Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries*

Ufuk Akcigit    Harun Alp
University of Pennsylvania and NBER    University of Pennsylvania

Michael Peters
Yale University and NBER

[PRELIMINARY AND INCOMPLETE, COMMENTS WELCOME!]

May 5, 2015

Abstract

Firm dynamics in poor countries show striking differences to those of rich countries. While some firms indeed experience growth as they age, many firms are simply stagnant in that they neither exit nor expand. We interpret this fact as a lack of selection, whereby producers with little growth potential survive because innovating firms do not expand enough to force them out of the market. To explain these differences we develop a theory, whereby contractual frictions limit firms’ acquisition of managerial time. If managerial effort provision is non-contractible, entrepreneurs will benefit little from delegating decision power to outside managers, as they spend most of their time monitoring their managerial personnel. As the return to managerial time is higher in big firms, improvements in the degree of contract enforcement will raise the returns of growing large and thereby increase the degree of creative destruction. To discipline the quantitative importance of this mechanism, we incorporate such incomplete managerial contracts into an endogenous growth model and calibrate it to firm level data from India. Improvements in the efficacy of managerial delegation can explain a sizable fraction of the difference between plants’ life-cycle in the US and India.

Keywords: Development, growth, selection, competition, firm dynamics, contracts, management, entrepreneurship.

JEL classification: O31, O38, O40

*We thank the seminar and conference participants at the Productivity, Development & Entrepreneurship, and Macroeconomics Within and Across Borders Meetings at the NBER Summer Institute, the NBER Development Program Meeting, Yale, LSE, OECD/NBER Conference on Productivity and Innovation, Minnesota Macro, Penn State, Kauffman Entrepreneurship Conference, London Macroeconomics Conference, Cologne Workshop on Macroeconomics, SKEMA Growth Conference, and UPenn Growth Reading Group for helpful comments. In particular we like to thank our discussants Francesco Caselli, John Haltiwanger, Chang-Tai Hsieh, Rasmus Lentz and Yongs Shin. We furthermore received very valuable feedback from Abhijit Banerjee, Nick Bloom, Matthew Cook, Jeremy Greenwood, Daniel Keniston, Pete Klenow, Sam Kortum, Aart Kraay, Giuseppe Moscarini, Luis Serven, and Silvana Tenreyro. Chang-Tai Hsieh and Pete Klenow have kindly provided some of their data for this project. Akcigit gratefully acknowledges financial support from the World Bank and thanks the Cowles Foundation and the Becker Friedman Institute for the great hospitality during some parts of this project.
1 Introduction

The process of firm dynamics differs vastly across countries. While firms in rich countries experience rapid growth conditional on survival, firms in poor countries remain small and do not grow as they age. Hsieh and Klenow (2014) for example show that the average firm in the US has grown by a factor of five by the time it is 30 years old. In contrast, firms in India see very little growth as they age, making 30 year old firms barely bigger than new entrants. A salient regularity underlying this aggregate numbers is the plethora of small firms in underdeveloped economies, which are entirely stagnant. A case in point is the Indian manufacturing sector, where 80% of firms have at most two workers. More strikingly, this stock of micro-firms is essentially independent of age, i.e. not only are most entering firms in India small, but even for firms at age 30, 75% never grow beyond the size of two workers. This is very different in developed economies like the US, which is characterized by a pronounced “up-or-out” phenomenon, where firms also enter small but then either exit or expand. This process of selection, whereby stagnant producers are replaced quickly, is all but missing in poor countries. In this paper we provide a theory why this is the case.

To do so, we introduce two novel ingredients into an otherwise standard model of endogenous growth. To meaningfully speak about selection, we first assume that firms are heterogenous in their ability to expand. While able entrepreneurs have the necessary skills to grow their business by stealing market share of other producers, stagnant entrepreneurs lack such skills and only survive as long as they are not driven out of the market by their more dynamic competitors. The abundance of small producers in poor countries is therefore not due to these firms being constrained in their expansion possibilities, but rather a reflection of the fact that other firms with growth potential are not expanding enough to replace them quickly. To understand the lack of selection in an economy like India, one therefore has to have a theory why the returns to expansion for able entrepreneurs might be low.

As a second ingredient we explicitly introduce a need for managerial delegation into the theory. If workers require managerial oversight but the entrepreneur’s time to provide managerial services is limited, firms have to delegate decision power to outside managers as they expand. The returns to growing large are therefore crucially dependent on the availability of managerial skills in the economy. If there are plenty of people with managerial human capital, able entrepreneurs have high incentives to expand as they can efficiently delegate decision power along the expansion path. If however, managerial human capital is scarce or contractual frictions prevent the seamless hiring of managerial personnel, entrepreneurs anticipate that they will run into managerial bottlenecks once they expand their firm to a size, which they cannot manage on their own. This will reduce their desire to expand and hence endogenously limit the degree of selection in the economy.

Both the notion of heterogeneity in entrepreneurial growth abilities and of managerial bottlenecks in poor countries have recently been subject to a growing empirical literature. Hurst and Pugsley (2012) show that there are heterogenous types of entrepreneurs in the US economy, a majority of which intentionally chooses to remain small. Similar findings are reported in Schoar
Firm Dynamics in Developing Countries

(2010) and Decker et al. (2014), who classify entrepreneurs as *transformative* versus *subsistence* entrepreneurs, where the former ones operate with the intension to grow and the latter ones want to keep their businesses small and often within their families. For the case of developing countries, LaPorta and Shleifer (2014) and LaPorta and Shleifer (2008) provide evidence for a duality between small, informal and large, formal firms and also argue that the ”decline of informality is the result of replacement of inefficient informal firms by efficient formal ones” (LaPorta and Shleifer, 2014, p. 121)

That developing countries might suffer from a scarcity of managerial resources has recently been argued in a series of papers by Nick Bloom and John van Reenen. Bloom et al. (2013) for example argue that textile firms in India are severely constrained in their managerial resources, which prevents them from expanding. In particular, they show that the delegation of decision-rights hardly extends to managers outside the family and that the number of male family members is the dominant predictor of firm size. Using data across countries, they also provide evidence that managerial practices differ across countries (Bloom and Van Reenen (2007, 2010)), that both human capital and contractual imperfections are likely to be important to explain the limits to managerial delegation in poor countries Bloom et al. (2009) and that developed countries have firms, which are both larger and delegate more managerial tasks to outsiders Bloom et al. (2012).

Our theory combines these two strands of the literature by arguing that limits to managerial delegation might cause a lack of selection. This idea goes at least back to the seminal work of Penrose (1959), who argues not only that managerial resources “create a fundamental and inescapable limit to the amount of expansion a firm can undertake at any time” but also that it is precisely this scarcity of managerial inputs that prevents the weeding out of small firms as “the bigger firms have not got around to mopping them up” ((Penrose, 1959, p. 221)). We formalize this intuition in a firm-based model of endogenous growth in the spirit of Klette and Kortum (2004). In the model, firm dynamics are determined through creative destruction la Aghion and Howitt (1992), whereby successful firms expand by replacing other producers. As in Acemoglu et al. (2013) we allow for heterogeneity in entrepreneur’s growth potential. While high-type firms have the ability to grow by investing in technological improvements, low-type firms are lacking that potential and remain small. Hence, life-cycle growth generates selection, whereby the growth of “good” firms eliminates “bad” firms that would have otherwise remained in the economy.

High types’ incentives to grow are directly linked to the supply of managerial time they can acquire. Firms can be run by their owners. As the firm owner’s time endowment is limited, growing owner-run firms run into span of control problems in the spirit of Lucas (1978), which ultimately reduce the returns of growing large. To overcome such decreasing returns, the firms can augment the owner’s time endowment by delegating managerial decisions to outside managers. We are particularly interested in the efficiency with which such delegation can take place in different countries. Not only are the returns to delegation dependent on the human capital of outside managers but, as stressed by the empirical literature mentioned above however, managerial hiring is often subject to contractual frictions. If for example managerial effort provision is non-contractible,
outside managers have to be monitored by the owner, which itself requires valuable time. Hence, underdeveloped contractual institutions could reduce the returns to managerial delegation.

The model makes tight predictions how the delegation environment affects the process of firm-dynamics, the equilibrium firm-size distribution and degree of selection. The model predicts a threshold firm size, below which firms are only run by their owners. As long as firms do not delegate decision rights, firm profits have decreasing returns as the entrepreneur’s time is a fixed factor in production. This dictates that growth incentives are declining in size. Once, the marginal value of delegation is sufficiently high, firms start to hire outside managers to overcome the decreasing returns. In fact, the model predicts that the firm’s value function becomes linear, the slope of which is determined by the net return of delegation and shapes the dynamic incentives to firm growth. Because the model has an analytic solution, we can derive a set of robust comparative statics results. In particular, increases in the returns to delegation, which could e.g. stem from an increase in managerial human capital, an improvement in the contractual environment or better technology, which is complementary with outside managers, will increase the expansion incentives for large firms, raise average firm size, induce tougher selection in that stagnant firms exit quickly, make the life-cycle of firms steeper and reduce the number of firms running their business without managerial personnel. These predictions are qualitatively consistent with the cross-country data if richer countries have higher returns to delegation.

We then analyze the quantitative importance of this mechanism using firm-level data from the manufacturing sector in the US and India. Our theoretical model has the convenient feature that the delegation environment is summarized by a single parameter, which we can directly calibrate to the data. Hence, for the calibration exercise, we do not have to take stand on whether delegation benefits in the US are high because outside managers have high human capital or because the system of contract enforcement is more efficient. To identify such delegation benefits, we use data on managerial hiring in both the US and India. We use the calibrated model to quantify the effect of cross-country variation in the delegation environment to the implied process of firm-dynamics. If Indian firms were able to hire managers as seamlessly as firms in the US, the gap in lifecycle growth between India and the US would have been reduced by 30% to 50% depending on the firm age. As returns to expansion increase thanks to more effective managerial delegation possibilities, growing firms increases the degree of selection in the economy. Stagnant firms start to exit much more frequently but those that survive grow faster generating a steeper life-cycle profile. We then dig deeper into the fundamental determinants of the delegation environment. Using cross-country data on managerial employment shares, the quality of legal institutions and on managerial human capital, we show that while 46% to 70% of the higher delegation benefits in the US stem from a more efficient court system, human capital differences between US and Indian managers account for relatively lower portion of these explained differences in the efficiency of delegation.
**Related Literature**  
On the theoretical side, this paper provides a new theory of firm dynamics in developing countries.\(^1\) While many recent papers have aimed to measure and explain the static differences in allocative efficiency across firms,\(^2\) there has been little theoretical work explaining why firm dynamics differ so much across countries. A notable exception is the work by Cole et al. (2012), who argue that cross-country differences in the financial system will affect the type of technologies that can be implemented. Like them, we let the productivity process take center stage. However, we turn to the recent generation of micro-founded models of growth, in particular Klette and Kortum (2004), which have been shown to provide a tractable and empirically successful theory of firm-dynamics (Lentz and Mortensen (2005, 2008), Acigiz and Kerr (2010), Acemoglu et al. (2013)).

We focus on inefficiencies in the interaction between managers and owners of firms to explain the differences in firms’ demand for expansion. Caselli and Gennaioli (2013) also stress the negative consequences of inefficient management, but focus on static misallocation on the market for control, whereby (untalented) firm-owners might not be able to sell their firms to (talented) outsiders. Our economy does not have any exogenous heterogeneity in productivity, but we argue that managerial frictions within the firm reduce growth incentives and hence prevent competition from taking place sufficiently quickly on product markets. Such within-firm considerations are also analyzed in Powell (2012), who studies an economy where firms (“owners”) need to hire managers subject to contractual frictions. In contrast to our theory, firm productivity is constant, i.e., there is no interaction between contractual frictions in the market for managers and firms’ innovation incentives.

The remainder of the paper is organized as follows. In Section 2 we describe the theoretical model, where we explicitly derive the link between firms’ delegation decisions under contractual frictions and their innovation incentives. In Section 3 we take the model to the data. We first calibrate the model to the Indian micro data and then use the cross-country data on the aggregate importance of managerial employees, their human capital and quality of legal institutions to assess the quantitative importance of our mechanism. Section 4 concludes.

**2 Theory**

We consider a model of firm-dynamics in the spirit of Klette and Kortum (2004). This framework, which we describe in more detail below, is a natural starting point, in that it offers a tractable formalization of the firm-level growth process, which can take to the micro-data. In particular, it puts the role of selection and creative destruction at center stage. Firms spend resources to expand by stealing marketshare of their competitors. Producers that are unsuccessful in growing are being

---

\(^1\) An overview of some regularities of the firm size distribution in India, Indonesia and Mexico is contained in Hsieh and Olken (2014).

\(^2\) The seminal papers for the recent literature on misallocation are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). As far as theories are concerned, there is now a sizable literature on credit market frictions Buera et al. (2011); Moll (2010); Midrigan and Xu (2010), size-dependent policies Gurer et al. (2008), monopolistic market power Peters (2013) and adjustment costs Collard-Wexler et al. (2011). A synthesis of the literature is also contained in Hopenhayn (2012) and Jones (2013).
replaced by their more innovative rivals and leave the economy. Hence, the degree of selection is fully endogenous. We augment this framework with three ingredients:

1. We assume that entrepreneurs are heterogeneous in their growth potential,

2. we explicitly allow for both managerial services and production labor to be inputs for production and model individuals’ occupation choice to be employed as workers or managers and

3. we suppose that entrepreneurs are endowed with a fixed amount of managerial time and can choose to delegate managerial tasks to outside managers to increase the supply of managerial services within the firm.

The first item allows us to meaningfully speak about a process of selection, whereby stagnant producers with little growth potential are forced out of the market by their dynamic rivals. The last two items give a specific role for managerial delegation to determine how quickly this process takes place. In particular, they will imply that in order to sustain expansion incentives, entrepreneurs have to delegate eventually to not run out of managerial time. How efficiently this process of delegation takes place will depend on the delegation environment, in particular the human capital of outside managers and the ease at which contracts with managerial personnel can be written.

Preferences and Technology We consider a closed economy, where a final good is a Cobb-Douglas composite of a continuum of intermediate goods and produced under perfect competition. Specifically,

\[
\ln Y_t = \int_0^1 \ln y_{jt} d\bar{j},
\]

where \(y_{jt}\) is the amount of product \(j\) produced at time \(t\). Hence, there is a measure one or intermediate products and to save notation we will drop the time subscript \(t\) whenever it does not cause any confusion.

The production of intermediate goods is conducted by heterogeneous firms and necessitates both production workers and managers. In particular, the firm \(f\) can produce good \(j\) according to

\[
y_{jf} = q_{jf} \mu(e_{jf}) l_{jf},
\]

where \(q_{jf}\) is the firm-product specific production technology, \(l_{jf}\) is the number of workers employed for producing intermediate good \(j\) and \(e_{jf}\) denotes the amount of managerial services firm \(f\) allocated towards the production of good \(j\). \(\mu(e_{jf})\) is a production function translating managerial services into productivity units and we will specify a particular function form below.

As in Klette and Kortum (2004), firms compete in the product space for the right to produce. As the production technologies \(q_{jf}\) are firm-specific, product \(j\) will be produced by the firm that has the highest productivity to produce it. At each point in time, a firm is a collection of product
lines in which it has access to the best technology. Figure 1 illustrates examples of two firms in the economy. In this example, firm $f_1$ owns 5 lines and $f_2$ owns 3. Firms can expand into a new product line $j'$ by introducing a better version of what the current incumbent in $j'$ produces and we will describe the details of firms’ innovation environment in Section 2.2, when we turn to the dynamics. Before doing so, we will now describe the static allocations, which determine firms’ profits and hence their incentives to expand.

The figure depicts two examples of firms in this economy. While firm one is the current quality leader in five products, firm two only has three products in its portfolio.

2.1 Static Allocations

At each point in time $t$, there is a set of active firms $F_t$. Each intermediate good $j$ is produced by a firm that has the highest productivity in line $j$. One of the new ingredients of our model, which is also the main focus of our analysis, is the provision of the managerial effort in production. In order to keep the focus on the determination of the managerial effort, we skip the details of the competition between firms and refer to Section 5.1 in the Appendix where we formally characterize the entire static equilibrium of the economy. In particular, we show that firm $f$’s profit in product line $j$ (after paying for production workers $l_{fj}$) is given by

$$\pi_{fjt} = e_{fj}^\sigma Y_t. \quad (2)$$

Hence, profits are a simple power-function of managerial effort parametrized by the elasticity $\sigma$ (which is a property of the function $\mu$ in (1)). Intuitively, because managerial effort increase physical productivity (see (1)), higher managerial effort will allow firms to sustain higher mark-ups over competing firms. Note that firm $f$’s profits do not depend on its physical productivity $q_{fit}$ but only on the amount of managerial effort is allocates to the production of product $j$.

Managerial effort is provided both by the entrepreneur herself and outside managers. In particular, we assume that there is a measure 1 of individuals who can work either as production workers or managers. Letting the number of production workers and managers hired by firm $f$ be $l_{ft}$ and
m_{ft}$, respectively, labor market clearing requires that
\[ 1 = \int_{f \in F_t} (l_{ft} + m_{ft}) \, df. \tag{3} \]

Finally, in the Appendix we also show that aggregate output in this economy is given by
\[ Y_t = Q_t M_t L_P^t, \tag{4} \]
where $L_P^t = \int_{f \in F_t} l_{ft} \, df$ denotes the amount of production workers, $Q_t$ is the usual Cobb-Douglas composite of individual efficiencies
\[ \ln Q_t = \int_{j=0}^{1} \ln q_{jt} \, dj. \]
and $M_t$ is an endogenous TFP term based on managerial effort, which summarizes the aggregate effects of the distribution of mark-ups in this economy. In particular,
\[ M_t = \left[ \int_{j=0}^{1} [1 - (e_{jft})^\sigma] \, dj \right]^{-1}. \tag{5} \]

As we are mostly interested in the implication for firm-dynamics, we will not focus on the aggregate consequences of mark-ups in this paper. We are rather interested in the consequences of (limits to) managerial delegation on firms’ expansion incentives. This is where we turn now.

**Managerial Effort and Delegation** At the heart of our theory is the allocation of managerial efforts $e_{jft}$ across firms, which determine the profitability of being active in product line $j$ (see (2)). We assume that managerial services can be provided by the entrepreneur herself and by outside managers. In particular, we assume that each entrepreneur has a fixed endowment of managerial time $T > 0$, which she can provide as managerial effort into her firm. If an entrepreneur owns a firm with $n$ product lines and decides to run her firm alone, then she will have $e_j = T/n$ units of managerial effort per product line.$^3$

Equation (2) then directly implies that the normalized profit from each product line is $\bar{\pi}_{jt} \equiv \pi_{jt}/Y_t = (T/n)^\sigma$. Similarly, the (normalized) value of the firm is simply given by the aggregate profits of all product lines the firms owns. Hence,
\[ V^{self}(n) = n \bar{\pi}_j = n \times (T/n)^\sigma = T^\sigma n^{1-\sigma}. \tag{6} \]

(6) has a simple but important implication: while the value of a firm is increasing in the number of products, it does so at a decreasing rate. This is due to the fact that the owner has a fixed time endowment $T$ and runs into span of control problem as in Lucas (1978). Figure 2 illustrates the value of a firm that is run only by its entrepreneur. The fact that $V^{self}(n)$ is concave has important dynamic implications for firms’ expansion incentives: as marginal returns are declining.

$^3$That she will want to spread her $T$ units of managerial time equally across all product lines follows directly from the concavity of $\pi$ (see (2))
in n, the incentives to grow and to break into new markets decline as firms grow. With the managerial endowment $T$ being a fixed factor, the entrepreneur’s managerial services are being spread thinner and thinner as her firm expands, which causes growth incentives to decline. To prevent this scarcity of managerial services to be a drag on growth incentives, the entrepreneur can decide to bring outside managers into the firm. It is this process, which we refer to as delegation.

**Figure 2: Value of an Owner-run Firm**

![Value of an Owner-run Firm](image)

The figure depicts the value function of a firm without managerial delegation $V_{self}^{self}(n)$, see (6).

The entrepreneur can decide to delegate some of the managerial tasks to outside managers. The cost of hiring an outside manager is the wage rate $w$ that will be paid to the manager. The benefit is that outside managers will add to the firm’s endowment of available managerial time and hence generate managerial services $e$, which translate into higher profits. The demand for outside managers stems from the net increase managerial resources they bring to the firms. We denote the benefit of delegation per manager by $\xi$ which is in terms of managerial effort units $e$. In other words, an owner that has $n$ product lines over which she distributes her time equally, and hires $m_j$ managers per each line $j$ has a total managerial effort in line $j$ equal to $e_j = T/n + \xi m_j$ per line.

We think of $\xi$ as a country-specific parameter. First of all, $\xi$ clearly depends on the human capital of outside managers. Hence, the benefits of delegation might be low, simply because potential managers do not the necessary skills to organize the process of production efficiently. While some countries might have plenty of people who are able to supervise dozens of production workers and deal with multiple input suppliers, such skills might be scarce in many poor countries of the world. Secondly, we think of $\xi$ as being affected by technology. In particular, the efficient provision of managerial services by outside managers might be complimentary with technologies to document work-flows and to make such knowledge available to other decision makers within the firm. Finally, $\xi$ could depend on the contractual environment. If contractual imperfections are severe, entrepreneurs might need to spend substantial amounts of their own time to monitor their managerial personnel.

At this point, we are agnostic about the exact determinants of $\xi$. We will rather take it as a country-specific parameter and calibrate it directly within our model. It is only then that we dig deeper in its determinants and use cross-country data on human capital and contractual institutions.
to decompose it in various components. To fix ideas, in the following subsection, we provide a micro-founded game to illustrate how the level of $\xi$ might depend on such country characteristics. However, we do not have to commit to this particular game for our quantitative analysis. Readers, who are not interested in the particular micro-foundation can take $\xi$ as given and skip to the next section, where we solve for firms’ optimal delegation strategies.

**Limits to Delegation: A Simple Example** Consider the following set-up, which provides a simple micro foundation for the delegation benefits $\xi$. Suppose that both managers and entrepreneurs each have one unit of time at their disposal. While latter can provide $T$ units of effort during that time interval, managers can provide $\eta$ units of effort. Hence, $\eta$ can be thought of a measure of the efficiency of outside managers, which can determined by both human capital and the technology, which outsider managers use to perform their duties.

Additionally, assume that managerial effort provision is subject to contractual friction. For simplicity, we assume that the manager can decide to either provide effort, in which case his contribution to the firms’ time endowment is $\eta$, or shirk, in which case he adds no usable services to the firm:

$$e_{j}^{\text{manager}} = \begin{cases} \eta & \text{if work,} \\ 0 & \text{if shirk.} \end{cases} \quad (7)$$

Crucially, we assume that the managers’ effort choice is not perfectly contractible but that the entrepreneur has to monitor the manager and has to rely on the legal system in case she catches the manager shirking. Specifically, assume that if the owner spends $s$ units of time in monitoring a manager, she will catch a shirking manager with probability $\phi s$. Here $\phi$ simply captures the quality of the monitoring technology. Whenever the manager shirks and the entrepreneur catches him, the owner can go to court and sue the manager for the managerial wage $w$. To model differences in the efficiency of the legal system, we assume that in such case, the court (rightly) decides in the owner’s favor with probability $\kappa \in [0, 1]$. Hence one can think of $\kappa$ as the proxy for the rule of law in the country. Finally, the demand for shirking arises because shirking carries a private benefit $bw$, where $b < 1$.\(^4\)

It is straight-forward to characterize the equilibrium of this simple game. If the entrepreneur spends $s$ units of time monitoring the manager, the manager does not shirk if and only if

$$w \geq bw + w (1 - \kappa \phi s),$$

where $(1 - \kappa \phi s)$ is the probability that the manager is not required to return his remuneration despite having shirked. This incentive constraint implies that the manager puts effort as long as $s \geq \frac{b}{\kappa \phi}$. Clearly the owner will never employ a manager without inducing effort.\(^5\) Hence, the owner

\(^4\)The necessity to the the private benefit being proportional to the wage arises in order to make the contract stationary.

\(^5\)It is clearly never optimal for the entrepreneur to hire an outside manager without monitoring sufficiently to prevent shirking (see (7)).
will spend

\[ s = \frac{b}{\kappa \phi} \]  

units of time monitoring the manager. Note that expression (8) has two implications. First of all, monitoring and the strength of the legal system are substitutes as \( \frac{\partial s}{\partial \kappa} < 0 \). Secondly, improvements in the efficiency of monitoring \( \phi \), will also free up entrepreneurial resources to provide sufficient incentives.

Given the equilibrium of this game, the net \textit{benefit of delegation} in terms of managerial effort is simply the manager’s effort \( \eta \) \textit{minus} the owner’s time in monitoring. Hence, the overall amount of managerial effort in product line \( j \) is given by

\[ e_j = \frac{T}{n} - m_j s + m_j \eta = \frac{T}{n} + \left( \eta - \frac{b}{\phi \kappa} \right) \times m_j \]  

Note that the benefit of delegation is increasing in managerial human capital \( \eta \), in the state of the contractual environment \( \kappa \) and in the monitoring technology \( \phi \). Note also that the whole purpose of delegation is to increase the firms’ managerial resources. Firms will never hire a manager if \( \xi (\kappa, \eta, \phi) < 0 \). Hence, whenever managers are sufficiently unproductive, firms will never want to hire outside managers. Similarly, whenever the quality of legal system is sufficiently low, there will not be any managerial demand as owners need to spend more of their own human capital to prevent opportunistic behavior of managers than they gain in return. For the following we will therefore assume that \( \xi > 0 \).

\textbf{Optimal Delegation} \quad \text{Given the net benefit of delegation } \xi, \text{ we can now solve the entrepreneur’s static delegation problem. Consider a firm with } n \text{ products. Given (2), the owner maximizes the total profits of the firm by choosing the optimal number of outside managers. Hence,}

\[ V(n) = \sum_{j=1}^{n} \max_{m_j \geq 0} \left\{ \left( \frac{T}{n} + \xi m_j \right)^{\sigma} Y_t - w_t m_j \right\} \]  

This expression is intuitive. The owner allocates \( T/n \) units of her time on each product line. In addition, by hiring \( m \) outside managers, she can obtain an overall managerial efficiency of \( T/n + m_j \xi \) but has to pay \( mw \) overall to managers.\(^7\) The solution to firms’ optimal delegation decision is provided in the following proposition.

\textbf{Proposition 1} \quad \text{Consider the maximization problem in (11) and let the normalized wage rate be}\n
\(^6\)Note that we do not require that \( s < T \), i.e. we do not require the owner to perform the monitoring himself. We rather think of managerial efficiency units to be perfect substitutes so that other managers can be

\(^7\)Note that in this maximization problem we already imposed the optimality condition that the owner is going to allocate the same amount of time in each line \((T/n)\).
\( \omega \equiv \frac{w_t}{Y_t} \). Define the delegation cutoff

\[
n^* \equiv T \left( \frac{\omega}{\sigma \xi} \right)^{\frac{1}{1-\sigma}}.
\] (12)

Then the following is true:

1. Small firms with \( n < n^* \) do not hire any outside managers and are run only by their owners,

2. The demand for outside managers per product line is given by

\[
m(n) = \frac{T}{\xi} \times \max \left\{ 0, \left( \frac{1}{n^*} - \frac{1}{n} \right) \right\}.
\] (13)

3. The optimal amount of managerial services per product line \( e(n) \) is given by

\[
e(n) = T \times \max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\},
\] (14)

**Proof.** This follows trivially from the first order condition of (11). ■

**Figure 3:** Delegation in Different-sized Firms  **Figure 4:** Impact of an Increase in Net Benefit \( \xi \)

Proposition 1 characterizes the demand for outside managers - both on the intensive and the extensive margin. First of all, small firms do not hire outside managers. In particular, the delegation cutoff, \( n^* \) is increasing in owner’s time \( T \) and the monetary cost of hiring a manager \( \omega \) and it is decreasing in the net benefit of delegation \( \xi \). Secondly, similar comparative statics hold for the intensive margin as well: conditional on hiring, ample managerial resources by the entrepreneur herself (\( T \)) will reduce the demand for outside managers and larger delegation benefits (\( \xi \)) will induce firms to delegate more. Note also that larger firms hire more managers conditional on
Firm Dynamics in Developing Countries

hiring: As $T$ is a fixed factor, larger firms will have a higher demand for managerial demand (per product!), simply because less and less of the entrepreneur’s own time can be allocated to each individual product line. Finally (14) directly focuses on the endogenous choice of managerial resources $e_j$ and summarizes the economic rationale of delegation in this model: by hiring outside managers, large firms (with $n > n^*$) can elicit the same amount of managerial resources as firms of size $n^*$. The intuition for these results are provided in Figures 3 and 4. If the owner runs a small firm ($n = 1$ in Figure 3), the owner can invest her entire managerial time on that one product, which lowers the marginal return from hiring an outside manager. This can be seen from the fact that the slope at $m = 0$ is lower than wage rate, hence a small firm does not hire an outside manager. A large firm with $n > n^*$, on the other hand, hires outside managers until the marginal return from the last manager is equal to the market wage rate. That determines the equilibrium number of managers hired by an $n-$product firm.

Proposition 1 also has an interesting mathematical structure in that the delegation cutoff $n^*$ is a simple sufficient statistic for both the delegation benefits $\xi$ and the general equilibrium variable $\omega$. This sufficiency of $n^*$ is useful because it will allow us to characterize the equilibrium process of firm-dynamics through a single aggregate variable, $n^*$. To understand why, consider Proposition 2, where we characterize firms’ value function $V(n)$. The value function crucially determines the expansion incentives of firms.

**Proposition 2** Consider the firms’ value function $V(n)$ defined in (11) and let $n^*$ be defined as in (12). Let $\tilde{V} = \frac{V}{Y}$ be the equilibrium normalized value of an $n-$product firm. Then:

$$
\tilde{V}(n) = \begin{cases} 
\tilde{V}_{self}(n) & \text{for } n < n^* \\
\tilde{V}_{manager}(n) & \text{for } n \geq n^*
\end{cases}
$$

where $\tilde{V}_{self}(n)$ is the value of owner-run $n$-product firm,

$$
\tilde{V}_{self}(n) = T_{\sigma} \times n^{1-\sigma},
$$

and $\tilde{V}_{manager}(n)$ is the value of a delegating $n$-product firm,

$$
\tilde{V}_{manager}(n) = T_{\sigma} \times \left[ \sigma(n^*)^{1-\sigma} + \frac{1 - \sigma}{(n^*)^{\sigma}} n \right]
$$

**Proof.** Directly follows from substituting (13) into (11) and from the definition of $n^*$ in (12) \(\blacksquare\)

Proposition 2 has a crucial implication: the value function is linear in firm size $n$ once firms start delegating! Hence, entrepreneurs can overcome the diminishing returns by delegating to outside managers. The efficiency of the delegation technology directly parametrizes the slope of this value function, i.e., the incremental gain from firm growth, directly depends on the net benefit of delegation $\xi$ through the endogenous delegation cutoff $n^*$. Specifically, the marginal return of
adding an additional product to the firms’ portfolio is given by

\[ \tilde{V}'(n) = (1 - \sigma) \max \left\{ \left( \frac{T}{n} \right)^{\sigma}, \left( \frac{T}{n^*} \right)^{\sigma} \right\}, \] (16)

i.e. delegation of authority effectively puts a floor on the slope of the value function. In particular, the better the delegation technology (\( \xi \)), the lower the delegation cutoff \( n^* \) and the more will the marginal product be valued.\(^8\)

Figures 5 and 6 illustrate Proposition 2. Consider first Figure 5. When a firm is run only by the owner, the firm runs into diminishing returns in size \( n \), as in Lucas (1978). By delegating authority, on the other hand, the firm manages to keep the supply of managerial services growing and hence prevents the returns to growth from declining. Specifically, once once the firm size hits the delegation cutoff \( n^* \), the value function becomes linear as in the baseline version of Klette and Kortum (2004). Figure 6 illustrates an increase in the benefits of delegation. If delegation becomes more efficient through an increase in outside managers’ human capital or through improvements in the contractual environment, both the delegation cutoff declines and the slope of the value function increases. In fact, Proposition 2 shows that the extensive margin of delegation and marginal returns to expansion are tightly linked: the both depend only on the endogenous variable \( n^* \). To study more formally how these static benefits of delegation shape the incentives to expand, the firm-size distribution, the ensuing pattern of life-cycle growth and the degree of equilibrium selection, we will now embed the static economy from above in a dynamic environment.

2.2 Dynamics

Our model is a model of creative destruction. Firms grow by stealing products from their competitors and decline in size if other producers replace them as the most productive producer of a particular product. Firms exit the economy when they lose their last product and cease to produce.

\(^8\)Note also that \( V_{\text{manager}}(n^*) = V_{\text{self}}(n^*) = T^\sigma (n^*)^{1-\sigma} \), so that the value function is continuous.
New firms enter the economy by replacing existing firms as the producers of a particular product. Hence, aggregate growth, the existing firms’ life-cycle and the resulting processes of exit and entry are all endogenous and linked to firms’ growth incentives. In particular, the model gives a precise role for the lack of selection in countries with frictions to delegate. Stagnant, low-type producers only exit the economy when they are replaced by their innovative competitors. Hence, for low-type producers to exit the economy quickly, other firms have to have sustained incentives to grow. If imperfect legal systems or scarce managerial human capital prevent firms to delegate authority however, growth incentives decline rapidly, which allows inefficient firms to survive. Efficient managerial delegation will therefore foster selection and creative destruction by allowing good firms be become even larger. To put it differently: If an economy is characterized by firms that do not grow, it might not be the case that these firm are constrained - it might rather be a sign that other producers in the economy are not growing sufficiently quickly to replace them.

**Entry** In order to focus on the process of selection (or lack thereof) via product market competition, we assume that an exogenous measure \( z \) of entrepreneurs enters the economy at each point in time. This can be thought of as an exogenous flow of business ideas to outsiders, who enter the economy as new entrepreneurs. Importantly, entrants are heterogeneous in their growth potential and are either of high or low types. Formally, upon entry, each new entrant draws a firm type \( \theta \in \{\theta_H, \theta_L\} \) from a Bernoulli distribution, where

\[
\theta = \begin{cases} 
\theta_H & \text{with probability } \alpha \\
\theta_L & \text{with probability } 1 - \alpha 
\end{cases}
\]

The firms type \( \theta \) determines its innovation productivity or growth potential.

**Incentives to expand** Firms are endowed with an innovation technology, which allows them to expand their product of production. In particular, firms can spend resources to try to replace other firms as the producer of product \( j \). Formally, if a firm of type \( \theta \) with \( n \) products in its portfolio invests \( R \) units of the final good, it generates a flow rate of innovation of

\[
X(R; \theta, n) = \theta \left[ \frac{R}{Y} \right]^\zeta n^{1-\zeta},
\]

i.e. with flow rate \( X(R; \theta, n) \) it improves the productivity of a randomly selected product and replaces the existing firm. Hence, \( \theta \) parameterizes the efficiency with which firms can expand and \( \zeta \) determines the convexity of the cost function. For simplicity we assume that \( \theta_L = 0 \), i.e. low types will never be able to grow and we can focus on the high types’ decisions. This polar case is conceptually useful because it stresses that low types are never supposed to grow. Hence, the sole difference in firm-dynamic across countries will stem from the innovation incentives for high types and it will be high types appetite for expansion that will determine the degree of selection, i.e. the
time it takes for entering low types to be replaced. The other terms in the innovation technology are scaling variables required in many models of endogenous growth.\(^9\)

Firms spend resources to expand to maximize profits. In particular, the optimal flow rate of expansion is implicitly defined by

\[
X_n = \arg \max_X \left\{ X \left[ V_t(n + 1) - V_t(n) \right] - Y_t \left[ \frac{X_n}{\theta n^{1-\varsigma}} \right]^{\frac{\varsigma}{1}} \right\},
\]

where \(V_t(n)\) is the firms’ value function defined in Proposition 2 above. The first term is the expected profit of expanding with flow rate \(X\) and the last term is simply the cost function of innovation that is implied directly by the production function in (17).\(^{10}\) (18) has a simple solution. In particular, the optimal innovation rate per product line, \(x_n = X_n/n\) is given by

\[
x_n = \theta^{\frac{1}{1-\varsigma}} \varsigma^{\frac{1}{1-\varsigma}} \times \left( \frac{V(n + 1) - V(n)}{Y} \right)^{\frac{\varsigma}{1-\varsigma}}.
\]

(19)

Naturally, the incentives to grow depend on the marginal returns of doing so, \(V(n + 1) - V(n)\). It is this marginal return that links firms’ innovation incentives to the delegation environment. Using (16) to approximate \(V'(n) \approx V(n + 1) - V(n)\), (19) implies that

\[
x_n(n^*) = A \times \max \left\{ n^{-\lambda}, (n^*)^{-\lambda} \right\},
\]

(20)

where \(A \equiv (1 - \sigma) \theta^{\frac{1}{1-\varsigma}} \varsigma^{\frac{1}{1-\varsigma}} T^{\frac{\varsigma}{1-\varsigma}}\) is constant in a stationary equilibrium and \(\lambda = \frac{\varsigma}{1\varsigma}\).

(28) is the crucial equation of this paper in that it explicitly expresses the incentives to expand as a function of the delegation environment, which is fully summarized by the delegation cutoff \(n^*\). It is via (28) that delegation frictions will have dynamic implications. Consider Figure 7, which depicts the optimal innovation effort for firms of different sizes. Focus first on the red line, which corresponds to an environment where delegation benefits are low. As the benefits of delegation are small, the delegation cutoff is large and only very large firms (with \(n > n^*_L\)) start to delegate. Before delegation takes place, production is characterized by decreasing returns, which induces rapidly declining innovation incentives. Hence, this is an economy where large firms will not have

---

\(^9\)Note that we scale firms’ innovation costs by \(Y_t\). We do so for two reasons. Because we denote innovation costs in terms of the final good, a growing scaling variable is required to keep the model stationary. We could have done so by using \(Q_t\) as a scale. As profits grow at rate \(Y_t\) (see (2)), this would have introduced scale effects in that firms’ optimal innovation rate would have depended on the size of the labor force and the average mark-up in the economy (see (48)). By using \(Y_t\) as a scale, we abstract from these general equilibrium feedbacks, which simplifies the exposition. However, we show explicitly in Section 5.4 in the Appendix that our theoretical results do not depend on this choice and that or calibration remains unchanged. We also assume that firms’ innovation costs depend on the number of products \(n\) to generate deviations from Gibrat’s Law solely through incomplete delegation. In particular, if the value function was linear (as in Klette and Kortum (2004)), the specification in (17) would imply that firm growth was independent of size.

\(^{10}\)The fact that the probability of the firm losing a product does not feature in (18) is due to the fact that firms are assumed to be short-lived and that our continuous time formulation implies that events of expansion and destruction cannot happen simultaneously.
an incentive to grow and most firms will stay small. In the aggregate this will mean that there will be little creative destruction and selection as such frictions will exclusively harm innovative entrepreneurs. Stagnant producers that never contemplated to expand were never planning to delegate decision power anyways. Now suppose that the benefits of delegation increase. This will lower the delegation cutoff to \( n^*_H \) and increase innovation incentives. Importantly: only large firms’ expansion incentives will be affected. As innovative high-types are more likely to be large, efficient managerial delegation technologies will foster expansion by high-types and hence force low-type firms to exit faster. Hence, improvements in the efficiency of delegation will increase the extent of selection. Empirically this will manifest itself as a steeper increase in firm-size as firms age.

**Figure 7: Innovation Incentives and Firm Size**

**Flow Equations and the Stationary Distribution** The key force that pushes firms out of the market is creative destruction: Firms lose products if they are replaced by either new entrants or successful incumbents. To study the aggregate consequences of selection, we need to keep track of the share of product lines belonging to high and low types respectively. To do so, we have to introduce some notation. We will focus on a stationary environment, where both the number of firms and the respective firm-size distribution is constant. Let us denote the (endogenous) number of high- and low-type firms by \( F^H \) and \( F^L \) respectively and let \( \nu^j_n \) be the (endogenous) share of firms of type \( j \) with \( n \) products. As there is a measure one of products, it will be the case that

\[
1 = F^H \sum_{n=1}^{\infty} n\nu^H_n + F^L \sum_{n=1}^{\infty} n\nu^L_n = F^H \sum_{n=1}^{\infty} n\nu^H_n + F^L, \tag{21}
\]

where the second equality stems from the fact that there will not be any low-types with more than one product as they never grow.

To characterize the stationary equilibrium, note that firms’ innovation incentives \( x_n \) will be constant, i.e. they will be a function of firm-size but they will not be time-dependent. Given this schedule of innovation intensities, we can construct the entire process of firm dynamics. In particular, let us denote the aggregate rate of creative destruction, i.e., the rate at which the producer of a given product is replaced, by \( \tau \). Creative destruction can happen by new entry
(which occurs at rate $z$) or through the expansion of incumbent firms, whereby incumbents with $n$ products expand at rate $x_n$ (per product). Therefore,

$$\tau \equiv F^H \sum_{n=1}^{\infty} x_n n \nu^H_n + z.$$  \hspace{1cm} \text{(22)}

As usual, we can determine the steady-state values for the number of firm and the distribution of high-types from the economy-wide flow equations. These are given by the following set of equations:

\begin{align*}
\text{STATE:} & \quad \text{OUTFLOW} = \text{INFLOW} \\
F_L & : \quad F_L^\tau = (1 - \alpha) z \\
F^H \nu^H_1 & : \quad F^H \nu^H_1 \tau = \alpha z \\
\nu^H_{n \geq 1} & : \quad \nu^H_n [\tau + x_n] = \nu^H_{n-1} [n-1] x_{n-1} + \nu^H_{n+1} \tau [n+1]
\end{align*}  \hspace{1cm} \text{(23)}

The first line concerns the number of low-type firms in the economy. In particular, the left-hand side denotes the total number of low-type products that exit the economy (which happens at the rate $\tau$) and the right-hand side shows the number of low-type one-product firms that enter the economy. Together with (22), this directly implies that the number of low-types is given by

$$F_L = \frac{(1 - \alpha) z}{\tau},$$  \hspace{1cm} \text{(24)}

which already stresses the importance of creative destruction. Holding the amount of entry constant, the number of surviving low types will be small whenever creative destruction is severe. As creative destruction is only ignited by high-types expansion incentives embodies in $x_n$ (see (22)), (24) directly shows that the abundance of small firms in poor countries might largely be driven by the fact that transformative entrepreneurs might not be willing to grow. It is this mechanism, which we refer to as selection.

The second line in (23) similarly ensures that the number of high-type firms is constant. Note that the total mass of one-product high-type firms is given by $F^H \nu^H_1$, a fraction $\tau$ of which exit in each instant. Finally, the third line specifies the outflows and inflows for all high-type product lines with $n \geq 1$. The outflow from each product line can happen in two ways: Either the current producer of the product line will lose one of its $n$ product lines at the total rate of $n \tau$, or it will come up with a new innovation at the rate $X_n = nx_n$ in which case the respective firm will expand into a $(n+1)$−product firm. Likewise, the inflow can occur in two ways: Either firms with $n-1$ lines grow to being an $n$−line firm (which happens at the rate $(n-1)x_{n-1}$) or firms with $(n+1)$ products lose one product against another competitor (which happens at rate $(n+1)\tau$).

While the above discussion focused on the cross-sectional aspects of the innovation environment, the model also delivers a tractable theory of the firms’ life-cycle. Given the equilibrium rate of destruction $\tau$ and rate of expansion $x_n$, we can express the evolution of the size of any given high-type firm with $n$−products. Within a small time interval $dt$ the number of products will evolve as
Firm Dynamics in Developing Countries

follows:

\[
\begin{align*}
n(t + dt) = & \begin{cases} 
n(t) + 1 \text{ with probability } nx_n dt, \\
n(t) - 1 \text{ with probability } n\tau dt, \\
n(t) \text{ with probability } 1 - n(x_n + \tau) dt. 
\end{cases}
\end{align*}
\]

(25)

This implies that the expected (unconditional) growth rate is hence given by

\[
g_n = \lim_{dt \to 0} E \left[ \frac{n(t + dt) - n(t)}{n(t) dt} \right] = x_n - \tau = A \times \max \left\{ n^{-\lambda}, (n^*)^{-\lambda} \right\} - \tau
\]

(26)

(26) shows precisely why limits to delegation are plausibly related to the shallow life-cycle profile in poor countries: if bottlenecks in managerial hiring cause the delegation cutoff \( n^* \) to be large, firms' optimal innovation incentives \( x_n \) are declining in size until firms start the delegate. This particular deviation from Gibrat’s Law implies that large firms will grow at a lower rate than small firms, so that age is less of a predictor of size.

Similarly, consider a low-type firm, which by construction has only a single product. As this firms loses its only product at rate \( \tau \), the probability of that firm still being around in \( t \) years, is simply given by

\[
P[\text{survival until } t | \text{low type}] = e^{-\tau \cdot t}.
\]

(27)

While all low type firms exit economy eventually, (27) stresses that this weeding-out process runs its course faster, the higher the rate of creative destruction \( \tau \). The fact that stagnant firms in poor countries seem to survive for a long time is therefore consistent with the view that efficient firms generate too little creative destruction to drive them out of the market quickly.

We now have all the ingredients to formally define and characterize a stationary equilibrium in this economy. The definition is standard and formally stated in the Appendix. In particular, we require that firms' behave optimally, that the endogenous firm-size distribution is consistent with firms' innovation choices and that markets clear. More interestingly, we economy allows to characterize the stationary firm-size distribution in closed form, which makes the interaction between the delegation environment and the resulting firm-size distribution very concise.

**Proposition 3** Consider the economy above and let the delegation-cutoff \( n^* \) be given. Then there is a unique stationary equilibrium. High-types' innovation rates are given by

\[
x_n(n^*) = A \times \max \left\{ n^{-\lambda}, (n^*)^{-\lambda} \right\},
\]

(28)

where \( A \equiv (1 - \sigma)\theta^{\frac{1}{1-\sigma}} \xi^{\frac{\varsigma}{\varsigma-1}} T^{\frac{\varsigma}{\varsigma-1}} \) and \( \lambda = \frac{\varsigma}{1-\varsigma} \) are constants, the firm-size distribution of high types is given by

\[
\nu^H_n = \frac{n^{-1} \prod_{j=1}^{n} (\frac{x_n}{x_j})}{\sum_{s=1}^{\infty} s^{-1} \prod_{j=1}^{s} (\frac{x_j}{x_s})},
\]

(29)
the number of high- and low-type firms is given by

\[ F^H = \frac{\alpha z}{\tau} \times \left[ \sum_{s=1}^{\infty} s^{-1} \frac{\tau}{x^s} \prod_{j=1}^{s} \left( \frac{x_j}{\tau} \right) \right] \]

\[ F^L = \frac{(1 - \alpha) z}{\tau}, \]

the endogenous rate of creative destruction is given by

\[ \tau = z \times \left[ \alpha \sum_{s=1}^{\infty} \prod_{j=1}^{s} \left( \frac{x_j}{\tau} \right) + 1 \right]. \]

The equilibrium delegation-cutoff \( n^* \) is implicitly defined from the labor-market clearing condition

\[ 1 = L^P + M^D = \frac{Y_t}{\gamma w_t M_t} + \sum_{n=1}^{\infty} m_n n F^H v^H_n + m_n F^L, \]

where \( m_n \) denotes managerial demand given in Proposition 1 (see (13)). Under regularity conditions, there is a unique \( n^* \) consistent with (33).

Proposition 3 essentially characterizes the equilibrium analytically and contains three main results. First of all, we find it useful, to characterize the equilibrium firm-size distribution for a given delegation-cutoff \( n^* \). This clarifies the margin through which the delegation environment matters: in our economy \( n^* \) only matters for the endogenous firm-size distribution via high-types’ innovation incentives. In particular, (29), (30), (31) and (32) only depend on the innovation schedule \( \{x_n(n^*)\}_n \), which in itself depends on the delegation cutoff \( (n^*) \) explicitly. Secondly, the equilibrium value for \( n^* \) can be calculated from the labor-market clearing condition. This is not surprising, as \( n^* \) is directly linked to the equilibrium wage (see (12)). Finally, our main structural parameter of interest, the delegation benefits \( \xi \), only affect firms’ innovation incentives and hence the distribution of firms via \( n^* \). Hence, variation in delegation benefits across say countries, will affect \( n^* \) via the allocation of individuals into production workers and managers and then have implications for the implied process of firm-dynamics. It this property that allows us to directly use the empirical variation in the occupational distribution to identify \( \xi \) and then gauge the aggregate implications.

The fact that \( n^* \) parametrizes both the schedule of innovation intensities \( [x_n] \) allows us to make tight predictions about the comparative statics. To do so, let \( \varphi_n \) be the number of products being produced by firms with \( n \) products in their portfolio. Hence,

\[ \varphi_n(n^*) = \begin{cases} F^L + F^H v^H_n & \text{if } n = 1 \\ F^H v^H_n & \text{if } n > 1 \end{cases} \]
where we explicitly note the dependence on $n^*$.\footnote{Note that $\sum_n \varphi_n(n^*) = 1$ as the product space has measure one.} From (34) we can also define the share of products produced by firms with at most $n$ products, $\Phi_n(n^*)$, and the share of products produced by high type firms, $\chi^H(n^*)$. These are given by

\begin{align}
\Phi_n(n^*) &= \sum_{j=1}^n \varphi_n(n^*) \tag{35}
\chi^H(n^*) &= \sum_n nF^H(n^*)\nu^H_n(n^*) = 1 - F^L(n^*). \tag{36}
\end{align}

Proposition 3 and (34), (35), and (36) then imply the following comparative static results.

**Proposition 4** Consider the setup above. Then the following is true:

1. The innovation schedule $x_n(n^*)$ is decreasing in $n^*$ for all $n \geq n^*$,
2. The rate of creative destruction $\tau(n^*)$ is decreasing in $n^*$,
3. The number of low-type firms $F^L(n^*)$ is increasing in $n^*$,
4. The share of the product space produced by high-type firms $\chi^H(n^*)$ is decreasing in $n^*$,
5. The distribution of products $\Phi_n(n^*)$ is decreasing in $n^*$ in a FOSD sense,
6. Average firm size is decreasing in $n^*$.

Furthermore, the equilibrium delegation cutoff $n^*$ is decreasing in the benefits of delegation $\xi$.

**Proof.** See Appendix.

Proposition 4 summarizes the economic effects of the delegation environment via firms’ expansion activities and the endogenous adjustments through selection and creative destruction. The "prime-mover" of an increase in delegation, i.e. a decrease in $n^*$, is an increase in high-types appetite for expansion. As high-type firms expand on behalf of their rivals, the rate of destruction $\tau$ will increase. This will force low-type firms to exit a faster rate (see (27)), thereby reducing both their number and the share of products they produce in equilibrium. Furthermore, improvements in firms’ delegation possibilities will affect the firm-size distribution in an asymmetric way: as small firms do not delegate decision power anyway, large firms’ expansion rates will increase relative to small firms and the distribution of firms’ growth rates will more closely resemble Gibrat’s Law (see (26)). This implies that the firm-size distribution will shift towards larger firms and that average firm-size increases. Hence, qualitatively, Proposition 4 is consistent with the main stylized facts about firms in rich and poor countries and provides a unifying mechanism to interpret this cross-country evidence: firms in rich countries might have access to a better delegation environment via the abundance of managers with the appropriate human capital, a well-functioning system of contract enforcement or better complimentary technologies, which raise the value of delegation.
Wether or not this mechanism is also quantitatively able to explain the differences across countries is the subject of the next sections.

3 Quantitative Exercise

We now take the model to the data to gauge the quantitative importance of this mechanism. Our strategy is as follows. We first calibrate the model to various moments from the US. In particular, we target different moments of the process of firm-dynamics of the US manufacturing sector so that the model matches the US life-cycle by construction. As stressed in the theory, the crucial parameter for firms’ demand for delegation is the management multiplier $\xi$, which we discipline by forcing the model to match the US managerial employment share. We think of this multiplier as a reduced form parameters at the country level, which depends on various country-characteristics - in our particular micro-foundation it depends on the strength of the legal system $\kappa$, the level of managerial human capital $\eta$ and the efficiency of the monitoring technology $\phi$. Hence, from the micro-variation within a country, we cannot identify these individual components but only $\xi$ itself. To quantify the explanatory power of this delegation margin, we hence proceed in three steps. We first recalibrate the entire model to microdata from the manufacturing sector in India. This allows us to use the model to analyze the importance of selection in shaping the differences in firm-performance between India and US. We then study the implications of $\xi$ for the life-cycle of manufacturing plants. In particular, we will use the calibrated model to perform the counterfactual exercise of studying the Indian economy with the US level of delegation benefits and vice versa. Finally, we try decompose the calibrated delegation benefits $\xi$ into various measurable components. To do so, we treat $\xi$ as a country-fixed effect and calibrate it the observed cross-sectional distribution of managerial employment shares across the world. That is, for each country we calibrate $\xi$ to perfectly match the particular countries share of managerial personnel. The resulting change in the implied firm-dynamics is - through the lens of the model - the causal effect of changes in delegation benefits and hence answers the question what fraction of the observed difference in firm performance our mechanism can explain, once it is disciplined to be consistent with the managerial employment shares across countries. This exercise, is deliberately silent on the underlying source of variation in these benefits of delegation and is hence not a policy parameter with well-defined counterfactual experiments. To make progress in that dimension, we than decompose such delegation benefits into various components. In line with our micro-foundation, we focus on the variation induced by differences in the legal system, by managerial human capital and by the development of the financial system. Exploiting the cross-country variation in these three fundamentals and the calibrated delegation benefits, we can then further decompose the explanatory power of the legal system on firms’ life-cycle through its effect on delegation benefits and also consider counterfactual exercises, like improving India’s legal system holding human capital constant.
3.1 Data

Here we briefly describe the main data sources for our analysis. A more detailed description is contained in Section 5.7 in the Appendix.

**US Data** To calibrate our model to the US economy, we rely on publicly available data from the Business Dynamics Statistics (BDS). The BDS are provided by the U.S. Census Bureau and are compiled from the Longitudinal Business Database (LBD), which draws on the Census Bureau’s Business Register to provide longitudinal data for each establishment with paid employees. Importantly for us, the BDS uses a unified treatment of establishments and firms. While an establishment is a fixed physical location where economic activity occurs, firms defined at the enterprise level such that all establishments under the operational control of the enterprise are considered part of the firm. The BDS contain information on the cross-sectional relationship between age and size (which we refer to as the life-cycle), exit rates and exit rates by age conditional on size. The latter will be a crucially moment to identify the importance of heterogenous types in the US economy.

The definition of age is straightforward for the case of establishments. There, the year of birth is defined as the year an establishment first reports positive employment in the LBD. Establishment age is then computed by taking the difference between the current year of operation and the birth year. For firms, the procedure is slightly different. First, a firm is assigned an initial age by determining the age of the oldest establishment that belongs to the firm at time of birth. Then, firm age accumulates with every additional year. This implies that mergers or acquisitions to not lead to abrupt changes in firm age. Table 1 provides some summary statistics about the size-distribution of firms and plants in the US. The average manufacturing firm in the US has 55 employees, while the average plant only 45. It is also the case that large firms have multiple establishments (firms with more than 1000 employees have on average 13) so that large firms account for more than half of total employment. There is less concentration at the plant level in that plant with more than 1000 employees account for less than one fifth of aggregate employment in manufacturing in the US.

We augment the firm- and plant-level data by additional information pertaining to the importance of managerial personnel in the US economy. We rely on two data sources. We first focus on individual level micro data from the US census, which contains detailed information on earning and occupational categories. This allows us to measure the importance of managers in both employment and factor payments in the manufacturing sector in the US. Secondly, we use the US Product and Income Accounts (NIPA) to measure corporate profits and employee compensation for US manufacturing firms, which will be helpful to identify the elasticity of managerial effort $\sigma$.

**Indian Micro Data** We are using two major sources of data about Indian manufacturing establishments. The first source is the Annual Survey of Industries (ASI) and the second is the National Sample Survey (NSS). The ASI is an annual survey of manufacturing enterprises. It covers all plants employing ten or more workers using electric power and employing twenty or more workers
Table 1: Descriptive Statistics: US Micro Data

<table>
<thead>
<tr>
<th>Size</th>
<th>Firms</th>
<th></th>
<th>Establishments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>96219</td>
<td>2.37</td>
<td>1.51</td>
<td>14.51</td>
</tr>
<tr>
<td>5-9</td>
<td>55408</td>
<td>6.75</td>
<td>2.47</td>
<td>1.43</td>
</tr>
<tr>
<td>10-19</td>
<td>45591</td>
<td>13.82</td>
<td>4.17</td>
<td>1.10</td>
</tr>
<tr>
<td>20-49</td>
<td>39813</td>
<td>31</td>
<td>8.17</td>
<td>1.04</td>
</tr>
<tr>
<td>50-99</td>
<td>16438</td>
<td>68.7</td>
<td>7.48</td>
<td>1.18</td>
</tr>
<tr>
<td>100-249</td>
<td>10468</td>
<td>142.59</td>
<td>9.88</td>
<td>1.52</td>
</tr>
<tr>
<td>250-499</td>
<td>3610</td>
<td>291.65</td>
<td>6.97</td>
<td>2.48</td>
</tr>
<tr>
<td>500-999</td>
<td>1859</td>
<td>524.38</td>
<td>6.45</td>
<td>3.82</td>
</tr>
<tr>
<td>1000+</td>
<td>2750</td>
<td>2905.49</td>
<td>52.9</td>
<td>13.31</td>
</tr>
<tr>
<td>Aggregate</td>
<td>272156</td>
<td>55.5</td>
<td>100</td>
<td>6.93</td>
</tr>
</tbody>
</table>

Notes: This table contains summary statistics for US manufacturing firms and plants in 2005. The data is taken from BDS.

without electric power. For our analysis we use only the cross-sectional data in 2005 to make it comparable to our sample of the NSS. For an economy like India, the ASI covers only a tiny fraction of producers, as most plants employ far fewer than twenty employees. To overcome this oversampling of large producers in the ASI, we complement the ASI data with data from the NSS. The National Sample Survey is a survey covering different aspects of socio-economic life in India. Every five years, however, Schedule 2.2 of the NSS surveys a random sample of the population of manufacturing establishments without the minimum size requirement of the ASI. While these producers are (by construction) very small, they still account for roughly 30% of aggregate output in the manufacturing sector. We use the NSS data for the year 1994/95 and merge it with the ASI using the sampling weights provided in the data. In terms of the data we use, we mostly focus on the employment side. In particular, we draw on the information on age and employment to study the cross-sectional age-size relationship. For a more detailed description and some descriptive statics, we refer to Hsieh and Olken (2014).

Cross Country Data To discipline the benefits of delegation across countries and identify the underlying fundamental determinants we draw on three sources of cross-country data. First we use census information disseminated by IPUMS-International. This data provides us with consistent measures of occupational categories, sectors of employment educational attainment for millions of workers around the world. We aggregate this micro-data at the country level and focus on the cross-country variation in managerial employment shares and human capital. We augment this data by standard information on measures of development from the Penn World Tables and by the World Bank Global Financial Development database, which provides various measures of the state
3.2 Identification

In the model, we have eight parameters to calibrate: \( \sigma, \xi, \alpha, \theta, z, \gamma, \beta, \) and \( T. \) Given the non-linear nature of the model, we cannot get analytical solutions for all firm moments that we would like to match. Nevertheless, in this section we will provide a heuristic description of where each parameter is identified from.

- We identify \( \sigma \) from manager compensations. Recall that the profit maximization of a firm is given by

\[
V(n) = n \max_{m \geq 0} \left\{ (\frac{T}{n} + m\xi)^\sigma - \omega m \right\},
\]

where \( \omega \equiv w/Y \) is the normalized wage payment. The optimality condition is such that

\[
(\frac{T}{n} + m\xi)^{\sigma-1} \sigma \xi = \omega
\]

which implies

\[
\sigma = \frac{m\omega \left( \frac{T}{n} + m\xi \right)}{\frac{\sigma \omega}{1 - \sigma}} = \frac{\text{manager compensation}}{\text{profit}} \frac{(\frac{T}{n} + m\xi)}{m\xi}.
\]

Therefore we will use \( M_1 : \frac{\text{manager compensation}}{\text{profit}} \) as one the target moments. Note that when \( T \) is small or \( n \) is large (i.e, \( T/n \to 0 \)), as it will be the case for the US economy, \( \sigma \) is exactly equal to the target moment in our model.

- Aggregate growth rate in our model is equal to

\[
M_2 : g = \tau \ln \gamma,
\]

therefore we will use the aggregate growth rate to identify \( \gamma. \)

- In order to identify the delegation benefit \( \xi \) and owner’s time \( T, \) we will use information on managers in an economy. Recall that manager hiring decision, both at the extensive and intensive margins, is affected by \( \xi \) and \( T \) as follows:

\[
m_n = \begin{cases} 
0 & \text{if } n < \left[ T \left( \frac{\omega}{1 - \sigma} \right) \right] \\
\left( \frac{\sigma}{\omega} \right)^{\frac{1}{1-\sigma}} \xi^{\frac{1}{1-\sigma}} - \frac{T}{\xi n} & \text{otherwise}
\end{cases}
\]

Therefore we will target the fraction of firms that do not hire managers to capture information on the extensive margin and also the fraction of managers in the total workforce to capture information in the extensive margin. Let \( F_n^* \) denote the measure of firms that is of type...
s ∈ \{H, L\} and has n product lines. Then these moments can be expressed as
\[
M_3 : \frac{\text{firms without managers}}{\text{all firms}} = \frac{F^L + \sum_{n=1}^{n^*} F^H_n}{F^L + F^H}, \quad \text{and} \quad M_4 : \frac{\text{managers}}{\text{total workforce}} = \sum_{n=n^*}^{\infty} F^H_n m_n
\]
where \(F^s\) denotes the total measure of firms of type \(s\). Here moment \(M_3\) captures the fraction of firms with no manager and \(M_4\) captures the fraction of managers in the total workforce (recall that the total measure of labor is unity).

- Entry rate in our model corresponds to
\[
M_5 : \text{entry rate} = \frac{z}{F^L + F^H}.
\]
Clearly targeting \(M_5\) will inform us about the value of \(z\).

- Now we focus on firm type parameters \(\alpha\) and \(\beta\). Note that without firm types, all dynamic moments such as firm growth and firm exit depend only on firm size. Once firm size is controlled, firm age does not have any implications when firm types are abstracted from. However, from the empirical studies (Haltiwanger et al. 2001) we know that firm exit rate decreases strongly in firm age, especially for the smallest firms. This is a result of firm selection.

In addition, conditional on survival, average employment over a firm’s life-cycle also depends strongly on the selection in the economy. Besides, share of high-types \(\alpha\) affects the equilibrium measure of firms. Therefore to be able to match these different margins in the data, we adopt \(\alpha\) and \(\beta\) and target the following selection moments
\[
M_6 : \frac{\bar{n}(a = 26+)}{\bar{n}(a = 1-5)} \quad \text{and} \quad M_7 : \frac{\text{exit rate}(a = 16 - 20 \mid n = 1)}{\text{exit rate}(a = 1 - 5 \mid n = 1)}.
\]
Moment \(M_6\) captures the ratio of average firm size of all the firms that are older than 26 to average firm size of all firms whose age is less than or equal to 5. Likewise, \(M_7\) focuses on the ratio of exit rates of 1-product firms that are between 16-20 years old to those that are younger than 5. Note that the fact that we focus on only 1-product firms effectively controls for firm size and focuses on the age cohort where the selection is strongest both in the data and in the model.

- Finally, we target profitability to discipline the innovation incentives.

### 3.3 Calibrating the Model to the US

We now turn to the US economy and calibrate the model to the US Manufacturing establishments. Table 2 reports the targeted moments and the model-generated counterparts.
Table 2: Moments Targeted for the U.S.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$. Share of manager compensation</td>
<td>0.51</td>
<td>0.50</td>
</tr>
<tr>
<td>$M_2$. Aggregate growth rate</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$M_3$. Employment share of no-manager firms</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>$M_4$. Share of managers in workforce</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$M_5$. Entry rate</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$M_6$. Mean employment for 21-25 year old firms</td>
<td>2.61</td>
<td>2.62</td>
</tr>
<tr>
<td>$M_7$. Relative exit rate (age:16-20 to age:1-5 ratio)</td>
<td>1.45</td>
<td>1.45</td>
</tr>
<tr>
<td>$M_8$. Employment share of 21-25 year old firms</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>$M_9$. Profitability</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Overall the model matches the moments very closely. Table 3 reports the resulting parameters.

Table 3: Parameter Calibration for the US

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>Benefit of delegation</td>
<td>Manager share</td>
<td>0.41</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Curvature of efficiency</td>
<td>Life-cycle</td>
<td>0.51</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of high type</td>
<td>Empl. share of old firms</td>
<td>0.22</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Innovativeness</td>
<td>Growth rate</td>
<td>2.52</td>
</tr>
<tr>
<td>$z$</td>
<td>Entry flow rate</td>
<td>Rate of entry</td>
<td>0.06</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Innovation step size</td>
<td>Profitability</td>
<td>1.10</td>
</tr>
<tr>
<td>$\beta$</td>
<td>High-type replacement</td>
<td>Age vs exit profile</td>
<td>0.45</td>
</tr>
<tr>
<td>$T$</td>
<td>Managerial endowment</td>
<td>Share of self-employed</td>
<td>0.01</td>
</tr>
</tbody>
</table>

We estimate the share of high-type entrepreneurs to be 0.22 which implies three out of four new entrepreneurs never expands. The estimate of $\beta$ implies that almost half of the attempts are deflected by high-type incumbents. Finally, $T$ is estimated to be very small which implies that owner’s own time has minimal impact on firm performance since in the US economy owner’s can rely on outside entrepreneurs when they want to expand.

These parameters help the model mimic the life-cycle profile of US firms very closely. Figure 8 plots the life-cycle dynamics both in the data and in the model.

The positive slope in the life cycle reflects the selection that takes place in the background. In particular, high-type firms find it profitable to expand into new markets since they are not worried about the span of control problem. This is thanks to the delegation possibilities in the US.

To see the strength of the selection mechanism, we now focus on the composition of a cohort over time. As high-type firms expand, they push out the low-type firms from the economy as illustrated in Figure 9. Even though a cohort starts with 22% high-type firms, by the age of 40 this share increases to almost 100%.
3.4 Calibrating the Model to the Indian Microdata

Our goal is to study the implications of differences in delegation benefits across India and the US economy. To this end, this section re-estimates the model using Indian data. The resulting parameters and moments are reported in Tables 4 and 5.

Table 4: Moments Targeted for India

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$. Share of manager compensation</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>$M_2$. Aggregate growth rate</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$M_3$. Employment share of no-manager firms</td>
<td>0.78</td>
<td>0.75</td>
</tr>
<tr>
<td>$M_4$. Share of managers in workforce</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$M_5$. Entry rate</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>$M_6$. Mean employment for 21-25 year old firms</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>$M_7$. Relative exit rate (age:16-20 to age:1-5 ratio)</td>
<td>1.10</td>
<td>1.09</td>
</tr>
<tr>
<td>$M_8$. Employment share of 21-25 year old firms</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>$M_9$. Profitability</td>
<td>0.17</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 5: Parameter Calibration for India

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>benefit of delegation</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>curvature of efficiency</td>
<td>0.22</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>share of high type</td>
<td>0.03</td>
</tr>
<tr>
<td>$\theta$</td>
<td>innovativeness</td>
<td>1.26</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>entry flow rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>innovation step size</td>
<td>1.27</td>
</tr>
<tr>
<td>$\beta$</td>
<td>high-type replacement</td>
<td>0.27</td>
</tr>
<tr>
<td>$T$</td>
<td>managerial endowment</td>
<td>0.01</td>
</tr>
</tbody>
</table>
As seen in Table 4, the model does again a very good job in matching the salient features of the Indian data. This is also seen in Figure 10, where we compare the entire implied life-cycle profile with the one observed in the data. The model essentially matches the observed flat life-cycle - in fact the non-calibrated age bins are matched at least as good as target moment of the 21-25 year old firms in Table 4.

**Figure 10: Life Cycle of Indian Firms**

<table>
<thead>
<tr>
<th>Age</th>
<th>0-5</th>
<th>6-10</th>
<th>11-15</th>
<th>16-20</th>
<th>21-25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Employment</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
</tr>
</tbody>
</table>

**Figure 11: Share of One-product Firms in India**

We can now use the model to understand better the importance of selection or the lack thereof in comparison to the US economy. The theory stresses that the life-cycle profile in Figure 10 masks the heterogeneity across firms, whereby some producers do grow - but not sufficiently to affect the aggregate life-cycle profile. To see this mechanism in the calibrated model, consider Figure 11, where dotted-red line shows the share of one-product firms in India as a function of the cohort’s age and the solid-black line plots the US counterpart for comparison. The remarkable result is that while this share in the US declines from 100% down to 20% by the age of 40, it declines only to 85% in India. This difference occurs due to the fact that high-type firms in the US are growing rapidly and pushing out the low-type firms whereas this does not take place in so starkly in India.

Another way of looking at this selection mechanism is to look at the share of high-type firms in a cohort. Figure 12 plots this share both for the US and Indian firms. There are two results that stand out in this figure. First, the share of high-type firms in the US is significantly bigger among the entering cohort. Second, high-type firms grow faster in the US creating a much stronger selection. The end result is that by the age of 40, the share of high-type firms increases to 100% whereas this share raises only to 50% in India. Clearly this picture is affected by the initial composition of the cohort. Therefore, to abstract from this initial difference, Figure 13 simulates a counterfactual US economy as if the initial type distribution in the US was equal to the Indian distribution while keeping all other parameters at the US levels, i.e., it replaces $\alpha_{US}$ with $\alpha_{IND}$. What it shows is that even if the initial share of high-type firms was as low as the Indian level, high-type firms would have taken over the whole cohort by the age of 40, thanks to the very strong selection that takes
Comparing Selection in the Data and in the Model  

We now compare the selection predicted by the model to the selection that we observe in the data. Figure 14 plots the share of small firms in the US and in India, normalizing the initial share to 1. What is remarkable in this figure is that the selection generated in our model for India and the US is extremely close to what the data generates. Please note that this is not a moment that we target in our estimation analysis, therefore this figure implies that the selection mechanism, which is at the heart of our analysis, generates empirically realistic dynamics.

Figure 14: Share of Small Firms (Data vs Model)
3.5 Quantitative Implication of the Delegation Mechanism

How much of the difference in firms’ life-cycle between India and US is due to a more seamless mode of delegation in the US? More specifically, how would their life cycle have looked like if the Indian firms had exactly the same delegation benefits as the US firms and vice versa?

To answer this question: (1) we simulate the Indian economy by giving the Indian firms the US level of delegation benefit, i.e., we replace \( \xi_{\text{IND}} \) with \( \xi_{\text{US}} \) in Figure 10; (2) we simulate the US economy with Indian level of delegation benefit, i.e., we replace \( \xi_{\text{US}} \) with \( \xi_{\text{IND}} \) in Figure 8. The results are plotted in Figures 15 and 16.

**Figure 15:** Life Cycle of Indian Firms with \( \xi_{\text{US}} \)  
**Figure 16:** Life Cycle of US Firms with \( \xi_{\text{IND}} \)

Figure 15 shows that when Indian firms are given the US level of delegation benefit, they expand more rapidly and create more selection in Indian economy. As a result, the life-cycle profile becomes steeper and closes the gap in the average size of 26+ firms by 32% which is quite sizable. Figure 16 mirrors this exercise in the US economy by giving the US firms \( \xi_{\text{IND}} \). The result is a big reduction in the average size of the firms. For instance, this change closes the gap between 26+ year-old Indian and US firms almost entirely. These two exercises show that the differences in delegation possibilities could go a long way in explaining quantitatively the empirical life-cycle differences in India and the US.

3.6 Extension 1: Decomposing the Benefits to Delegation into Fundamentals

So far, we have shown that the differences in delegation benefits \( \xi_{\text{us}} \) and \( \xi_{\text{ind}} \) have quantitatively very important impacts on life cycle dynamics. Our analysis did not need to take any stand on the exact determinants of the delegation benefits. In this section, our goal is to go one step further and attempt to understand the institutional components of \( \xi \). In other words, we try to explicitly decompose this “country fixed effect” into the different components using the structure of the theory. Our focus is on three of the most popular institutions that the previous literature has
considered: the rule of law, human capital and financial development.

Our empirical strategy in this section is as follows. Our main source of identification for $\xi$ stems from the aggregate importance of managerial workers. Using the occupational identifiers in the international census data, we can measure this managerial employment share for a broad cross-section of countries. In Figure 17 we show the cross-sectional variation in the importance of managerial personnel across countries. In particular, managerial employment is much more common in more developed economies. This systematic variation in the importance of managers will discipline our quantitative analysis.

**Figure 17: Managerial Employment Across Countries**

As a baseline, we are going to impose the identification assumption that the *entirety* of the cross-sectional dispersion in managerial employment is driven by underlying differences in delegation benefits $\xi$. Under this assumption we can identify $\xi$ for each country given the data on managerial employment shares shown in Figure 17. Keeping all other parameters at the US levels of Table 3, we can assign a unique level of delegation benefit $\xi_c$ to each country $c$.

Figure 18 maps each country $c$ to its model-implied delegation benefit $\xi_c$ using its share of managers in the workforce. Likewise, Figure 19 plots the model’s implication for cross-country relationship between the managerial employment share and implied “steepness” of the life-cycle profile through the lens of the model, i.e. the size of 30 year old firms relative to new entrants. Recall that the slope of this locus is identified through the mapping between the observable managerial share and the benefits of delegation, which are identified from the firm-level data in India. As seen clearly in the Figure: given the variation of the importance of managerial personnel across countries, the model is able to deliver sizable differences in the life-cycle across firms.

---

12Conceptually, this exercise is very similar to the quantitative strategy adopted by Buera et al. (2011) in the context of a model with credit market frictions. There, the sole variation across countries is assumed to lie in the quality of the financial system. To discipline the variation in financial systems across countries, Buera et al. (2011) target the cross-country variation in the debt-to-GDP ratio, which is an endogenous outcome of the model. In our case, we envision the source of variation stemming from delegation benefits and identify $\xi$ from the cross-country variation in managerial employment shares, which is also an endogenous outcome of the model.
Given the inferred value of delegation benefits $\xi_c$, we now decompose explicitly this “country fixed effect” into the different institutional components: Rule of law, human capital, and financial development. We adopt a reduced form approach, where we simply run the following cross-country regression

$$
\xi_c = \beta_0 + \beta_1 \times ROL_c + \beta_2 \times HC_c + \beta_3 \times FD_c + \beta_4 \times \ln(y_c) + \epsilon_c.
$$

(37)

where $ROL_c$ proxies the legal system by the World Banks rule of law measure, $HC_c$ denotes the measured human capital, $FD_c$ denotes the financial development proxied by the domestic lending as a fraction of GDP, and $\ln(y_c)$ is country $c$’s log GDP per capita, which we include in the regression to control for the general stage of development.

In order to implement the decomposition contained in (37), we have to decide on an appropriate measure of human capital. We present the results using two different measures. As a benchmark we consider the average level of human capital, which we construct from the educational attainment of the entire population in the respective country (taken from the Census data). As an alternative we consider the human capital of managerial workers. The results are contained in Table 6 below.

These results show that the cross-sectional variation in managerial share across countries is robustly correlated with the quality in legal institutions, even after the level of development, human capital and financial development are controlled for.

Given the estimated parameters $\hat{\beta}_1$ through $\hat{\beta}_4$, we can use (37) to decompose the variation in inferred delegation benefits $\xi_c$ into its different components. For instance, we can study the impact of Indian-level of rule of law on US firms by simply giving $\hat{\xi}(ROL_{\text{ind}}, HC_{\text{us}}, FD_{\text{us}}, y_{\text{us}})$ to the US firms where

$$
\hat{\xi}(ROL_{\text{ind}}, HC_{\text{us}}, FD_{\text{us}}, y_{\text{us}}) = \hat{\beta}_0 + \hat{\beta}_1 \times ROL_{\text{ind}} + \hat{\beta}_2 \times HC_{\text{us}} + \hat{\beta}_3 \times FD_{\text{us}} + \hat{\beta}_4 \times \ln(y_{\text{us}}).
$$

Figure 20 introduces one additional Indian-level of institutional parameter at a time to the US firms and plots the resulting life cycles.

The solid black line at the very top and the dashed red line at the very bottom show the
**Table 6: Managerial Employment Share**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rule of law</td>
<td>0.044*</td>
<td>0.036*</td>
<td>0.042*</td>
<td>0.066***</td>
<td>0.062**</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.023)</td>
<td>(0.020)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>human capital</td>
<td>0.023</td>
<td>0.022</td>
<td>0.023</td>
<td>0.023</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.015)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>private credit</td>
<td>0.006</td>
<td>0.021***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bank branches</td>
<td></td>
<td>0.011</td>
<td></td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td></td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td>bank deposits</td>
<td></td>
<td>-0.001</td>
<td>-0.021</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.015)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>46</td>
<td>45</td>
<td>37</td>
<td>43</td>
<td>35</td>
</tr>
<tr>
<td>R2</td>
<td>0.60</td>
<td>0.60</td>
<td>0.61</td>
<td>0.64</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Notes: All specifications control for log GDP per capita.

**Figure 20: Decomposing the Life Cycle**

---

model’s predictions for the US and India, respectively. More precisely, they show average plants’ lifecycles of plants given the inferred delegation benefits $\hat{\xi}(ROL_{IND}, HC_{IND}, FD_{IND}, y_{IND})$ and $\hat{\xi}(ROL_{US}, HC_{US}, FD_{US}, y_{US})$ while keeping all other remaining parameters at their US levels reported in Table 3. The three remaining dashed lines inbetween refer to the decompositions of the life-cycle. Starting from the top, the second light-colored dashed line represents the hypothetical US economy with the Indian rule of law, i.e, $\hat{\xi}(ROL_{IND}, HC_{US}, FD_{US}, y_{US})$; the third line adds Indian level of human capital on top, $\hat{\xi}(ROL_{IND}, HC_{IND}, FD_{US}, y_{US})$; and finally, the fourth line replaces the financial development measure on top such that, $\hat{\xi}(ROL_{IND}, HC_{IND}, FD_{IND}, y_{US})$.

Table 7 reports the share of average life-cycle difference accounted by each institution by age of the cohort.

This tables shows that the impact of the human capital and rule of law are highest throughout
the life cycle of a cohort. In any given age, human capital accounts for 40% of the difference whereas the rule of law accounts for 30%. Financial development explains 15% of the differences and the rest is attributed to other institutional variables that is captured by the GDP per capita term in the regression.

3.7 Extension 2: Firm-level Estimation

So far our analysis has utilized establishment-level data from India and the US, following the tradition of Hsieh and Klenow. In this section, we will reestimate our model using firm-level data from the US. The new targets are reported in Table 8.

### Table 8: Moments Targeted for US Firms

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$. Share of manager compensation</td>
<td>0.51</td>
<td>0.47</td>
</tr>
<tr>
<td>$M_2$. Aggregate growth rate</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$M_3$. Employment share of no-manager firms</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>$M_4$. Share of managers in workforce</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$M_5$. Entry rate</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>$M_6$. Mean employment for 21-25 year old firms</td>
<td>3.45</td>
<td>3.45</td>
</tr>
<tr>
<td>$M_7$. Relative exit rate (age:16-20 to age:1-5 ratio)</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>$M_8$. Employment share of 21-25 year old firms</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>$M_9$. Profitability</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Among these moments, the main change is observed on the life-cycle moment. While establishments grow 2.6 times relative to their entry size by the age of 25, this number is 3.45 for the US firms. Our model delivers again a very good fit with respect to all moments. The resulting parameters are listed in Table 9. The resulting parameters are very similar to what we had estimated using the establishment data. In particular, the share of high-type firms is again around 1/4 and the benefit of delegation is very much in the same ballpark.

Another important result from this exercise is that the selection margin works also very strongly here. When we do our main analysis of giving the US-level of delegation benefits to Indian firms and vice versa, the results are very comparable to what we had before, as shown in Figures 21 and 22.
3.8 Extension 3: Role of Different Margins of the Model

Up to now we only allowed for a single source of heterogeneity across countries. In this section, we will analyze the role of different mechanisms on the implied firms dynamics in more detail. For this, we will examine the partial effects of various channels of the model. This will turn out to be a useful exercise to see which ingredients are quantitatively important for the model’s implications.

**Entry** We first start with the entry intensity $z$, which could presumably vary across countries. Figure 23 plots the employment share of firms 26+ years old against various values of the entry rate, keeping all other parameters at their Indian values. A seen in Figure 23 more entry implies more competition and hence lowers firm size. This is because incumbent firms are losing their product lines at a faster rate to new entrants. Hence, for the entry rate to explain the differences in life cycle growth between Indian and US firms, it had to be the case that entry in the US was much lower. Not only do we not know of any evidence for this to be true, but Figure 23 also shows that the quantitative effect of a higher entry rate is relative minor. Differences in the rate of entry are therefore unlikely to play a major role - at least within a model like ours.

**Entrepreneurial Talent** We next turn to $\alpha$, the probability of being a high type en-
entrepreneur. Maybe the low life-cycle growth profile among Indian plants stems simply from the fact that there are too few entrepreneurs with high innovation potential. In Figure 24 we keep all other parameters at their Indian values and vary only $\alpha$. Initially, this indeed contributes to a higher employment share of older firms. However, this positive effect fades away quickly, so that the life-cycle growth is essentially non-responsive to changes in the share of innovative firms. In fact, for large values of $\alpha$, the average employment of large firms even start to decrease. This is due to the general equilibrium nature of the model. In our model growth is driven by selection, whereby successful firms managed to innovate and thereby steal market share from their less fortunate competitors. A higher share of innovative types therefore implies more competition among high type incumbents, so that firms also lose products quickly. To generate a steeply increasing life-cycle profile there cannot be too many innovative firms - there have to be sufficiently many marginal firms in the economy, which can be replaced easily. While this margin is likely to be more important then the entry rate analyzed above, average employment does not increase sufficiently to make it the sole candidate for explaining the differences between Indian and US firms. Simply having more talented entrepreneurs does not contribute much to the dynamism of economy unless the managerial environment allows for seamless delegation and thereby sustained incentives to grow large.

4 Conclusion

This paper studies the reasons behind the stark differences in firm dynamics across countries, as documented in Hsieh and Klenow (2014). We focus on manufacturing firms in India and analyze the stagnant firm behavior. We show that the poor life-cycle behavior in India could be explained by the lack of firm selection, wherein firms with little growth potential survive because innovative firms do not expand sufficiently quickly to replace them. Our theory stresses the role of imperfect managerial delegation as the main cause for the insufficient expansion by the firms with growth
potentials. We show that if the provision of managerial effort is non-contractible, the benefits of delegation are low and firms will run quickly into decreasing returns. This in turn will reduce the returns to innovation. Improvements in the degree of contract enforcement will therefore raise the returns to growth and increase the degree of creative destruction. This argument is in line with the empirical findings of Bloom and Van Reenen (2007, 2010). Quantitatively, such limits to delegation can explain very sizable fraction of the difference in life-cycle growth between the US and India.

References


5 Appendix

5.1 Static Equilibrium

On the demand side, we have a representative household with standard preferences

\[ U_0 = \int_0^\infty \exp (-\rho t) \ln C_t dt, \tag{38} \]

where \( \rho > 0 \) is the discount factor. Given the unitary intertemporal elasticity of substitution, the Euler equation along the balanced growth path is simply given by \( g = r - \rho \), where \( g \) is the growth rate of the economy and \( r \) is the interest rate.

Now consider the equilibrium in the product market. At each point in time, each product line \( j \) is populated by a set of firms that can produce this good with productivity \( q_{jf} \), where \( f \) identifies the firm. Recall that the production function was given in (1) as

\[ y_{jf} = q_{jf} \mu(e_{jf}) l_{jf}. \]

We will make sufficient assumptions on \( \mu(.) \), that the most productive firm (which we will sometimes refer to as the (quality) leader) will be the sole producer of product \( j \). Intuitively, while managerial slack can and will be a drag on efficiency, it can never reverse comparative advantage based on physical efficiency \( q \). More specifically, As the demand functions stemming from (2) have unitary elasticity, the optimal mark-up was infinity if there was no entry threat. Hence, we assume that there is a competitive fringe of firms, that can produce the product with efficiency \( q_{jt} \gamma \) for some \( \gamma > 1 \). For simplicity, we assume that fringe firms do not provide any managerial effort.

Given this structure the leader will always be forced to engage in limit pricing so that the equilibrium price for product \( j \) is given by

\[ p_{ij} = \frac{\gamma w_t}{q_{ij}}. \tag{39} \]

Equation (2) then implies that the demand for product \( j \) is given by

\[ y_{jt} = \frac{Y_t}{p_{ij}} = \frac{q_{jt} Y_t}{\gamma w_t}, \tag{40} \]

so that total sales are simply \( S_{jt} = p_{jt} y_{jt} = Y_t \), i.e., equalized per product. This, of course, does not imply that the distribution of sales is also equalized across firms; as some firms will (endogenously) have more products than other firms, the distribution of sales is fully driven by the distribution of products. This tight link between firm-level sales and firms’ product portfolios is not only analytically attractive but also conceptually useful in that it clarifies that our model attributes firm dynamics to a single mechanism
Similarly, the allocation of labor is simply

\[ l_{jft} = \frac{y_{jft}}{q_{jft} \mu(e_{ft})} = \frac{Y_t}{\mu(e_{jft}) \gamma w_t}, \]  

(41)
i.e., the allocation of labor across products depends on firms’ managerial choices. In particular, managerial efficiency is labor-saving. Intuitively, an increase in managerial effort increases profitability as it increases the firms’ sustainable mark-up. To see this, note that equilibrium mark-ups are given by

\[ \zeta_{jft} = \frac{p_{jft}}{MC_{jft}} = \frac{\gamma w_t}{q_{jft} \mu(e_{ft})} = \gamma \mu(e_{jft}), \]  

(42)
i.e., well-managed firms can keep their competitors at bay, sustain high prices and hence move up on their product demand curve. The resulting profit (before paying the managers) for producer \( f \) of variety \( j \) is then simply

\[ \tilde{\pi}_{jft} = \left[ \frac{\gamma w_t}{q_{jft}} - \frac{w_t}{q_{jft} \mu(e_{ft})} \right] q_{jft} Y_t = \frac{\gamma \mu(e_{jft}) - 1}{\gamma \mu(e_{jft})} Y_t, \]  

(43)
i.e., profits depend only on how well the respective firm can incentivize their managers. In particular, \( \tilde{\pi}_{jft} \) is increasing in \( e_{jft} \): better managerial practices increase mark-ups and hence profits per product. Equation (43) contains the main intuition about the interaction between contractual frictions and innovation incentives: As contractual frictions will be detrimental to the provision of managerial effort, firms will be unable to sustain high mark-ups as they grow. The marginal product will therefore be less profitable than the average product and incentives to break into new products will be low.

For analytical convenience we assume that

\[ \mu(e) = \frac{1}{\gamma} \times \frac{1}{1 - e^\sigma}, \]  

(44)
and we will make sufficient parametric conditions that \( e < 1 \). (44) leads to a markup of (see (42))

\[ \zeta_{jft} = \frac{1}{1 - e^\sigma}; \]  

(45)
Hence, if firms do not manage to elicit any managerial effort, their mark-up is zero as they effectively produce with the same technology as the competitive fringe. Using (44) and (43) implies

\[ \tilde{\pi}_{jft} = (e_{jft})^\sigma Y_t \]  

(46)
as required.
Substituting (40) into (2) we get that equilibrium wages are given by

\[ w_t = \frac{1}{\gamma} Q_t, \tag{47} \]

where \( Q_t \) is the Cobb-Douglas composite of individual efficiencies

\[ \ln Q_t \equiv \int_0^1 \ln q_{jt} \, dj. \]

Using (41), we get that

\[ l_{jft} = \frac{1}{Q(t)\mu(e_{jft})} Y_t, \]

so that total output is given by

\[ Y_t = Q_t M_t L_t^P, \tag{48} \]

where

\[ M_t = \left[ \int_0^1 \mu(e_{jft})^{-1} \, dj \right]^{-1} = \frac{1}{\gamma} \left[ \int_0^1 \left[ 1 - (e_{jft})^\sigma \right] \, dj \right]^{-1}. \tag{49} \]

Here, \( M_t \) is the endogenous TFP term based on managerial effort. In particular, increases in x-efficiency, i.e., managerial effort, will increase aggregate TFP.\(^{13}\)

### 5.2 Proof of Proposition 3

A stationary equilibrium in this economy is defined in the usual way.

**Definition 1** A stationary equilibrium consists of firms’ demand schedules for managers and production workers \([m_j, l_j]\), firms’ innovation rates \([x_j]\), measures of low and high type firms \((F^L, F^H)\), a distribution of high-type firms across products \(\nu^H_n\) and a delegation cutoff \(n^*\), such that

1. \([l_j, m_j, x_j]\) are consistent with firms’ profit maximization problem,
2. \((F^L, F^H)\) and \([\nu^H_n]_n\) are consistent with firms’ optimal innovation rates \([x_j]\) and the law of motion (23),
3. \(n^*\) is consistent with labor market clearing, i.e.

\[ 1 = \sum_{n=1}^\infty l(n)nF^H\nu^H_n + l(1)F^L + \sum_{n\geq n^*} m(n)nF^H\nu^H_n, \]

where \(l(n)n = l_{j(n)}\) if firm \(j\) has \(n\) products in its portfolio.

We now prove the existence of a stationary equilibrium in our economy. We proceed in two steps. First we will argue that there is a unique stationary distribution for a given \(n^*\). Then we

\(^{13}\)In this paper, we will mostly be concerned with the process of firm-dynamics and less with the stationary properties of managerial effort determining the distribution of mark-ups and hence \(M_t\). See Peters (2013) for a related model that focuses explicitly on the heterogeneity of mark-ups as source of misallocation across firms.
will show that there is a unique $n^*$ consistent with labor market clearing, taking the dependence of the stationary distribution on $n^*$ into account.

**Step 1: The Stationary Distribution given $n^*$** The stationary distribution is fully determined by $[x_n]$ and the parameters $(\alpha, z)$. The innovation intensities $[x_n]$ are given by (see (28))

$$x_n = A \times \max \left\{ n^{-\lambda}, (n^*)^{-\lambda} \right\}, \quad (50)$$

where $A$ and $\lambda$ are constants given in the Proposition. Hence, given $n^*$, $[x_n]$ are known. We will now construct the stationary distribution.

The stationary distribution is described by the following equations, which are provided in the main text (see (23) and (22)).

$$\tau = \sum_{n=1}^{\infty} nx_n \nu^H_n F^H + z \quad (51)$$

$$F^L\tau = (1 - \alpha) z \quad (52)$$

$$F^H\nu^H_1 \tau = \alpha z \quad (53)$$

$$\nu^H_n (\tau + x_n) = \nu^H_{n-1} (n - 1) x_{n-1} + \nu^H_{n+1} (n + 1) \tau. \quad (54)$$

Additionally, we have that $\nu^H_n$ is a proper distribution

$$\sum_{n=1}^{\infty} \nu^H_n = 1. \quad (55)$$

We need to find $F^H, F^L, [\nu^H_n]_{n=1}^{\infty}$. Let $\nu^H_1$ and $\tau$ be given. From (52) we get $F^L$ and from (53) we get $F^H$. From (54) and (55) we get $[\nu_n]_{2}^{\infty}$. Then we can use (51) to find $\tau$ and $\nu^H_1$. We now solve explicitly for these objects.

**Lemma 1** The distribution of high types takes the form

$$\nu_n = \frac{\prod_{j=1}^{n} x_j}{\tau^n} \frac{\tau}{x_n} \nu_1. \quad (56)$$

**Proof.** Substituting (56) in (54) yields

$$\frac{\prod_{j=1}^{n} x_j}{\tau^n} \nu^H_n (\tau + x_n) = \frac{\prod_{j=1}^{n-1} x_j}{\tau^{n-1}} \tau x_n \nu_1 + \frac{\prod_{j=1}^{n+1} x_j}{\tau^{n+1}} x_{n+1} \nu_1 \tau$$

$$\frac{\prod_{j=1}^{n-1} x_j}{\tau^{n-1}} (\tau + x_n) = \frac{\prod_{j=1}^{n-1} x_j}{\tau^{n-1}} \tau + \frac{\prod_{j=1}^{n} x_j}{\tau^{n-1}} \tau$$

$$\frac{\prod_{j=1}^{n-1} x_j}{\tau^{n-1}} (\tau + x_n) = \frac{\prod_{j=1}^{n-1} x_j}{\tau^{n-1}} (\tau + x_n).$$

44
Hence, if $\nu_n$ satisfies (56), it satisfies all the restrictions for (54). □

Hence, we can use (55) to get

$$1 = \sum_{n=1}^{\infty} \nu_n^H = \sum_{n=1}^{\infty} \prod_{j=1}^{n} \frac{x_j \tau}{x_n \nu_1}$$

$$= \nu_1 \sum_{n=1}^{\infty} \frac{1}{n} \prod_{j=1}^{n-1} \left( \frac{x_j}{\tau} \right)$$

$$= \nu_1 \sum_{n=1}^{\infty} \frac{1}{n x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right)$$

so that

$$\nu_1 = \left( \sum_{n=1}^{\infty} \frac{1}{n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \right)^{-1}. \quad (57)$$

Hence,

$$\nu_n = \frac{1}{n} \prod_{j=1}^{n} \frac{x_j \tau}{x_n \sum_{n=1}^{\infty} \frac{1}{n x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right)} \cdot (58)$$

Using (57) we then get from (53) that

$$F^H = \frac{\alpha z}{\tau} \frac{1}{\nu_1^H} = \frac{\alpha z}{\tau} \times \left[ \sum_{n=1}^{\infty} \frac{1}{n x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \right]$$

and (52) directly implies that

$$F^L = \frac{(1 - \alpha) z}{\tau}.$$

Hence, we only need to determine $\tau$, which we get from (51) as

$$\tau = \sum_{n=1}^{\infty} n x_n \nu_n^H F^H + z = \sum_{n=1}^{\infty} x_n \left( \frac{\prod_{j=1}^{n} x_j \tau}{\tau^n x_n} \right) \frac{\alpha z}{\tau} \frac{1}{\nu_1} + z$$

$$= \left[ \sum_{n=1}^{\infty} \alpha \left( \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \right) + 1 \right] z. \quad (59)$$

The RHS is increasing in $\tau$, the LHS is decreasing in $\tau$. Hence, there is a unique $\tau$, which is consistent with (59). Note also that this solution is consistent with the fact that the total product
space is of measure one. To see this note that

\[ 1 = \sum_{n=1}^{\infty} n F^H_n \nu_n^H + F^L = \sum_{n=1}^{\infty} \frac{\alpha z}{\tau} \nu_n^H + \frac{(1 - \alpha) z}{\tau} \]

\[ = \sum_{n=1}^{\infty} \frac{\alpha z}{\tau} \left( \frac{\prod_{j=1}^{n} x_j \tau}{\tau^n x_n} \right) + \frac{(1 - \alpha) z}{\tau} \]

Hence,

\[ \tau = z \left[ \alpha \sum_{n=1}^{\infty} \frac{\tau}{x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) + (1 - \alpha) \right] . \]

Using (59) we get that

\[ \alpha \sum_{n=1}^{\infty} \frac{\tau}{x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) + (1 - \alpha) = \alpha \sum_{n=1}^{\infty} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) + 1 \]

\[ \sum_{n=1}^{\infty} \frac{\tau}{x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) - 1 = \sum_{n=1}^{\infty} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \]

\[ 1 = \sum_{n=1}^{\infty} \frac{\tau}{x_n} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) - \sum_{n=1}^{\infty} \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \]

This proves the first part of Proposition 3, in particular (29) - (32)

**Step 2: Uniqueness of \( n^* \)** Now we will argue that there is a unique \( n^* \), which is consistent with labor market clearing, i.e. (33). Using (48) and (49) we get that

\[ L^P_t = \frac{Y_t}{Q_t M_t} = \frac{Y_t}{\gamma w_t M_t} = \frac{1}{\gamma w_t M_t} . \]
Labor market clearing requires that

\[ 1 = L^D + M^D = \frac{1}{\gamma \omega_t M_t} + M_D. \]  \hfill (61)

First note that

\[ M_D (n^*) = \sum_{n=1}^{\infty} m_n (n^*) \varphi_n (n^*) = \sum_{n \geq n^*} T (\frac{1}{n^*} - \frac{1}{n}) \varphi_n (n^*), \]  \hfill (62)

where \( \varphi_n \) is defined in (34). Now consider \( L_P^t \). From the definition of \( n^* \) (see (12)) we get that

\[ \omega = (n^*)^{1-\sigma} \frac{\xi \sigma}{T^{1-\sigma}}, \]

so that

\[ L^D (n^*) = \frac{1}{\gamma \omega_t M_t} = \frac{T^{1-\sigma}}{\gamma \sigma \xi} (n^*)^{-(1-\sigma)} \times \frac{1}{M_t}, \]  \hfill (63)

where

\[ M_t (n^*) = \frac{1}{\gamma} \left[ \sum_{n=1}^{\infty} (1 - e_n (n^*)^\sigma) \varphi_n (n^*) \right]^{-1}. \]  \hfill (64)

Hence, (61), (62), (63) and (64) imply that

\[ 1 = \frac{T^{1-\sigma}}{\sigma \xi} (n^*)^{-(1-\sigma)} \sum_{n=1}^{\infty} (1 - e_n (n^*)^\sigma) \varphi_n (n^*) + \sum_{n \geq n^*} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n (n^*). \]  \hfill (65)

This is one equation in one unknown \( n^* \). We first show that there exists at least one \( n^* \), which is consistent with (65).

**Lemma 2** There is at least one \( n^* \), which is consistent with (65).

**Proof.** Consider (65) and suppose that \( n^* \to \infty \). Then

\[ \lim_{n^* \to \infty} \left[ \sum_{n \geq n^*} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n (n^*) \right] \to 0. \]

Note also that

\[ (n^*)^{-(1-\sigma)} \left[ \sum_{n=1}^{\infty} (1 - e_n (n^*)^\sigma) \varphi_n (n^*) \right] \]

\[ < (n^*)^{-(1-\sigma)} \left[ \sum_{n=1}^{\infty} \varphi_n (n^*) \right] \]

\[ = (n^*)^{-(1-\sigma)}. \]

47
Hence, the LHS of (65) goes to 0 as $n^* \to \infty$. Now consider $n^* \to 0$. Then

$$\lim_{n^* \to 0} \left[ \sum_{n \geq n^*} \frac{T}{\xi} \left( \frac{1}{n} - \frac{1}{n^*} \right) \phi_n (n^*) \right] = \lim_{n^* \to 0} \left[ \frac{T}{\xi} \left( \frac{1}{n^*} \right) \sum_{n=1}^{\infty} \phi_n (n^*) \right] = \infty.$$ 

Similarly,

$$\lim_{n^* \to 0} \left[ (n^*)^{-(1-\sigma)} \left( \sum_{n=1}^{\infty} (1 - e_n (n^*)^{\sigma}) \phi_n (n^*) \right) \right] = \infty$$

as $e_n (n^*) < 1$. As the LHS of (65) is continuous, there is at least one $n^*$ consistent with (65). Now consider the uniqueness of $n^*$. The function $m_n (n^*)$ is weakly decreasing in $n^*$ and weakly increasing in $n$. An increase in $n^*$ will shift the distribution $\phi_n$ down (see Section 5.3 below for the proof), in the sense that $\tilde{n}^* > n^* \to \Phi_n (\tilde{n}^*) > \Phi_n (n^*)$,

where $\Phi_n (n^*)$ is defined in (35). As $m_n (n^*)$ is weakly increasing in $n$, an increase in $n^*$ will shift more mass on firms with less products. These firms do not hire managers. Hence, $M_D$ is decreasing in $n^*$, i.e.

$$\frac{\partial M_D (n^*)}{\partial n^*} < 0.$$ 

Now consider $L^D (n^*)$. The first term $\left( \frac{T^{1-\sigma}}{\xi \sigma \gamma} (n^*)^{-(1-\sigma)} \right)$ is decreasing in $n^*$ as an increase in $n^*$ will increase $\frac{w}{Y}$ and hence reduce labor demand. The effect on the second effect is more subtle. Note that

$$M_t (n^*)^{-1} = \gamma \left[ \sum_{n=1}^{\infty} (1 - e_n (n^*)^{\sigma}) \phi_n (n^*) \right]$$

$$= \gamma \left[ \sum_{n<n^*} \left( 1 - \left( \frac{T}{n} \right)^{\sigma} \right) \phi_n (n^*) + \sum_{n \geq n^*} \left( 1 - \left( \frac{T}{n^*} \right)^{\sigma} \right) \phi_n (n^*) \right]$$

$$= \gamma - \gamma T^{\sigma} \left[ \sum_{n<n^*} \left( \frac{1}{n} \right)^{\sigma} \phi_n (n^*) + \left( \frac{1}{n^*} \right)^{\sigma} \sum_{n \geq n^*} \phi_n (n^*) \right].$$

(66)

Holding $\phi_n$ fixed, the term in brackets is decreasing in $n^*$. However, an increase in $n^*$ will shift the distribution $\phi_n$ towards small firms, which will enable firms to increase their effort and hence markups. Hence the effect via $\phi_n$ will make the term in brackets increasing in $n^*$. Hence, theoretically it seems possible for (61) to have multiple solutions for $n^*$. Quantitatively, we could not find a parametrization where $L^D (n^*)$ was not decreasing in $n^*$.

### 5.3 Proof of Proposition 4

We are going to prove the different parts of the Proposition in turn.
1. Obvious from the definition of $x_n$ in (28)

2. $\tau$ is uniquely defined by (see (59))

$$
\tau = \left[ \sum_{n=1}^{\infty} \alpha \left( \prod_{j=1}^{n} \left( \frac{x_j}{\tau} \right) \right) + 1 \right] z.
$$

(67)

As $[x_n]_{n \geq n^*}$ is strictly decreasing in $n^*$ and $[x_n]_{n < n^*}$ is not a function of $n^*$, (67) directly implies that $\tau$ is decreasing in $n^*$

3. Follows directly from the fact that $\tau$ is decreasing in $n^*$ and from $F_L = \frac{(1-\alpha)z}{\tau}$

4. Follows directly from $\chi^H (n^*) = 1 - F_L (n^*)$ (see 36)

5. From (34) and we get that

$$
\Phi_n (n^*) = \sum_{j=1}^{n} \varphi_j (n^*) = F_L + \sum_{j=1}^{n} F^H \nu_j^H j.
$$

Using Proposition 3 we get

$$
\Phi_n (n^*) = \frac{(1-\alpha)z}{\tau} + \sum_{j=1}^{n} \frac{\alpha z}{\tau} \frac{\tau}{x_j} \prod_{r=1}^{j} \left( \frac{x_r}{\tau} \right)
\]

$$
= \frac{z}{\tau} \left[ 1 - \alpha + \alpha \left( 1 + \sum_{j=1}^{n-1} \prod_{r=1}^{j} \left( \frac{x_r}{\tau} \right) \right) \right]
\]

$$
= \frac{z}{\tau} \left[ 1 + \alpha \sum_{j=1}^{n-1} \prod_{r=1}^{j} \left( \frac{x_r}{\tau} \right) \right].
$$

Now consider $n_1^* < n_2^*$. By definition we have

$$
\lim_{n \to \infty} \Phi_n (n_1^*) = \lim_{n \to \infty} \Phi_n (n_2^*) = 1
$$

and

$$
\Phi_0 (n_1^*) = \Phi_0 (n_2^*) = 0.
$$

49
Then consider \( n < n_1^* \). It directly follows that

\[
\Phi_n(n_1^*) = \frac{z}{\tau_1} \left[ 1 + \alpha \sum_{j=1}^{n-1} \prod_{r=1}^{j} \left( \frac{x_r(n_1^*)}{\tau_1} \right) \right]
\]

\[
< \frac{z}{\tau_2} \left[ 1 + \alpha \sum_{j=1}^{n-1} \prod_{r=1}^{j} \left( \frac{x_r(n_1^*)}{\tau_2} \right) \right] = \Phi_n(n_2^*), \quad (68)
\]

because \( \tau_1 > \tau_2 \) and \( x_r(n_2^*) = x_r(n_1^*) = An^{-\lambda} \) for \( r < n_1^* \). Define the schedule

\[
q_n(n^*) \equiv \frac{x_n(n^*)}{\tau(n^*)}.
\]

This schedule has the following properties: there is \( n_1^* < \bar{n} < n_2^* \) such that

\[
q_n(n_2^*) > q_n(n_1^*) \text{ if } n < \bar{n}
\]

\[
q_n(n_2^*) < q_n(n_1^*) \text{ if } n < \bar{n}.
\]

(69)

Above we showed that \( x_r(n_2^*) = x_r(n_1^*) = An^{-\lambda} \) for \( r < n_1^* \), so that \( q_n(n_2^*) > q_n(n_1^*) \) for \( n \leq n_1^* \). Also: suppose we had \( q_n(n_2^*) > q_n(n_1^*) \) for \( n = n_2^* \). Then we had \( q_n(n_2^*) > q_n(n_1^*) \) for all \( n \), which is a contradiction as (67) would then imply that \( \tau_2 > \tau_1 \). Now note that we can always write for \( n > n_1^* \)

\[
\Phi_n(n_1^*) = \frac{z}{\tau_1} \left[ 1 + \alpha \sum_{j=1}^{n-1} \prod_{r=1}^{j} (q_n(n_1^*)) \right] = \frac{z}{\tau_1} \left[ 1 + \alpha \sum_{j=1}^{n_1^*-1} \prod_{r=1}^{j} (q_n(n_1^*)) + \alpha \sum_{j=n_1^*}^{n-1} \prod_{r=1}^{j} (q_n(n_1^*)) \right]
\]

\[
= \Phi_n(n_1^*) + \frac{z}{\tau_1} \alpha \sum_{j=n_1^*}^{n-1} \prod_{r=1}^{j} (q_n(n_1^*)),
\]

and for \( \Phi_n(n_2^*) \) accordingly. (69) and (68) then directly imply that

\[
\Phi_n(n_2^*) > \Phi_n(n_1^*) \text{ for all } n.
\]

Suppose there was \( n_1^* < \hat{n} < \infty \) with \( \Phi_{\hat{n}}(n_2^*) = \Phi_{\hat{n}}(n_1^*) \). Then

\[
\Phi_{\hat{n}}(n_1^*) = \Phi_{\hat{n}}(n_1^*) + \frac{z}{\tau_1} \alpha \sum_{j=n_1^*}^{\hat{n}-1} \prod_{r=1}^{j} (q_n(n_1^*)) = \Phi_{\hat{n}}(n_2^*) + \frac{z}{\tau_2} \alpha \sum_{j=n_1^*}^{\hat{n}-1} \prod_{r=1}^{j} (q_n(n_2^*)},
\]

50
so that

\[
\frac{z}{\tau_1} \sum_{j=n_1^*}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_1^*)) - \frac{z}{\tau_2} \sum_{j=n_1^*}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_2^*)) = \Phi_n(n_2^*) - \Phi_n(n_1^*) > 0.
\]

Hence,

\[
\frac{1}{\tau_1} \sum_{j=n_1^*}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_1^*)) > \frac{1}{\tau_2} \sum_{j=n_1^*}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_2^*))
\]

(70)

But then consider any \( \hat{n} = \hat{n} + (\hat{n} - n_1^*) \). Then

\[
\Phi_{\hat{n}}(n_1^*) = \Phi_{\hat{n}}(n_1^*) + \frac{z}{\tau_1} \sum_{j=\hat{n}}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_1^*))
\]

\[
\Phi_{\hat{n}}(n_2^*) = \Phi_{\hat{n}}(n_2^*) + \frac{z}{\tau_2} \sum_{j=\hat{n}}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_2^*))
\]

so that

\[
\Phi_{\hat{n}}(n_1^*) - \Phi_{\hat{n}}(n_2^*) = z\alpha \left[ \frac{1}{\tau_1} \sum_{j=\hat{n}}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_1^*)) - \frac{1}{\tau_2} \sum_{j=\hat{n}}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_2^*)) \right].
\]

(71)

As \( q_n(n_2^*) \) is declining relative to \( q_n(n_1^*) \) (see (69)), (70) and (71) imply that

\[
\frac{1}{\tau_1} \sum_{j=\hat{n}}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_1^*)) - \frac{1}{\tau_2} \sum_{j=\hat{n}}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_2^*)) = \frac{1}{\tau_1} \sum_{j=\hat{n}}^{\hat{n} + (\hat{n} - n_1^*) - 1} \prod_{r=1}^{j} (q_n(n_1^*)) - \frac{1}{\tau_2} \sum_{j=\hat{n}}^{\hat{n} + (\hat{n} - n_1^*) - 1} \prod_{r=1}^{j} (q_n(n_2^*))
\]

\[
> \frac{1}{\tau_1} \sum_{j=n_1^*}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_1^*)) - \frac{1}{\tau_2} \sum_{j=n_1^*}^{\hat{n} - 1} \prod_{r=1}^{j} (q_n(n_2^*))
\]

> 0.

Hence, \( \Phi_{\hat{n}}(n_1^*) > \Phi_{\hat{n}}(n_2^*) \), which implies that

\[
\lim_{n\to\infty} \Phi_n(n_1^*) > \lim_{n\to\infty} \Phi_n(n_2^*).
\]

Hence, \( \Phi_n(n_2^*) > \Phi_n(n_1^*) \) for all \( n \), which proves the proposition.

6. The fact that average firm size is increasing in \( n^* \) follows directly from the fact that \( \Phi_n(n^*) \) is increasing in \( n^* \) as the product space has size one.
5.4 Using \( Q_t \) as a scale for the innovation cost function

5.5 Identification of the Model

We will now discuss the identification of our model. The analytical results for the stationary firm-size distribution makes our identification approach transparent. In total there are 7 parameters to identify

\[(T, \theta, \sigma, \xi, \alpha, \gamma, z).\]

In Proposition 3 we showed that the process of firm-dynamics and the stationary distribution is fully determined by the share of high-types \( \alpha \), the entry rate \( z \) and the expansion schedule \([x_n]_n\), which itself was given by

\[x_n = A \times \max\left\{ n^{-\sigma} \frac{\zeta}{1-\zeta}, (n^*)^{-\sigma} \frac{\zeta}{1-\zeta} \right\}. \tag{72}\]

As we set \( \zeta \) exogenously, \([x_n]_n\) is fully parametrized by three objects \((A, n^*, \sigma)\). Note that \( n^* \) is endogenous (but constant in a stationary equilibrium) and that \( A \) depends on \( T \) and \( \theta \). For now, suppose \( \sigma \) is known. From the firm-level data we can hence at most identify the four parameters

\((n^*, \alpha, z, A)\).

We do so using the following four pieces of information:

1. The entry rate

\[M_5 = \frac{z}{FL + FH} \rightarrow z\]

2. The relative size of firms age 20-25 relative to young firms

\[M_6 = \frac{\bar{n} (\bar{a} = 21 - 25)}{\bar{n} (\bar{a} = 1 - 5)} \rightarrow A\]

3. The aggregate employment share of firms without managerial personnel

\[M_3 = \frac{\sum_{n=1}^{n^* - 1} l_n n \varphi_n (n^*)}{\sum_{n=1}^{\infty} l_n n \varphi_n (n^*)} \rightarrow n^*\]

4. The relative exit rate of young versus old firms conditional on size, i.e.

\[M_7 = \frac{\text{exit}(a = 16 - 20|n = 1)}{\text{exit}(a = 1 - 5|n = 1)} \rightarrow \alpha\]

These four moment will identify the parameters uniquely. We then use two moments related to managerial occupations, namely the managerial employment share and the compensation of managers relative to corporate profits to identify \( \sigma \) and \( \frac{T}{\xi} \). Consider first \( \sigma \). The total compensation
for managerial personnel relative to aggregate profits is

\[
\frac{wM_D}{\Pi} = \frac{\sum_{n=1}^{\infty} \omega m_n \varphi_n (n^*)}{\sum_{n=1}^{\infty} \pi_n \varphi_n (n^*)} = \frac{\sum_{n=1}^{\infty} \omega m_n \varphi_n (n^*)}{\sum_{n=1}^{\infty} \pi_n \varphi_n (n^*)}.
\]

Using that

\[
m_n = \frac{T}{\xi} \times \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}
\]

and

\[
\tilde{\pi}_n = \left( T \max \left\{0, \frac{1}{n^*}\right\}\right)^{\sigma},
\]

we get that

\[
\frac{wM_D}{\Pi} = \frac{\sum_{n=1}^{\infty} \omega \left( \frac{T}{\xi} \times \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right) \varphi_n (n^*)}{\sum_{n=1}^{\infty} T^{\sigma} \left( \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right)^{\sigma} \varphi_n (n^*)}
\]

\[
= \frac{\sum_{n=1}^{\infty} \left( \frac{n^*}{T} \right)^{1-\sigma} \sigma \xi \left( \frac{T}{\xi} \times \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right) \varphi_n (n^*)}{\sum_{n=1}^{\infty} \left( \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right)^{\sigma} \varphi_n (n^*)}
\]

\[
= \sigma \frac{\sum_{n=1}^{\infty} \left( n^* \right)^{1-\sigma} \left( \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right) \varphi_n (n^*)}{\sum_{n=1}^{\infty} \left( \max \left\{0, \frac{1}{n^*} - \frac{1}{n}\right\}\right)^{\sigma} \varphi_n (n^*)}.
\]

(73)

Hence, conditional on \(n^*\) and the firm-size distribution, which fully determines \(\varphi_n (n^*)\), (73) only depends on \(\sigma\). Now the firm-size distribution does depend on \(\sigma\) and hence we need to iterate on (73), taking the dependence of \(\varphi_n\) on \(\sigma\) into account. Note that neither \(T\) nor \(\xi\) enter in (73).

Hence, we can first calibrate \(\sigma\) and then simply set \(T\) and \(\xi\).

To do so, we consider the labor market clearing condition. Managerial demand \(M_D (n^*)\) is given by

\[
M_D (n^*) = \sum_{n \geq n^*} \frac{T}{\xi} \left( \frac{1}{n^*} - \frac{1}{n}\right) \varphi_n (n^*).\]

(74)

With the total size of the labor force being normalized to unity and \(n^*\) and \(\varphi_n (n^*)\) being calibrated, we can use (74) to solve for \(\frac{T}{\xi}\) as

\[
\frac{T}{\xi} = MES \times \left[ \sum_{n \geq n^*} \left( \frac{1}{n^*} - \frac{1}{n}\right) \varphi_n (n^*) \right]^{-1},
\]

(75)

where \(MES\) is the managerial employment share in the data. Then we can use the calibrated \(n^*\) and \(A\) to find the deep primitive parameters \(\theta\) and \(\xi\). In particular

\[
n^* = T \left( \frac{\omega}{\sigma \xi} \right)^{\frac{1}{1-\sigma}} = \frac{T}{\xi} \left( \frac{\omega}{\sigma} \right)^{\frac{1}{1-\sigma}}
\]

(76)

\[
A = (1 - \sigma) \theta^{\frac{1}{1-\sigma}} \xi^{\frac{1}{1-\sigma}} T^{\frac{\xi}{1-\sigma}}.
\]

(77)
Note that

\[
\omega = \frac{w}{Y} = \frac{\gamma^{-1}Q_t}{M_tL_t^P} = \frac{\sum_{n=1}^{\infty} (1 - \rho_n) \varphi_n(n^*)}{L_t^P} = 1 - \frac{T^\sigma \sum_{n=1}^{\infty} (\max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\})^\sigma \varphi_n(n^*)}{(1 - M_t^P)} = 1 - \left( \frac{T}{\xi} \right)^\sigma \xi^\sigma \sum_{n=1}^{\infty} (\max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\})^\sigma \varphi_n(n^*) \frac{(1 - \rho_n \psi_n(n^*))}{(1 - MES)}.
\]

(78)

As \( \sigma \) is already calibrated and \( \frac{T}{\xi} \) is known from above, (76) and (78) imply that

\[
\left( n^* \frac{1}{T/\xi} \right)^{1-\sigma} = \frac{1}{\sigma} \xi^{-\sigma} \omega = \frac{1}{\sigma} \xi^{-\sigma} \frac{1 - \left( \frac{T}{\xi} \right)^\sigma \xi^\sigma \sum_{n=1}^{\infty} (\max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\})^\sigma \varphi_n(n^*)}{(1 - MES)} = \frac{1}{\sigma} \xi^{-\sigma} - \left( \frac{T}{\xi} \right)^\sigma \sum_{n=1}^{\infty} (\max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\})^\sigma \varphi_n(n^*) \frac{(1 - \rho_n \psi_n(n^*))}{(1 - MES)}.
\]

Hence, \( \xi \) is given by

\[
\xi^{-\sigma} = \sigma \left( n^* \frac{1}{T/\xi} \right)^{1-\sigma} (1 - MES) + \left( \frac{T}{\xi} \right)^\sigma \sum_{n=1}^{\infty} (\max \left\{ \frac{1}{n}, \frac{1}{n^*} \right\})^\sigma \varphi_n(n^*). \tag{79}
\]

From (79) and \( \frac{T}{\xi} \) we obviously get \( T \). (77) then implies that

\[
\theta = \left( A \frac{1}{1-\sigma} \right)^{1-\zeta} \left( \frac{1}{\zeta} \right)^\zeta \left( \frac{1}{T} \right)^\zeta. \tag{80}
\]

Finally, we can solve for the step-size \( \gamma \) to fit the aggregate growth rate of TFP (and output)

\[
g_Q = \ln(\gamma) \tau, \tag{81}
\]

where \( \tau \) is also fully determined. This gives us a simple sequential algorithm to calibrate the model, which is summarized in Table 10 below. Note that (78) imposes the labor market-clearing condition \( 1 = L_t^P + M_t^D \), i.e. the calibrated parameters satisfy labor market clearing by construction.\(^{14}\)

5.6 Identification of the quantitative model (\( \beta < 1 \))

For our quantitative exercise, we extend our baseline model slightly by allowing for potentially differential rates at which high- and low types exit. We will show that the model is still identified

\(^{14}\)To see that the size of the labor force \( L \) is indeed normalization, suppose there were \( L \) workers. (74) would then imply that the managerial employment share, \( MES \), was given by

\[
MES = \frac{M_t^D(n^*)}{L} = \frac{T}{\xi L} \times \sum_{n \geq n^*} \left( \frac{1}{n^*} - \frac{1}{n} \right) \varphi_n(n^*).
\]
Parameter | Moment
--- | ---
σ | Managerial compensation
A | Life-cycle
n* | Empl. share of non-managerial firms
z | Entry rate
α | Age-exit relationship

<table>
<thead>
<tr>
<th>Step 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2</td>
<td>see (74)</td>
</tr>
<tr>
<td>Step 3</td>
<td>see (79)</td>
</tr>
<tr>
<td>see (80)</td>
<td>see (80)</td>
</tr>
</tbody>
</table>

**Table 10: Calibration Strategy**

along the same lines as in Section 5.5. Suppose that high types lose their products only with rate \( \beta \) conditional on some other firm innovating in their product line. Let the net rates of innovation, entry and destruction be \( (\hat{\tau}_H, \hat{\tau}_L, \hat{x}_n, \hat{z}) \). These are given by

\[
\begin{align*}
\hat{\tau}_H &= \beta \tau \tag{82} \\
\hat{\tau}_L &= \tau \tag{83} \\
\hat{x}_n &= \Gamma \times x^H_n \tag{84} \\
\hat{z} &= \Gamma \times z \tag{85}
\end{align*}
\]

and

\[
\Gamma = 1 - \chi + \beta \chi, \tag{86}
\]

where

\[
\chi = F^H H \sum_{n=1}^{\infty} \nu^H_n n
\]

is the share of high-type products and

\[
\tau = F^H \sum_{n=1}^{\infty} \nu^H_n x_n n + z. \tag{87}
\]

Hence, (74) would give us \( \frac{T}{\xi L} \). (76) and (78) would be given by

\[
\begin{align*}
n^* &= T \left( \frac{\omega}{\xi L} \right)^{\frac{1}{\sigma}} = \frac{T}{\xi L} \left( \xi L \right)^{\frac{\sigma}{\omega L}} \left( \frac{\omega L}{\sigma} \right)^{\frac{1}{\sigma}} \\
\omega L &= 1 - \left( \frac{T}{\xi L} \right)^{\sigma} \sum_{n=1}^{\infty} \left( \max \left\{ \frac{1}{n_{\pi}}, \frac{1}{n^*} \right\} \right)^{\sigma} \varphi_n (n^*) (1 - MES)
\end{align*}
\]

These equations can be solved for \( \xi L \) and hence \( T \). (80) then still determines \( \theta \).
is the gross rate of creative destruction. We still get the following flow equations:

1. The number of low types has to be constant

\[ F^L \hat{\tau}^L = (1 - \alpha) \hat{z}, \]

2. The number of high types has to be constant

\[ F^H \nu^H \hat{\tau}^H = \alpha \hat{z}, \]

3. The distribution of high types satisfies

\[ F^H \nu^H \left( \hat{\tau}^H + \hat{z}^H_n \right) n = F^H \nu^H_{n-1} (n-1) \hat{x}^H_n + F^H \nu^H_{n+1} (n+1) \hat{\tau}^H \quad \text{for } n \geq 2. \]  \hspace{1cm} (88)

Note first that (88) is mathematically exactly the same expression as above. Hence, Lemma (1) still applies so that

\[ \nu^H_n = \frac{\prod_{j=1}^n \hat{x}^H_j \hat{\tau}^H}{(\hat{\tau}^H)^n \hat{x}^H_n \nu_1}. \]  \hspace{1cm} (89)

We still have that (57) holds true so that

\[ \nu_1 = \left( \sum_{n=1}^{\infty} \frac{1}{n} \hat{\tau}^H \prod_{j=1}^n \left( \frac{\hat{x}^H_j}{\hat{\tau}^H} \right) \right)^{-1}. \]  \hspace{1cm} (90)

This determines the distribution as a function of \((\hat{x}^H_n, \hat{\tau}^H)\). We also still have that

\[ F^L = \frac{(1 - \alpha) \hat{z}}{\hat{\tau}^L}, \]

\[ F^H = \frac{1}{\nu^H_1} \frac{\alpha \hat{z}}{\hat{\tau}^H}. \]

The stationary share of high types is

\[ \chi = \sum_{n=1}^{\infty} \nu^H_n F^H n = \sum_{n=1}^{\infty} \frac{\alpha \hat{z} n \nu^H_n}{\hat{\tau}^H \nu_1} = \frac{\alpha \hat{z}}{\hat{\tau}^H} \sum_{n=1}^{\infty} \left( \frac{\prod_{j=1}^n \hat{x}^H_j \hat{\tau}^H}{(\hat{\tau}^H)^n \hat{x}^H_n} \right) \]

\[ = \frac{\alpha \hat{z}}{\hat{\tau}^H} \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{\prod_{j=1}^n \hat{x}^H_j}{(\hat{\tau}^H)^n} \right) \right] \]

\[ = \frac{\alpha \Gamma z}{\beta \tau} \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{\prod_{j=1}^n (\Gamma x_j)}{(\beta \tau)^n} \right) \right]. \]

The flow rate of success \(\Gamma\) is

\[ \Gamma = 1 - \chi + \chi \beta. \]
Finally, we have the definition of \( \tau \) (see (87)) that

\[
\tau = \sum_{n=1}^{\infty} x_n F^H H n + \sum_{n=1}^{\infty} x_n \alpha \frac{\nu^H}{\nu_1} H n + z = \sum_{n=1}^{\infty} x_n \frac{\alpha G}{\tau H} \left( \prod_{j=1}^{n} \frac{\hat{x}_j^H}{\tau_H} \right) + \left[ \sum_{n=1}^{\infty} x_n \frac{\alpha G}{\tau} \left( \prod_{j=1}^{n} \frac{\hat{x}_j^H}{\tau_H} \right) + 1 \right]
\]

Hence, given the gross rates \((z, x_n)\) and the parameters \((\alpha, \beta)\) we get three equations in the unknowns \((\tau, \Gamma, \chi)\) which are

\[
\tau = z \left[ \alpha \sum_{n=1}^{\infty} \left( \prod_{j=1}^{n} \frac{\Gamma x_j}{(\beta \tau)^n} \right) + 1 \right] \quad (91)
\]

\[
\Gamma = 1 - \chi + \beta \chi \quad (92)
\]

\[
\chi = \frac{\alpha G z}{\beta \tau} \left[ 1 + \sum_{n=1}^{\infty} \left( \prod_{j=1}^{n} \frac{\Gamma x_j}{(\beta \tau)^n} \right) \right] \quad (93)
\]

These determine \((\tau, \Gamma, \chi)\). We can then directly construct the net rates in (82)-(85), which then fully determine \((F^H, F^L, \nu^H)\). Hence, the entire distribution of firms and the number of firms boils down to solving three equations in three parameters (91)-(93), which we get in closed form as a function of “fundaments” \((z, x_n)\).

Note that \(x_n\) is not exactly given by (72). The reason is that innovations are only successful with the (endogenous) probability \(\Gamma < 1\). Hence, \(x_n\) is now given by

\[
x_n = \tilde{A} \times \max \left\{ n^{-\beta \tau} \frac{\xi}{(n^*)^{-\beta \tau} \frac{\xi}{\tau}} \right\} = \Gamma \frac{\xi}{\tau} \times A \times \max \left\{ n^{-\beta \tau} \frac{\xi}{(n^*)^{-\beta \tau} \frac{\xi}{\tau}} \right\}, \quad (94)
\]

where \(\tilde{A}\) is endogenous but constant in a stationary equilibrium. This shows that the firm size distribution and the dynamics are fully determined from the five parameters (again for given \(\sigma\))

\[
(n^*, \tilde{A}, z, \alpha, \beta).
\]

To identify \(\beta\), we add one moment to the four moment used above. In particular, we also consider the aggregate employment share of firms of age 20-25 relative to young firms. This moment is informative about \(\beta\), because conditional on the life-cycle, it is determined by the relative size of old cohorts. As high-type firms are more likely to be old, it is informative about the relative rate of exit, which is exactly \(\beta\).
The identification of the remaining parameters \((T, \xi, \sigma, \theta)\) is exactly the same as before. In particular, (73), (75) and (79) still apply, which identify \((T, \xi, \sigma)\). To identify \(\theta\), we than use (94), to get that
\[
\tilde{A} = \Gamma (1 - \sigma) \theta^{1 - \zeta} \xi^{1 - \zeta} T^{\zeta \sigma},
\]
which implies that
\[
\theta = \left( \frac{\tilde{A}}{(1 - \sigma) \Gamma} \right)^{1 - \zeta} \left( \frac{1}{\zeta} \right)^{\zeta} \left( \frac{1}{T} \right)^{\zeta \sigma}.
\]
As \(\tilde{A}\) is calibrated and \(\Gamma\) is known in equilibrium (see (92)), this equation identifies \(\theta\).
5.7 Data

HERE: DESCRIPTION OF MANAGERIAL CLASSIFICATION AND NIPA DATA Hence, we can calculate the share of managers, their human capital and the share of self-employed for around 70 countries of the world. We try to conform with our measure of managers to our theory. Our main source of information relies on the harmonized occupational titles according to ISCO (International Standard Classification of Occupations). We identify managers as members of the occupation “Legislators, senior officials and managers”. As our theory stresses frictions in delegation between entrepreneurs and outside managers, we then use more detailed occupational data to reclassify all self-employed as non-managers. Additionally, we also drop all government officials. To calculate human capital, we adopt the usual approach to translate observed years of schooling into human capital unit using Mincerian returns.\textsuperscript{15} Hence, we get a measure of human capital at the individual level so that we can calculate human capital stocks both at the country level and by narrowly defined occupational groups (within countries). For our empirical exercise, we are going to use two measures of human capital across countries. As a benchmark, we are going to use the human capital by managerial workers as the empirical counterpart of the theories parameter $\eta$. While this measure is attractive in that it provides us with information about human capital conditional on working as a manager, the interpretation of counterfactual exercises is less straight-forward, as these require us to take a stand on the human capital of the marginal non-manager. We will therefore also show the results of simply using the average human capital stock within countries. We further augment this data with data on the rule of law by the World Bank and match it to the Penn World Tables to exploit additional controls at the country level.

5.8 Further Empirical Evidence

In this section we present further empirical evidence for the main ingredients of our theory. We look at three distinct but complimentary data sources. First of all we look at the country-level cross-sectional relationship between managerial employment shares and different measures for the quality of legal institutions. Secondly, we use the variation across sectors in an exercise akin to RAJAN ZINGALES to provide some evidence that underdeveloped contractual institutions are most harmful for sectors that are intensive in managerial personnel. Finally, we use the micro data from India to look at particular implications of our theory. We want to point out that we do not interpret any of these regressions causally as we do not have access to a convincing source of exogenous variation. In contrast, we interpret these findings as equilibrium relationships, which are consistent with our model and plausibly other theories as well.

Data Across Indian Firms Additionally, we use the information on managerial personnel and family size to provide some direct evidence on friction in managerial hiring. As there is no data on family size for firms in the ASI, we will have to limit our attention to the NSS sample, when

\textsuperscript{15}See Appendix for details.
exploiting this information. To compare the implications of our theory with data from the US, we also complement these data sets using the summary statistics on the US economy from Hsieh and Klenow (2014).

**Data Across Indian States** We augment our cross-country exercise with some cross-state variation across India. As we are particularly interested in the link between the prevailing legal system and firms’ subsequent delegation decisions. Both the Indian microdata on firms and the Indian version of the census contain information about the state, where the respective firm or individual is located in. Additionally, we extract information on the general level of trust between people at the state-level from the World Value Surveys. While this variable is not directly aimed at eliciting the (perceived) quality of the prevailing legal environment, it fits well into our theoretical framework as long as trust reduced the required time, the owner needs to spend to incentivize outside managers. See also Bloom et al. (2012), who also use this variable.

### 5.9 Stylized Facts

In this section, we first summarize the predictions of the theoretical model and then provide the empirical results.

**Prediction 1** *Everything else equal, the probability of hiring an outside manager and, conditional on hiring, the number of outside managers is (i) increasing in firm size n, (ii) decreasing in the owner’s time T, and (iii) increasing in the rule of law κ.*

**Prediction 2** *Average firm size n (i) increases in the owner’s time T, (ii) increases in the rule of law κ, and (iii) the positive relationship between firm size n and the owner’s time T becomes weaker as the rule of law κ improves.*

**Prediction 3** *Firm growth decreases in firm size, more so when the rule of law κ is weaker.*

**Cross-country Correlations** Our model predicts that as the benefit of delegation increases in a country, firms will demand more managers both at the extensive and intensive margins as in Prediction 1 and therefore we should observe a bigger fraction of workforce working as managers. To this end, Table 11 regresses the fraction of managers in a country on the rule of law and the average human capital in the population. Column 1 includes rule of law, Column 2 human capital and Column 3 includes both at the main independent variables. All regressions control for the GDP per capita, to absorb the most basic form of heterogeneity across countries.

In all the regressions, the rule of law measure appears to be highly significant and positive as the theory predicts. This relationship is not simply driven by the general level of economic development of the countries or the fact that human capital, i.e. the average years of schooling, is higher in richer countries. Our theory also predicts that the level of managerial human capital should increase the
demand for managers. As seen in columns 2 and 3, this impact does not come out significantly once we control for a countries’ GDP per capita even though the sign of the variable is as expected.

Empirical support for Prediction 2, especially on the positive link between the rule of law and average firm size, has already been provided by several papers and while we test this prediction using the cross-state variation in legal system within India, we lack the firm-level micro-data in different countries to test this prediction in the international context. Bloom et al. (2012), for instance, document that trust on legal system in the cross-country data correlates very significantly with average firm size. Likewise Kumar et al. (1999) find similar positive link between average firm size and the strength of the legal system among European countries. Laeven and Woodruff (2007) establishes a causal link between the quality of legal institutions and average firm size using Mexican firms. La Porta et al. (1997) show that the sales of the largest 20 publicly traded companies as a share of GDP highly correlates with the level of trust prevailing in a country.

**Cross-state Correlations in India** There are a number of predictions that are easier to study in micro-level establishment data. Especially predictions related to owner’s time can be tested in the Indian establishment level data. The NSS data contains information on the size of the family of the owner of the firm. In the theory, $T$ referred to the owner’s time endowment, which had a comparative advantage *within* the establishment - it could neither be sold nor on the market, nor was there any need to monitor. As long as family members require less monitoring time than outside managers, we can think of family size as inducing variation in the time endowment $T$. Additionally, we use our state level measures of trust, which - through the lens of the model - we interpret as inducing variation in the legal environment $\kappa$. Table 12 reports the correlations regarding Prediction 1 using Indian micro data.

Column 1 regresses the manager hiring decision on size of the firm proxied by log employment,
Table 12: Cross-state Correlations

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log employment</td>
<td>0.037***</td>
<td>0.036***</td>
<td>-0.436***</td>
<td>-0.436***</td>
<td>-0.005***</td>
<td>-0.004***</td>
</tr>
<tr>
<td>Log household size</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.242***</td>
<td>0.242***</td>
<td>-0.353**</td>
<td>-0.353**</td>
</tr>
<tr>
<td>Trust</td>
<td>0.013**</td>
<td>0.295**</td>
<td>-0.196*</td>
<td>-0.196*</td>
<td>0.131***</td>
<td>0.131***</td>
</tr>
<tr>
<td>Log employment × Trust</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.131*</td>
<td>0.131*</td>
</tr>
<tr>
<td>Log household size × Trust</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.353**</td>
<td>-0.353**</td>
</tr>
<tr>
<td>Log assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.131***</td>
<td>0.131***</td>
</tr>
<tr>
<td>State Dummies</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Firm dummy</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Observations</td>
<td>170,404</td>
<td>170,404</td>
<td>20,700</td>
<td>2,284</td>
<td>2,232</td>
<td>152,808</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.09</td>
<td>0.09</td>
<td>0.53</td>
<td>0.74</td>
<td>0.79</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Notes: Data comes from ASI and NSS. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10. All regressions include a constant term which is not reported. Additional controls are as follows. Columns (1) and (3): 4-digit sector dummies, rural/urban dummy, the age of the establishment, and state dummies. Columns (2) and (4): 4-digit sector dummies, rural/urban dummy, the age of the establishment, and state-level GDP per capita. Size of the household and state dummies to absorb state-specific heterogeneities. When we introduce state dummies, we are not able to measure the effect of trust since it varies at the state level. As pedicured by the theory, bigger firms are more likely to employ managers, but firms with larger families abstain from hiring outside managerial personnel (holding firm-size constant). In column 2 we explicitly introduce the state-level trust variable and exclude the state dummies. This leaves the coefficients from Column 1 unchanged but shows that trust is indeed correlated with an increase in managerial demand at the firm-level. In columns 3 to 6, we then turn to Predictions 2 and 3. Both of these predictions refer to firms who actually delegate. Hence, columns 3 to 6 consider only firms, who report to hire outside managers. In columns 3 we consider all firms (both in the ASI and in the NSS) and show that there is a robust positive correlation between firm-size and regional trust. Columns 4 and 5 look more detailed into the mechanism, by again considering the variation in family size (which forces us to focus on firms in the NSS). Bloom et al. (2013) show that the number of male family members is one of the best predictors of firm size in India. Motivated by this observation, Column 4 correlates firm size with household size which is also interacted with regional trust measure. The interesting result here is that the positive correlation between firm size and family size becomes weaker when the regional trust gets stronger. This is consistent with the view that family ties do in fact provide a comparative advantage in regions, where interactions with outside managers are hampered by low quality legal institutions and little trust. One potential concern could be that family wealth might generate a spurious correlation between family size and
firm size. Hence column 5 introduces log assets to proxy for family wealth. Even though the point estimates decrease in absolute value, the main message of column 4 remains the same. Note that both columns 4 and 5 explicitly introduce state fixed effect, so that we cannot identify the main effect of the regional trust but only the interaction with the size of the household. Column 6 finally turns to Prediction 3 and shows that firm growth decreases in firm size but that this negative relationship becomes weaker in high trust regions. To calculate the growth rate, we focus on the panel dimension of the ASI. As we have 8 years of the data, we calculate the annual growth rate and include firm fixed effects in the regression. This again precludes us from identifying the main effect of regional trust.

Overall, the empirical correlations in the data provides some suggestive evidence on the predictions of the theoretical model. In the next section we study the quantitative implications of the theoretical framework.