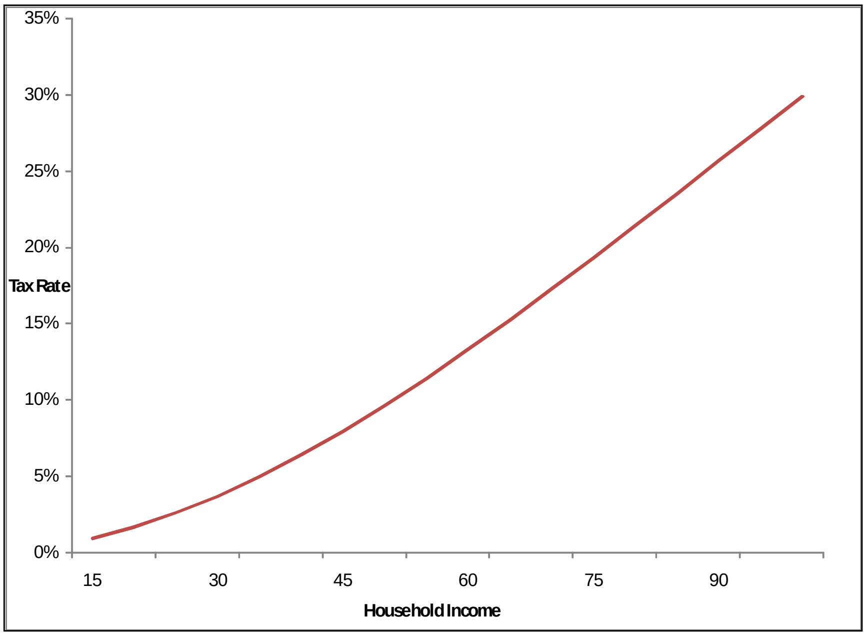

Figure 6: Income of the High-Income Group and Maximum Tax Rate

If productivities are not homogenous, and the productivity of the public school differs from that of its private competitors, we can use 39 to see that a public school that is strictly less productive can successfully deter private entry as long as:

$$
\begin{aligned}
\frac{\gamma_{public}}{\gamma_{private}} &\geq \frac{(1-\eta)^{1/\eta}(1-\tau)\eta Y_1}{\tau\frac{1}{3}(Y_1+Y_2+Y_3)} \Rightarrow \\
\frac{\gamma_{private}}{\gamma_{public}} &\leq \frac{\tau\frac{1}{3}(Y_1+Y_2+Y_3)}{(1-\eta)^{1/\eta}(1-\tau)\eta Y_1}
\end{aligned}
$$

The larger the tax rate, and the smaller the income level, the larger the productivity ratio that can persist in equilibrium without successful entry from the private schools.

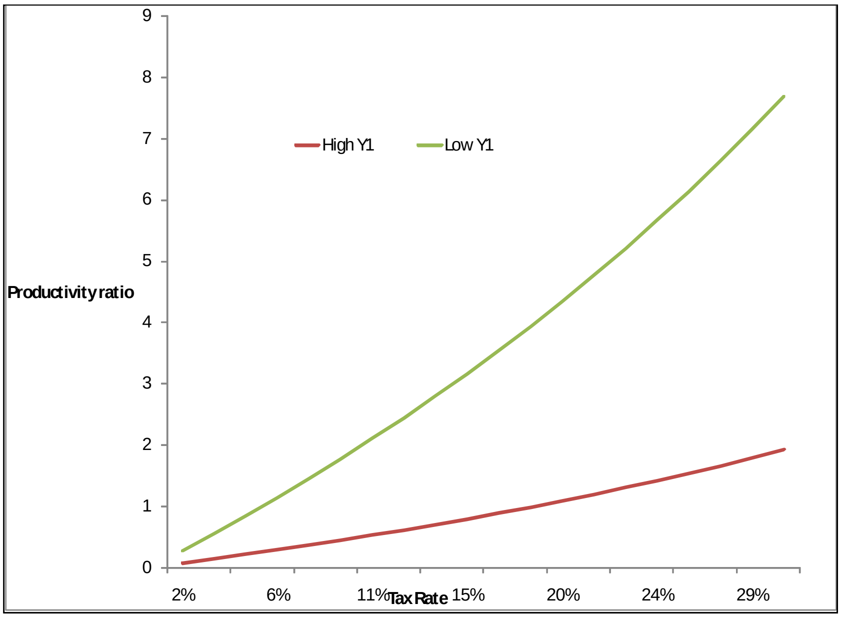

Figure 7: Taxes and the Max. Productivity Ratio with Deterred Entry

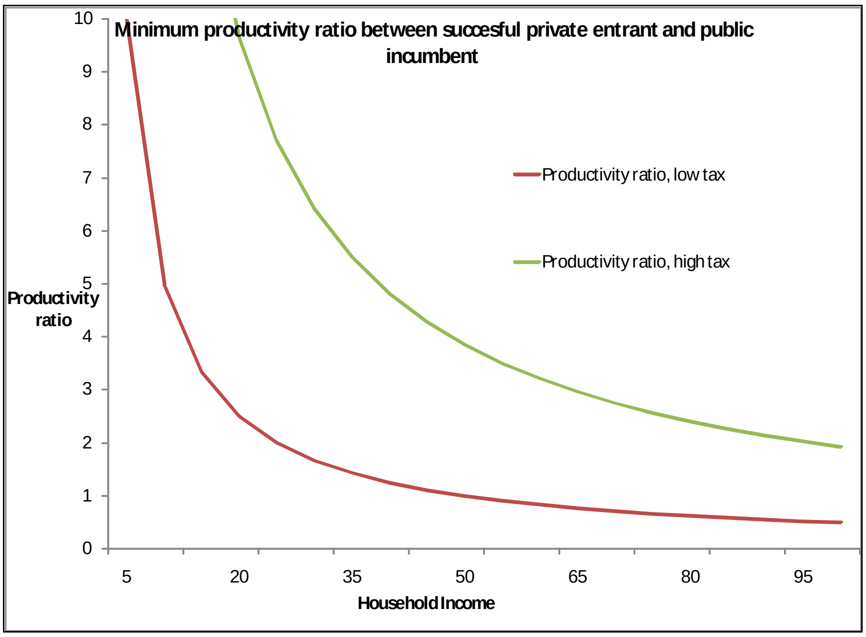

Figure 8: Income and the Max. Productivity Ratio with Deterred Entry

For the purposes of this paper, assume that the tax rate is set so that low-income and middle-income households choose to enroll their children in the public sector, while the unique private school will serve the high-income group. Using the indifference condition between public and private school qualities and the fact that groups are equally sized, the tax rate that allows for such an allocation with homogeneous productivities would satisfy:

$$\frac{2\eta(1-\eta)^{1/\eta}Y_2}{(Y^*)+2\eta(1-\eta)^{1/\eta}Y_2} < \tau < \frac{3\eta(1-\eta)^{1/\eta}Y_1}{(Y^*)+3\eta(1-\eta)^{1/\eta}Y_1} \tag{43}$$

, where $Y^* = Y_1 + Y_2 + Y_3$

Allowing for differences in productivity, assume that the most productive school, γ_1, is always private[15]. Also assume that, for the next 3 more productive entrepreneurs (those that range from γ_2 to γ_4), exactly two are public schools. Thus, in the set of more productive entrepreneurs, 2 of them operate as public schools and 2 as private[16].

To ensure that we always get an allocation where the low and middle income students go the public sector, regardless of which schools in the set γ_2 to γ_4 are public, it can be shown that the tax rate must satisfy:

$$\frac{2\eta(1-\eta)^{1/\eta}Y_2\gamma_2}{(Y^*)\gamma_4+2\eta(1-\eta)^{1/\eta}Y_2\gamma_2} < \tau < \frac{3\eta(1-\eta)^{1/\eta}Y_2\gamma_4}{(Y^*)\gamma_2+3\eta(1-\eta)^{1/\eta}Y_1\gamma_4} \tag{44}$$

To have a unique tax rate for all relevant comparisons across regime, we assume that productivities and incomes are always such that:

$$\frac{2\eta(1-\eta)^{1/\eta}Y_2\gamma_2}{(Y^*)\gamma_4+2\eta(1-\eta)^{1/\eta}Y_2\gamma_2} \leq \frac{3\eta(1-\eta)^{1/\eta}Y_2\gamma_4}{(Y^*)\gamma_2+3\eta(1-\eta)^{1/\eta}Y_1\gamma_4} \tag{45}$$

, so that there is at least one tax rate that solves 44.

Label the equilibrium qualities for each income group i in a regime where public schools are introduced as $q_i^{public\ regime}$. If public schools are rent-seekers, equilibrium qualities in the public and private sector are uniquely determined by the productivity of the private school outside the market. The stronger the competitive pressure from outside, the larger both public and private qualities.

If the tax rate satisfies 44, equilibrium qualities, then, will depend on which is the marginal private school outside the market:

$$\begin{aligned} \text{If } \gamma_2, \gamma_3 \text{ public} &: \quad q_1^{public\ regime} = \eta(1-\tau)Y_1\gamma_4 \\ \text{and } q_2^{public\ regime} &= \quad q_3^{public\ regime} = \eta(1-\eta)^{1/\eta}(1-\tau)Y_2\gamma_4 \end{aligned} \tag{46}$$

[15] This is not a crucial assumption, and results still hold without it.

[16] The process which determines which entrepreneurs are elegible to become public is not discussed here.

$$\begin{aligned} \text{If } \gamma_2, \gamma 4 \text{ public} \quad &: \quad q_1^{public\ regime} = \eta(1-\tau)Y_1\gamma 3 \\ \text{and } q_2^{public\ regime} \quad &= \quad q_3^{public\ regime} = \eta(1-\eta)^{1/\eta}(1-\tau)Y_2\gamma_3 \end{aligned} \tag{47}$$

$$\begin{aligned} \text{If } \gamma_3, \gamma 4 \text{ public} \quad &: \quad q_1^{public\ regime} = \eta(1-\tau)Y_1\gamma_2 \\ \text{and } q_2^{public\ regime} \quad &= \quad q_3^{public\ regime} = \eta(1-\eta)^{1/\eta}(1-\tau)Y_2\gamma_2 \end{aligned} \tag{48}$$

For a given tax rate, qualities in the public regime, both for private and public schools, are largest when competitive pressures from outside are stronger. This is, quality is the highest if public schools are relatively less productive than their private counterparts outside the market, as competitive pressures are the strongest when school γ_2 is outside the market. This also affects quality in the private sector, partially reverting the reduction in quality associated to the reduction in income due to taxation.

Notice that, for any taxes within the range, higher taxation and school funding actually has a perverse effect on quality. Higher public funding reduces the households' after-tax income, thus reducing the competitive pressure posed by the private entrepreneurs outside the market.

4.3.3 Vouchers

Keeping the same tax rate as in the previous example, assume now that public funding in the education sector is shifted towards a pure vouchers system. Initially, no payments on top are allowed, so a school must either operate by receiving vouchers or by full tuition from the parents. Any school is entitled to participate in the voucher system, and receives tax-financed payments per student enrolled. This eliminates the regulatory advantage enjoyed by the public school in the previous setup. In fact, in this simple setup, the distinction between public and private schools disappears, as both types of school now are exposed to the same competitive pressures[17].

As in the privately-funded system,only the more productive schools are able to profitably operate. The voucher sector will be served by schools γ_2 and γ_3,and the privately-paid sector will be served by school γ_1.

From the discussion in Section 2, we know that equilibrium qualities in the voucher sector will solely be determined by productivity of the marginal competitor, γ_4. In particular, equilibrium qualities must be such that the marginal competitor can not enter profitably.

[17] This is not always true. Public schools can still be subject to different regulations than private schools, or receive other sources of funding. In a dynamic setup, the introduction of a voucher system in a market previously served by public schools will usually place public schools with installed capacity against private entrepreneurs that have to make a potentially large fixed investment to begin operation. This still provides a competitive advantage to public schools, which may be exacerbated by imperfect capital markets.

In particular, the equilibrium qualities in the voucher sector are:

$$q_2^{voucher} = q_3^{voucher} = v\gamma_4 = \frac{\tau(Y_1 + Y_2 + Y_3)}{2}\gamma_4 \tag{49}$$

Given the production technology, and the fact that agents are homogeneous in terms of how costly it is to educate them, vouchers schools will not differentiate among themselves. Notice that the restriction on the tax rate set on 44 still ensures that middle-income agents are at least as well off in the voucher sector than if they shifted to the privately-paid education, and that high-income agents are better paying full tuition in the private sector.

As expected, the introduction of a voucher never decreases quality in tax-financed schools, and in general it will increase it. As discussed before, equilibrium public qualities could take 3 values, depending on the productivity of the public schools:

a) If public quality in the previous regime was $\eta(1-\eta)^{1/\eta}Y_2(1-\tau)\gamma_2$, such the second most productive school was left out of the market, quality cannot decrease as $v\gamma_4 = \frac{\tau(Y_1+Y_2+Y_3)}{2}\gamma_4 \geq \eta(1-\eta)^{1/\eta}Y_2(1-\tau)\gamma_2$ was the condition for productivities and the tax rate such that leaving school γ_2 out was feasible for school γ_4. For any tax strictly above the minimum threshold, such that $v\gamma_4 = \frac{\tau(Y_1+Y_2+Y_3)}{2}\gamma_4 > \eta(1-\eta)^{1/\eta}Y_2(1-\tau)\gamma_2$, qualities under the voucher system will be unambiguously bigger. School γ_4 is driven out of the market and school γ_3 is forced to increase its quality.

b) If public quality in the previous regime was $\eta(1-\eta)^{1/\eta}Y_2(1-\tau)\gamma_3$, such the second most productive school was left out of the market, for the same tax as in the previous case, quality strictly increases as $\frac{\tau(Y_1+Y_2+Y_3)}{2}\gamma_4 > \eta(1-\eta)^{1/\eta}Y_2(1-\tau)\gamma_2$. School γ_4 and school γ_3 is forced to increase its quality.

c) If public quality in the previous regime was $\eta(1-\eta)^{1/\eta}Y_2\gamma_4$, such the fourth most productive school was left out of the market, for the same tax as in the previous case, quality strictly increases as $\frac{\tau(Y_1+Y_2+Y_3)}{2}\gamma_4 > \eta(1-\eta)^{1/\eta}Y_2(1-\tau)\gamma_4$. The "public" schools survive, but are forced to increase their quality as the competitive pressure from school γ_4 increases.

Quality gains in publicly-funded schools are increasing in the tax rate, a reflection of the perverse effect of taxation on the previous regime. In fact, quality gains would still hold if all schools are equally productive. Notice that, for a given tax rate, quality gains relative to the public system are larger the more productive the initial set of public schools, as entry from more competitive private schools partially offsets the increase in competitive pressure. Gains from the adoption of a voucher system are smaller in that case because competitive pressures were already relatively strong in the public system.

What happens with quality in the private sector?

As in the case when all education was privately funded, the equilibrium quality (and price) provided by school γ_1 must be such that no school outside the market can profitably enter, and that no operating school chooses to deviate and serve the high-income households[18]. As discussed in Section 2.2, this is satisfied when q_1 is such that school γ_2 is indifferent between staying in the voucher sector or capturing the high-income group[19].

In particular,

$$\pi_2(q_2) = v - v\frac{\gamma_4}{\gamma_2} = \eta Y_1(1-\tau) - \frac{q_1}{\gamma_2} = \pi_2(q_1)$$

, from where:

$$q_1^{voucher} = [\eta Y_1(1-\tau)]\,\gamma_2 - v\left[\gamma_2 - \gamma_4\right] \tag{50}$$

Notice that quality in the private school can potentially decrease, as competitive pressures were strictly stronger in the previous regime when γ_2 was outside the market.

What happens if public schools are not rent-seekers, but rather supply all the quality they can given their resources? The adoption of the voucher and the potential increase in competitive pressures would not change their behavior. The voucher would be irrelevant if the public schools are at least as productive than their private counterparts, and would (at least on the margin) increase quality if they are replaced by more productive private schools, particularly if those schools are non-profit. Notice that, in that case - or in the general case where private schools are also altruistic- qualities within the publicly funded sector would differ, and there would rationing at the school that provides a higher quality.

4.3.4 Vouchers with payments on top (to be completed)

4.4 Simulations

Some simulations of the model, where all schools are non-altruistic, are presented below. Income groups are of the same size, and $Y_1 = 2Y_2 = 4Y_3$.The share of income spent in education is assumed to be $\eta = 0.4$ and the tax rate is set a the middle of the interval defined by 44. In all 3 cases, quality in the voucher schools is the same (from 49, γ_4 is the same for all simulations).

The first simulation involves the case where productivity across schools is homogenous. All quality gains from the adoption of a voucher come from increased competitive pressures in the public sector. Notice that quality for the

[18]Notice that this assumes that now all schools could potentially become private schools charging full tuition.

[19]This also ensures that the fully-paid private school makes more profits staying that way than receiving vouchers.

high-income group does not change with the adoption of the voucher, as competitive pressures on the public sector are constant (private schools get zero profits). If public schools were not rent-seekers, there would be no effect of the adoption of the voucher.

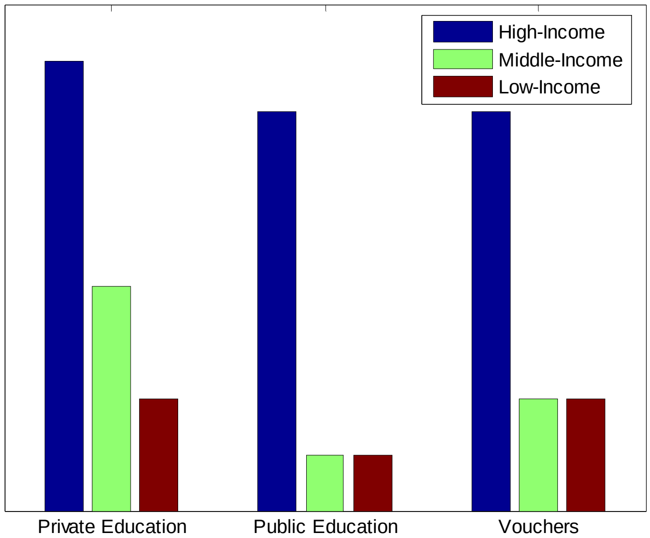

Figure 9: Equilibrium qualities with homogenous productivity

The second simulation involves the case where productivity across schools is heterogenous, and public schools in the second regime are relatively less productive. More efficient schools enter the market when a voucher is set in place, but competitive pressures do not increase as much as the marginal school is now less productive. This is reflected in the quality in the private sector, which takes a (minor) dip as the school can now provide a smaller quality without risking entry. If public schools were not rent-seekers, there would be no effect of the adoption of the voucher.

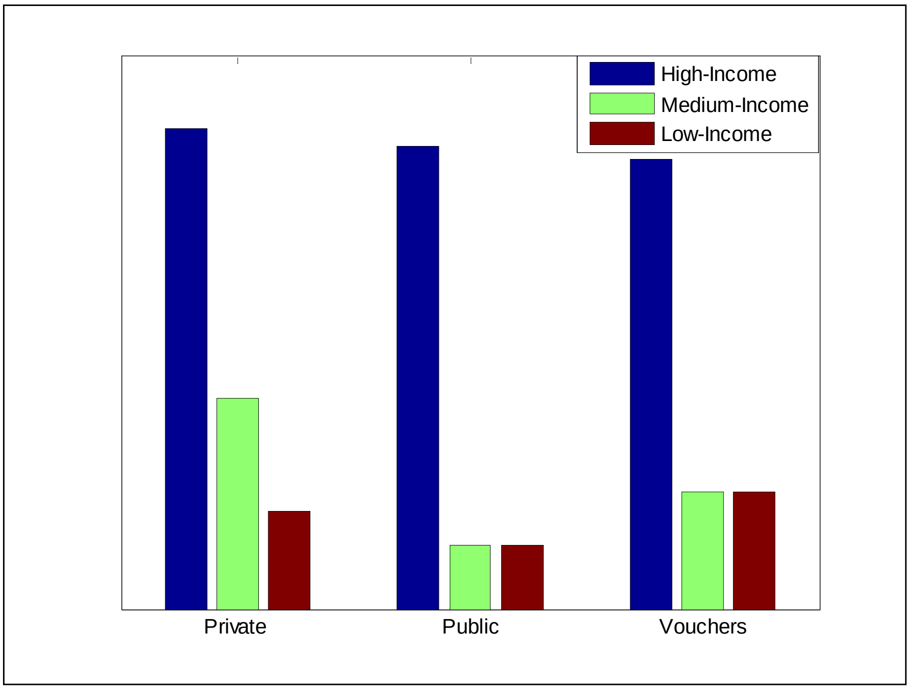

Figure 10: Equilibrium qualities with low-productivity public schools

The third simulation involves the case where public schools in the second regime have relatively high productivity. This is the case in which, given the tax rate, public education provides the smallest quality, as competitive pressures from (low productivity) private schools are the weakest. Notice that the same holds true for the private school: quality for the high-income group is significantly smaller when public education is adopted. The adoption of a voucher significantly increases competitive pressures on the public schools. The significant increase observed in the quality of the private school is caused by the assumption that, on the voucher system, "public" schools could become private if they wished to. If this was not true, quality in the private sector still could increase, as otherwise rich households would be better off moving to the voucher sector.

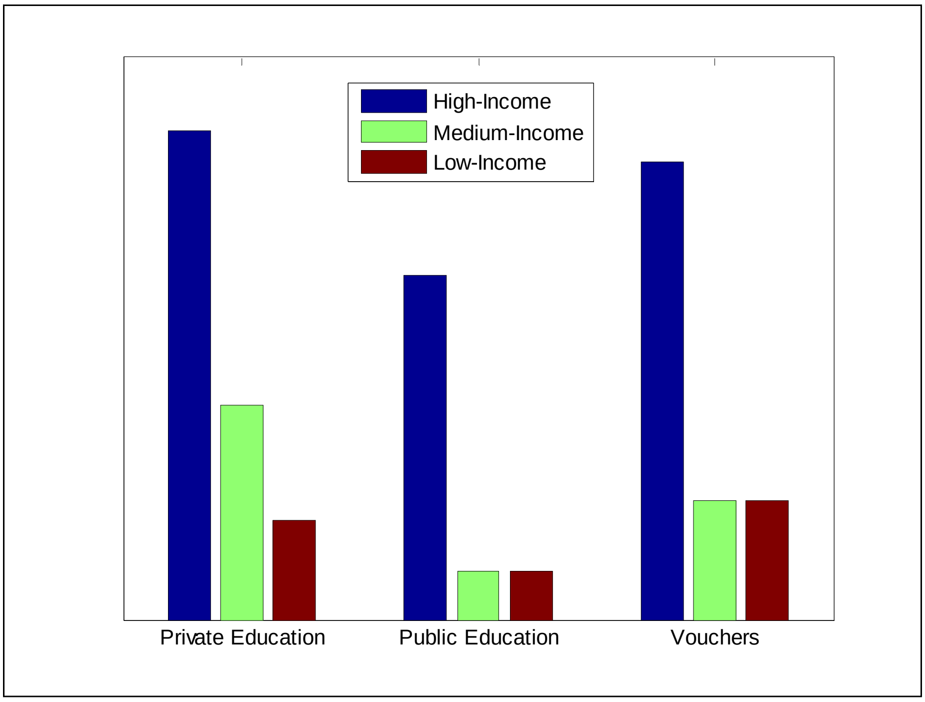

Figure 11: Equilibrium qualities with high productivity public schools

5 Future Extensions

The model presented in this paper serves as a starting point to analyze the operation of a voucher system with heterogeneous schools and students. The paper has emphasized the importance of competitive pressures and of the distribution of school and household characteristics in the determination of the equilibrium qualities and allocations.

However, there is still much to do. Relevant theoretical issues as scale or intertemporal dynamics were only discussed briefly, and were not incorporated in the general equilibrium setup. Including those elements is a natural extension of the current framework, significantly expanding the number of questions that can be dealt with this model. On the empirical side, Chile's experience with nation-wide vouchers, associated to an explosive development of the private sector in schooling, serves as a natural background against which to analyze and evaluate the model's main qualitative predictions.

This paper is part of a wider research agenda on the accumulation of human capital under different institutional frameworks. On a related paper, (Ferrada and Tapia, 2008), we analyze the macroeconomic implications of the institutional regime under which human capital accumulation takes place. We study the implications of each regime for aggregate production, growth, the distribution of income in a given cohort, and the degree of intergenerational mobility.

There are two additional papers that I plan as part of this line of research.

The first of them plans to analyze educational regimes in a model with endogenous growth and technology-adoption, where there is a human capital threshold below which agents cannot adopt (develop) a new technology. If the process of adoption-development poses a positive externality upon the rest of the economy, the analysis of the distributional effects of different institutional regimes changes. Regimes that focus more resources on the most productive, and that lead to more skewed to more skewed human capital distributions for a given mean, might be associated to further productivity gains.

The second project discusses the political economy of the human capital accumulation process, and how the institutions associated to it are affected by different interest groups. In particular, the paper plans to study how the solution to the equity-efficiency trade-off, and the relative weight given to distributional concerns, can shape education institutions under different political setups.

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