Human Capital Investment and Affirmative Action: A Structural Policy Analysis of US College Admissions

Brent Hickman
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Hickman (2012a) “Using Auction Theory to Study Human Capital Investment in Frictionless Matching Markets”

In labor markets with endogenous matching, human capital (HC) investment bears two separate returns:

1. First, as a productive input
   - The “productive channel” of investment incentives

2. Second, it determines the quality of one’s potential match partners by shaping the preferences of the other side of the market
   - The “competitive channel” of investment incentives
   - Actions of competitors have bearing on investment decisions too

EXAMPLE: College admissions and Affirmative Action (AA)
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EXAMPLE: College admissions and Affirmative Action (AA)
DEFINITION: preferential treatment of minorities in admissions decisions.

RATIONAL FOR AA: reduce inequality in market allocations/increase diversity on college campuses.

1. In 1996, 17.7% of new college freshmen were under-represented minorities
   - only 11.5% of enrollment in top quality quintile of colleges
   - 29.1% in bottom quintile

Related to this...

2. In 1996,
   median minority SAT = 19th percentile of non-minority SATs

CONTROVERSY: At what cost does AA promote equality?
DEBATE:

OPPONENTS OF AA:

ARGUMENT #1: If HC and college quality are complementary inputs, AA weakens socially desirable assortative matching.

PROONENTS OF AA:

REBUTTAL TO #1: Racial equality may be inherently desirable and worth some loss of market surplus.
DEBATE:

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ARGUMENT #2: Erodes incentives for minorities to invest by lowering the bar below market-based admission standards
  ● Weakening of the competitive channel of incentives

PROPONENTS OF AA:

REBUTTAL TO #1: Racial equality may be inherently desirable and worth some loss of market surplus

REBUTTAL TO #2: AA may increase minority investment by overcoming discouragement effects and providing access to better match partners
  ● Altering both productive and competitive channels of incentives
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Need a model to empirically evaluate relative merits of these arguments.
Research Questions

1. What role does AA play in allocation of seats in the college market as a whole?

2. What effect does AA have on human capital investment?
   - May alter both productive *and* competitive channels of incentives

3. What are implications of AA in American college admissions?
   - Pre-College HC investment?
   - College Placement?
   - Graduation Rates?
   - Welfare?
   - Inequality?

4. Are there alternative forms of AA that could do better than others?
   - **Admission Preference (AP)** (US system)
   - **Color-Blind Admissions (CB)**
   - **Quotas (Q)** (India, Sri Lanka, others)
Related Literature

Theory:

- **Many-to-Many Matching with fixed types:**
  - Gale and Shapley (1962); Azevedo and Leshno (2011)

- **Bilateral Matching:**
  - Coate and Loury (1993)

- **Contests with Complete Information:**
  - 2-Player: Schotter and Weigelt (1992); Fu (2006); Fain (2009)
  - $N$-Player: Franke (2009);
Related Literature

Empirical:

- **Admissions Impact of AA:**
  - Bowen and Bok (1998); Arcidiacono (2005); Howell (2010)
  - Estimate large counterfactual impact of AA ban

- **Mismatching and AA:**
    - AA causes minority dropout rates to increase
  - Long (2008); Rothstein and Yoon (2008); Chambers, Clydesdale, Kidder, and Lempert (2005)
    - Some mismatching, but magnitude small and benefits of AA large

- **PROBLEM W/ABOVE:** treating HC investment (SAT scores) as fixed
  - Ferman and Assunção (2011); Cotton, Hickman, and Price (2012)
  - Evidence that AA can alter investment behavior
Contributions

To develop a model that...

1. Tractably incorporates both productive and competitive channels of HC incentives into a many-to-many matching market
   - Auction theory!
   - Matching rights determined by centralized allocation mechanism that replicates the decentralized PAM of a frictionless matching market
   - Parsimonious representation of investment incentives
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2. Empirically tractable to study college market as a whole with existing data
   - Does not require matched student-school data
   - An advantage of empirical auctions tools
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2. Empirically tractable to study college market as a whole with existing data
   - Does not require matched student-school data
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3. Can be used to perform counterfactual analysis of
   - Aggregate HC investment patterns,
   - Matching equilibria (college enrollment patterns), and
   - Welfare implications
1. Ranking between Estimated US Admission Preference and Color-Blind system is ambiguous
   - AP results in lower inequality of investment, college placement, graduation rates, and income.
   - BUT, results in lower welfare, relative to CB

2. Quota is possibly superior to both:
   - Lower inequality than AP
   - Raises welfare, relative to AP in Kaldor-Hicks sense (more efficient)
   - ⇒ Any social preferences over total welfare and inequality that justify use of AP over CB must imply that Q is better!

**ALTERNATIVE PROPOSAL:** an Admission Preference that mimics outcomes under a Quota rule
Outline

- Theory Model
- (Semiparametric) Identification/Estimation
- Estimation Results
- Counterfactual Policy Experiments
AGENTS: $K$ heterogeneous students with privately-known cost types: 
$\theta \in [\underline{\theta}, \bar{\theta}]$
- Marginal cost of HC production
- Can reflect many factors: affluence, access to health-care, K-12 school quality, etc.

DEMOGRAPHICS: students belong to $\mathcal{M}$ or $\mathcal{N}$
- $|\mathcal{M}| + |\mathcal{N}| = M + N = K$
- For group $j$, $\theta \sim F_j(\theta)$, $j = \mathcal{M}, \mathcal{N}$

STRATEGIES: HC investment levels $s \in \mathcal{S} \subseteq \mathbb{R}_+$

COST OF INVESTMENT: incur a cost $C(s; \theta)$ to achieve $s$
- $\frac{\partial C}{\partial s} > 0; \quad \frac{\partial C}{\partial \theta} > 0$
- $\frac{\partial^2 C}{\partial s^2} \geq 0; \quad$ and
- $\frac{\partial^2 C}{\partial s \partial \theta} \geq 0$. 
**College Admissions as a Bayesian Game**

- **MATCH PARTNERS:** $K$ heterogeneous colleges, $\mathcal{P}_K = \{p_1, p_2, \ldots, p_K\}$
  - $p_k \in [\underline{p}, \bar{p}]$
  - $p_k$ represents common utility ranking of $k^{th}$ seat

- **MATCH UTILITY:** $U(p, s)$
  - $\frac{\partial U}{\partial p} > 0$; $\frac{\partial U}{\partial s} \geq 0$;
  - $\frac{\partial^2 U}{\partial s^2} \leq 0$; and
  - $\frac{\partial^2 U}{\partial p \partial s} \geq 0$.

- **TOTAL PAYOFF** to agent $i$ in group $j = \mathcal{M}$, $\mathcal{N}$ is
  \[
  \Pi(s_i, s_{-i}; \theta) = U[P_j(s_i, s_{-i}), s_i] - C(s_i; \theta_i),
  \]
  where $P_j(s_i, s_{-i})$ is a group-specific allocation mechanism.
Allocation Mechanisms:

- **Color-Blind**: simple PAM of $s$ with $\mathcal{P}_K$

- **Quota**: reserve a representative set of prizes for each group
  \[ \mathcal{P}_j = \{p_{j1}, p_{j2}, \ldots, p_{jK_j}\}, \quad j = M, N, \] and PAM w/in each group
    - Split game into two separate competitions

- **Admission Preference**: Markup for minority investment $\tilde{S}: S \rightarrow \mathbb{R}_+$
  - PAM by transformed investment levels
    \[ \tilde{s} = \{s_{N1}, \ldots, s_{NK_N}, \tilde{S}(s_{M1}), \ldots, \tilde{S}(s_{MK_M})\} \]
    \[ \tilde{S}(s) \geq s, \quad \tilde{S}'(s) > 0 \text{ (very general)} \]

Information:

- Agents know $\mathcal{P}_K$, $M$, $N$, $F_M$, $F_N$, and allocation rule $\mathcal{R} \in \{CB, Q, AP\}$ before making choices.
- Investment costs incurred before outcomes are determined
College admissions game is strategically equivalent to a multi-object, asymmetric **all-pay auction** with unitary demand and bid subsidies.

**Definition**

A *symmetric equilibrium* is a set of achievement functions

\[ \sigma_M(\theta), \sigma_N(\theta) \]

that simultaneously generate optimal grade choices for all competitors.

Achievement functions \(\Rightarrow\) HC distributions,

\[ G(s), G_M(s), G_N(s). \]

Results from auction theory (Athey (2001)) establish existence of a unique symmetric equilibrium that is monotonic in player types.
**PROBLEM:*** For large $M$ and $N$, the equilibrium of the finite game is analytically/computationally intractable.

Equilibrium expected payoffs in the $K$-player color-blind game:

\[
\Pi_i(s, \theta; K) = \sum_{k=1}^{K} U[p_{(k;K)}, s] \sum_{k_i \leq \min\{k, K_i\}, \atop k_j = k - k_i} \left[ \binom{K_i}{k_i-1} F_i\left(\sigma_i^{-1}[s;K]\right)^{K_i-k_i} \left[1-F_i\left(\sigma_i^{-1}[s;K]\right)\right]^{k_i-1} \right.
\]
\[
\times \left( \binom{K_j}{k_j} F_j\left(\sigma_j^{-1}[s;K]\right)^{K_j-k_j} \left[1-F_j\left(\sigma_j^{-1}[s;K]\right)\right]^{k_j} \right) [1-C(s; \theta)].
\]
**SOLUTION:** For large $K$, approximate the equilibrium by considering the limiting environment as $K \to \infty$.

Treat competitors and prizes as a continuum:

- Mass of group $\mathcal{M}$ denoted by $\mu \in (0, 1)$
- Prize rank ordering generated by a distribution $F_P$ on $\mathcal{P} = [\underline{p}, \overline{p}]$. 
Limiting Objective Functions:

**Limiting AP Mechanism:** Map transformed grade quantiles into prize quantiles

\[
P_{ap}^M(s; \theta) = F_p^{-1} \left[ (1 - \mu)G_N(\tilde{S}(s)) + \mu G_M(s) \right]
\]

\[
P_{ap}^N(s; \theta) = F_p^{-1} \left[ (1 - \mu)G_N(s) + \mu G_M(\tilde{S}^{-1}(s)) \right]
\]

- **Limiting Objective Function** (dropping superscripts):
  \[\Pi_j(s, \theta) = U[P_j(s), s] - C(s; \theta).\]

- **FOC:** \[U_1[P_j(s), s]P_j'(s) + U_2[P_j(s), s] = C'(s; \theta).\]

**Theorem**

The maximizers of the limiting objective functions approximate the symmetric equilibrium to arbitrary precision for large enough \(K\).
Qualitative Results (Relative to Color-Blind)

Assuming Likelihood Ratio Dominance (i.e., for measurable $T \subseteq [\theta, \bar{\theta}]$, we have $F_M(\theta) < F_N(\theta) \ \forall \theta \in T$),

QUOTA EQUILIBRIUM:

- High performing (low-cost) minorities decrease investment
- Low performing (high-cost) minorities increase investment
- Opposite for non-minorities
- Zero enrollment gap (by design)
Qualitative Intuition (Relative to Color-Blind)

Assuming Likelihood Ratio Dominance, $U[p,s] = p$, and $C(s;\theta) = \theta s$

**Fixed Grade Markup:** $\tilde{S}(s) = s + \Delta$
- All students decrease investment
- Relative to $N$, $M$ students lower investment by exactly $\Delta$
- No allocational effect among top colleges

**General Admission Preference**

$\tilde{S}(s)$ can perform better... An empirical question!
Outline

- Model
- Structural Estimation
- Estimation results/counterfactual policy experiments
**Structural Model Primitives:**

1. Type Distributions: $F_M(\theta), F_N(\theta), \mu$
2. Prize Distribution: $F_P(p)$
3. Costs: $C(s; \theta)$
4. Utility: $U(p, s)$
5. Markup Function: $\tilde{S}(s)$

**Associated Empirical Objects:**

1. Match Allocations: $F_{PM}(p)$ and $F_{PN}(p)$
2. HC Distributions: $G_M(s)$ and $G_N(s)$

**Data Year: 1996**

- **Prior:** reasonable to assume policy stable and known to agents
- **Post:** Legal changes to AA

*Hopwood v. Texas; AA ban in TX, CA, MI*
**College Data:** Sample of 1,314 4-yr degree institutions

- **USN&WR institutional quality measure** \( \{ Q_u \}_{u=1}^U \)

- **Freshman enrollment by race (IPEDS):** \( \{ M_u, N_u \}_{u=1}^U \)
  - \( \{ Q_u, M_u, N_u \}_{u=1}^U \), characterize the sample of prizes
    \[
    P_{K,K} = \{ p_k \}_{k=1}^K = \left\{ \{ p_{ui} \}_{i=1}^{M_u+N_u} \right\}_{u=1}^U, \quad p_{ui} = Q_u
    \]
  - \( \Rightarrow \) \( F_P, F_{PM}, F_{PN} \)

- **6-year graduation rates (IPEDS)**
  - Subset of 1131 schools
  - Utility function estimation

- **Average 10-yr salary for graduates (B&B 1993-2003 wave)**
  - Subset of 527 schools
  - Utility function estimation
Definition of Minority Group

MINORITIES: Black, Hispanic, American Indian/Alaskan Native
NON-MINORITIES: White, Asian/Pacific Islander, other

Table: Racial Representation Within Different Academic Tiers

<table>
<thead>
<tr>
<th>Tier (Quality Quintile)</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (top)</td>
<td>11.5%</td>
<td>88.5%</td>
</tr>
<tr>
<td>II</td>
<td>10.7%</td>
<td>89.3%</td>
</tr>
<tr>
<td>III</td>
<td>16.4%</td>
<td>83.6%</td>
</tr>
<tr>
<td>IV</td>
<td>20.5%</td>
<td>79.5%</td>
</tr>
<tr>
<td>V</td>
<td>29.1%</td>
<td>70.9%</td>
</tr>
</tbody>
</table>
HC INVESTMENT DATA:

- SAT scores & race for random sample of 92,153 (College Board)

$\Rightarrow G_M(s), G_N(s)$

Empirical SAT Distributions

![Empirical SAT Distributions Graph]

- POPULATION
- MINORITY
- NON-MINORITY
Empirical Procedure

1. Measure AA practices in US college market
   - Estimate grade markup $\tilde{S}(s)$

2. Given Step 1, estimate $F_M$, $F_N$, $U(p,s)$, and $C(s;\theta)$
   - Conditional on $(U,C)$, recover $F_M$ and $F_N$ nonparametrically
   - Parametrically estimate utility curvature parameters

3. Given Step 2, Impose alternative admissions mechanisms on the model (i.e., Color-Blind and Quota) and evaluate changes to investment, allocations, and welfare.
Estimating $\tilde{S}(s)$:

From the policy-maker’s perspective, allocations follow

\[
P_M(s) = F_P^{-1} \left[ (1 - \mu) G_N(\tilde{S}(s)) + \mu G_M(s) \right], \text{ and}
\]

\[
P_N(s) = F_P^{-1} \left[ (1 - \mu) G_N(s) + \mu G_M(\tilde{S}^{-1}(s)) \right].
\]

**IMPORTANT OBSERVATION:** $P_M(s) = P_N[\tilde{S}(s)]$ (*).

**DEFINITIONS:**

- Given a quantile rank $r \in (0, 1)$, define $s_N(r) \equiv G_N^{-1}(r)$
- Define $r_M(r) \equiv G_M\left(\tilde{S}^{-1}(s_N(r))\right)$ as the quantile rank of *de-subsidized* $s_N(r)$ in the minority score distribution.
Estimating $\tilde{S}(s)$:

\[(*) \Rightarrow F_{PM}^{-1}(r_M[r]) = F_{PN}^{-1}(r)\]

\[\Rightarrow G_M \left( \tilde{S}^{-1} [s_N(r)] \right) = F_{PM} \left( F_{PN}^{-1}[r] \right)\]

\[\Rightarrow G_N^{-1}(r) = \tilde{S} \left( G_M^{-1} \left[ F_{PM} \left( F_{PN}^{-1}[r] \right) \right] \right) .\]
Estimating \( \tilde{S}(s) \):

\[
(*) \Rightarrow F_{P_M}^{-1}(r_M[r]) = F_{P_N}^{-1}(r)
\]

\[
\Rightarrow G_M\left(\tilde{S}^{-1}[s_N(r)]\right) = F_{P_M}\left(F_{P_N}^{-1}[r]\right)
\]

\[
\Rightarrow G_N^{-1}(r) = \tilde{S}\left(G_M^{-1}\left[F_{P_M}\left(F_{P_N}^{-1}[r]\right)\right]\right).
\]

**Nonparametric Estimator:**

**Step 1:** Estimate \( \hat{G}_M, \hat{G}_N, \hat{F}_{P_M}, \hat{F}_{P_N} \)
- Kaplan-Meier
- Kernel-Smoothed

**Step 2:** for \( r \in [0, 1] \), we have \( \hat{S}\left[\tilde{s}(r)\right] = \hat{G}_N^{-1}(r) \), where

\[
\hat{s}(r) = \hat{G}_M^{-1}\left[\hat{F}_{P_M}\left(\hat{F}_{P_N}^{-1}[r]\right)\right].
\]
\( \tilde{S}(s) \) Estimation Results

**Estimation Results**

**SAT Scores vs. Markup Function Values**

- **45-degree line**
- **Empirical CDF Method**
- **Linear Approximation**
- **Kernel-Based Method**

**AVG Markup:**
- 69 points (orig SAT scale)

**AVG Markup for Top 5% Minority Students:**
- 102 points (orig SAT scale)

**Linear Approximation:**
- 6.8% grade inflation
- + 8 point additive bonus
Bootstrapped 95% Confidence Bounds

MARKUP FUNCTION ESTIMATION WITH 95% CONFIDENCE BOUNDS

- LOWER CONF. BOUND
- POINT ESTIMATE
- UPPER CONF. BOUND
- LINEAR APPROXIMATION
- 45-DEG LINE
In auction models, the equilibrium produces a mapping from types and type distributions into equilibrium actions; e.g.,

\[(\theta, F_M, F_N)\]

\[
\begin{align*}
\text{Unobserved} & \quad \rightarrow \quad s \\
\text{Observed} &
\end{align*}
\]

Guerre, Perrigne and Vuong (2000, *Econometrica*) discovered that this mapping can be re-worked to get

\[
\theta \quad \leftarrow \quad (s, G_M, G_N)
\]

**MAIN ADVANTAGE:** this method allows for recovery of \(F_M\) and \(F_N\) with no \textit{a priori} distributional assumptions.
Type Distribution Estimation à la GPV:

Recall that \( S_M \sim G_M(s) = 1 - F_M \left[ \gamma_M^{-1}(s) \right] \).

Thus, \textit{subsidized} minority test scores are distributed

\[
\tilde{S}(S_M) \sim \tilde{G}_M(s) = G_M \left( \tilde{S}^{-1}(s) \right) = 1 - F_M \left[ \gamma_M^{-1} \left( \tilde{S}^{-1}(s) \right) \right].
\]

Non-minority FOC:

\[
-U_1 \left[ \mathcal{P}_N(s),s \right] \frac{(1-\mu) f_N \left[ \gamma_N^{-1}(s) \right] (\gamma_N^{-1})'(s) + \mu f_M \left[ \gamma_M^{-1}(\tilde{S}^{-1}(s)) \right] (\gamma_M^{-1})'(\tilde{S}^{-1}(s)) \frac{d\tilde{S}^{-1}(s)}{ds}}{f_P \left( F_P^{-1} \left[ 1 - (1-\mu) f_N \left[ \gamma_N^{-1}(s) \right] + \mu f_M \left[ \gamma_M^{-1}(\tilde{S}^{-1}(s)) \right] \right) \right] } + U_2 \left[ \mathcal{P}_N(s),s \right] = C' \left( s; \gamma_N^{-1} \right)
\]
Type Distribution Estimation à la GPV:

The non-minority FOC can be re-written as

\[
U_1[\hat{P}_N(s), s] \frac{(1 - \mu)\hat{g}_N(s) + \mu\hat{g}_M(s)}{\hat{f}_P \left[ \hat{F}_P^{-1} \left( (1 - \mu)\hat{G}_N(s) + \mu\hat{G}_M(s) \right) \right]} U_2[\hat{P}_N(s), s] = C' \left( s; \hat{\theta} \right)
\]

(1)

where \( \hat{G}_M(s) = G_M \left( \tilde{S}^{-1}(s) \right) \) is the distribution of marked up non-minority SATs.
Match Utility Estimation

**Specification of \( U \):**

- \( U[p,s] = \rho(p,s)u(p,s) \)

- \( \rho(p,s) = \Pr[\text{Graduate}|p,s] \)
  - Estimate using school-level data on graduation rates and SAT scores
  - \( \rho = \beta_0 + \beta_1 p + \beta_2 p^2 + \beta_3 s + \beta_4 s^2 + \beta_5 ps + e_\rho \)
Match Utility Estimation

**Specification of $U$:**

- $U[p, s] = \rho(p, s)u(p, s)$

- $\rho(p, s) = \Pr[\text{Graduate} | p, s]$
  - Estimate using school-level data on graduation rates and SAT scores
  - $\rho = \beta_0 + \beta_1 p + \beta_2 p^2 + \beta_3 s + \beta_4 s^2 + \beta_5 ps + e_\rho$

- $u(p, s) =$ annual salary premium over high-school completers from going to college
  - COBB-DOUGLAS: $Sal - Sal_0 = Ap^{\alpha_1}s^{\alpha_2}$
  - Estimate using school-level data on average graduate salary, 10 years after graduation:
    
    $$\log(Sal - Sal_0) = \alpha_0 + \alpha_1 \log(p) + \alpha_2 \log(s) + e_u$$

- $Sal_0 = $26,133 (BLS, 2002 dollars)
## Match Utility Estimation

### Table: Conditional Graduation Probability Results: $\rho(p,s)$

<table>
<thead>
<tr>
<th></th>
<th>$N$</th>
<th>1131</th>
<th>1131</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>.7525</td>
<td>.7525</td>
</tr>
<tr>
<td>$p$</td>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$p^2$</td>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$s$</td>
<td></td>
<td>0.0077443</td>
<td>0.0055225</td>
</tr>
<tr>
<td>$s^2$</td>
<td></td>
<td>-0.0000487</td>
<td>-0.0000281</td>
</tr>
<tr>
<td>$p \times s$</td>
<td></td>
<td>0.0076061</td>
<td>0.0041589</td>
</tr>
<tr>
<td>const.</td>
<td></td>
<td>-0.175845</td>
<td>0.1889114</td>
</tr>
</tbody>
</table>
**Table: Cobb-Douglass Salary Premium Production Results: \( u(p,s) \)**

<table>
<thead>
<tr>
<th></th>
<th>( N )</th>
<th>( R^2 )</th>
<th>( p )</th>
<th>( s )</th>
<th>const.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>527</td>
<td>.1668</td>
<td><strong>0.5363185</strong> (0.05971)</td>
<td>–</td>
<td><strong>10.60251</strong> (0.0391162)</td>
</tr>
<tr>
<td></td>
<td>470</td>
<td>.1818</td>
<td><strong>0.6218095</strong> (0.108838)</td>
<td>0.2085263 (0.3405296)</td>
<td><strong>9.675291</strong> (1.622419)</td>
</tr>
</tbody>
</table>
Cost Function Identification

Now, FOC can be re-written as

$$\xi_j(s) = C'(s; \theta),$$

where $\xi_j$ is a known function. Even so, identification is difficult. Similar as in Guerre, Perrigne, and Vuong (Ecma, 2009) parametric restriction alone is not enough.

- Suppose $C(s; \theta)$ takes the form $\theta c(s)$, and suppose some $c(s)$ rationalizes the data; i.e., it produces a monotone mapping

  $$\gamma_j^{-1}(s) = \frac{\xi_j(s)}{c(s)} = \theta$$

- then any $\bar{c} = c(s)\phi(s)$, where $\phi(s)$ is positive and strictly increasing, will also produce a monotone map and hence, rationalize the data.

- Thus, need more structure to identify.
IDENTIFYING ASSUMPTIONS:

1. \( C(s; \theta) = \theta c(s) = \theta e^{\nu(s-s)}, \nu > 0. \)
   - The data reject a model with linear costs
   - \( C \) satisfies regularity conditions

2. Zero Surplus Condition:

   \[
   C(s; \theta) = U[p, s] \iff \bar{\theta} = \rho(p, s)u(p, s).
   \]

   - Conceptually similar to Campo, Guerre, Perrigne, and Vuong (2011).
   - The marginal type is indifferent between college and outside option

   \[
   \Rightarrow \nu = \frac{U_1(p, s)P_N'(s) + U_2(p, s)}{U(p, s)}.
   \]
Pseudo-Private Types:

\[ \hat{\mu} = 0.1765 \ [0.1628, 0.1895]; \quad \hat{\nu} = 0.1285 \ [0.0567, 0.2146] \]
COUNTERFACTUAL EXERCISE

1. Holding constant $\mu$, $F_M$, $F_N$, $U(p,s)$, $C(s;\theta,\nu)$, compute equilibrium for each alternative policy
   - Estimated US Admission Preference
   - Color-Blind
   - Quota

2. Points of Comparison:
   - College Placement Profiles
   - Investment
   - Graduation Rates
   - Surplus
   - Inequality

3. Confidence Intervals Computed via Nonparametric Bootstrap
Counterfactual Investment Changes

CB to AP

RACE GROUP QUANTILE

CHNG IN SAT POINTS W/95% CONF. INT.

NON–MINORITIES
MINORITIES

CB to Q

RACE GROUP QUANTILE

CHNG IN SAT POINTS W/95% CONF. INT.

NON–MINORITIES
MINORITIES

AP to Q

RACE GROUP QUANTILE

CHNG IN SAT POINTS W/95% CONF. INT.

NON–MINORITIES
MINORITIES

Brent Hickman University of Chicago ()

HC Investment and AA
## Table: Mean Investment by Group (in SAT Units)

<table>
<thead>
<tr>
<th>Group</th>
<th>AP</th>
<th>95% CI</th>
<th>N</th>
<th>AP</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>886</td>
<td>[879,889]</td>
<td>N</td>
<td>1037</td>
<td>[1019,1038]</td>
</tr>
<tr>
<td>CB:</td>
<td>873</td>
<td>[848,881]</td>
<td>CB:</td>
<td>1037</td>
<td>[1035,1038]</td>
</tr>
<tr>
<td>Q:</td>
<td>896</td>
<td>[891,913]</td>
<td>Q:</td>
<td>1036</td>
<td>[1030,1038]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Pop</th>
<th>AP</th>
<th>95% CI</th>
<th>Race Gaps</th>
<th>AP</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP:</td>
<td>1010</td>
<td>[993,1013]</td>
<td>AP:</td>
<td>151</td>
<td>–</td>
</tr>
<tr>
<td>CB:</td>
<td>1008</td>
<td>[1002,1011]</td>
<td>CB:</td>
<td>164</td>
<td>–</td>
</tr>
<tr>
<td>Q:</td>
<td>1011</td>
<td>[1008,1013]</td>
<td>Q:</td>
<td>140</td>
<td>–</td>
</tr>
</tbody>
</table>
Counterfactual Enrollment Changes

Counterfactual Enrollment Distributions by Race

CDF VALUES vs. SEAT QUALITY

- ADMISSION PREFERENCE, Non-Minority
- ADMISSION PREFERENCE Minority
- COLOR-BLIND, Non-Minority
- COLOR-BLIND, Minority
- QUOTA, Non-Minority
- QUOTA, Minority
### Counterfactual Enrollment % Change W/In Tier

#### Table: %-Changes in Enrollment, Relative to US Admission Preference

<table>
<thead>
<tr>
<th>Tier:</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>CB: -52.4</td>
<td>-17.2</td>
<td>-22.7</td>
<td>-1.6</td>
<td>+40.8</td>
</tr>
<tr>
<td></td>
<td>95% Conf. Int. [-64.1,-45.7] [-33.7,-0.8] [-48.4,-7.8] [-22.9,14.5] [27.3,65.3]</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Q:</td>
<td>+59.3</td>
<td>+52.6</td>
<td>+6.4</td>
<td>-14.5</td>
<td>-37.4</td>
</tr>
<tr>
<td></td>
<td>95% Conf. Int. [41.0,79.6] [33.0,67.9] [-7.2,27.5] [-22.6,-6.0] [-42.8,-29.9]</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$N$</td>
<td>CB: +6.5</td>
<td>+2.3</td>
<td>+4.5</td>
<td>+0.4</td>
<td>-16.0</td>
</tr>
<tr>
<td></td>
<td>95% Conf. Int. [5.4,8.2] [-0.1,4.8] [1.4,9.1] [-3.5,6.4] [-23.9,-12.2]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q:</td>
<td>-7.4</td>
<td>-6.9</td>
<td>-1.3</td>
<td>+3.8</td>
<td>+14.7</td>
</tr>
<tr>
<td></td>
<td>95% Conf. Int. [-9.1,-5.5] [-8.3,-4.8] [-4.4,1.8] [1.4,7.0] [9.8,19.7]</td>
<td></td>
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</tr>
</tbody>
</table>
Counterfactual Grad Rate Changes

### CB to AP

- **Race Group Quantile**: 0.1 to 0.9
- **Change in Grad Rate**: -10 to 15
- **Conf. Int.**: 95%
- **Categories**: Non-Minorities, Minorities

### CB to Q

- **Race Group Quantile**: 0.1 to 0.9
- **Change in Grad Rate**: -10 to 20
- **Conf. Int.**: 95%
- **Categories**: Non-Minorities, Minorities

### AP to Q

- **Race Group Quantile**: 0.1 to 0.9
- **Change in Grad Rate**: -5 to 10
- **Conf. Int.**: 95%
- **Categories**: Non-Minorities, Minorities
## Counterfactual Grad Rate Changes

### Table: Mean Graduation Rates by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>AP</th>
<th>95% CI</th>
<th>N</th>
<th>AP</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>0.47</td>
<td>[0.45,0.49]</td>
<td>N</td>
<td>0.57</td>
<td>[0.55,0.58]</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>CB</td>
<td>0.42</td>
<td>[0.40,0.44]</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Q</td>
<td>0.52</td>
<td>[0.50,0.54]</td>
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</tr>
<tr>
<td>Total Pop</td>
<td>AP:</td>
<td>0.55</td>
<td>95% CI: [0.53,0.56]</td>
<td>Race Gaps</td>
<td>AP:</td>
</tr>
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</tbody>
</table>
Counterfactual Surplus Distributions by Race

SURPLUS = Pr[Graduate|P,S] × Salary

- ADMISSION PREFERENCE, Non-Minority
- ADMISSION PREFERENCE Minority
- COLOR-BLIND, Non-Minority
- COLOR-BLIND, Minority
- QUOTA, Non-Minority
- QUOTA, Minority
Counterfactual Surplus Distributions by Race

Counterfactual Surplus Distributions

CDF VALUES

SURPLUS = Pr[Graduate|P,S] × Salary

ADMISSION PREFERENCE, Non−Minority
ADMISSION PREFERENCE Minority
COLOR−BLIND, Non−Minority
COLOR−BLIND, Minority
QUOTA, Non−Minority
QUOTA, Minority

Brent Hickman  University of Chicago  ()  HC Investment and AA
# Counterfactual Welfare

## Table: Mean Surplus by Group

<table>
<thead>
<tr>
<th>Group</th>
<th>AP</th>
<th>N</th>
<th>CB</th>
<th>Q</th>
<th>Total Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$10,524</td>
<td>$14,357</td>
<td>$8,797</td>
<td>$13,327</td>
<td>$13,681</td>
</tr>
<tr>
<td>95% CI</td>
<td>[7137,12103]</td>
<td>[10342,15952]</td>
<td>[5209,10095]</td>
<td>[9799,14741]</td>
<td>[9740,15284]</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$14,357</td>
<td>$10,524</td>
<td>$14,847</td>
<td>$13,848</td>
<td>$3,833</td>
</tr>
<tr>
<td>95% CI</td>
<td>[10342,15952]</td>
<td>[7137,12103]</td>
<td>[10822,16464]</td>
<td>[9820,15435]</td>
<td>–</td>
</tr>
<tr>
<td>CB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$14,847</td>
<td>$14,357</td>
<td>$6,049</td>
<td>$551</td>
<td>–</td>
</tr>
<tr>
<td>95% CI</td>
<td>[10822,16464]</td>
<td>[10342,15952]</td>
<td>[9820,15435]</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$13,848</td>
<td>$14,847</td>
<td>$551</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>95% CI</td>
<td>[9820,15435]</td>
<td>[10822,16464]</td>
<td>[9820,15435]</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Brent Hickman  University of Chicago ()  HC Investment and AA
## Table: Cross-Mechanism Surplus Changes

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>$\Delta_{AP-CB}$</th>
<th>95% CI</th>
<th>$\Delta_{AP-CB}$</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>$1,726$</td>
<td>[1314,2283]</td>
<td>-489</td>
<td>[-627,-377]</td>
</tr>
<tr>
<td>$N$</td>
<td>$4,529$</td>
<td>[4051,5112]</td>
<td>-998</td>
<td>[-1112,-867]</td>
</tr>
<tr>
<td>$Q$</td>
<td>$2,803$</td>
<td>[2244,3307]</td>
<td>-509</td>
<td>[-632,-367]</td>
</tr>
<tr>
<td>Total Pop</td>
<td>-98</td>
<td>[-181,-45]</td>
<td>-23</td>
<td>[-28,-1]</td>
</tr>
<tr>
<td></td>
<td>60% CI:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-$23</td>
<td>[-28,-1]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>95% CI:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$76$</td>
<td>[21,188]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Policy-Maker announces that SAT scores will be reassigned a value equal to their group percentile rank.
This implies the following Admission Preference rule:

\[ \tilde{S}(s) = G_{N}^{-1}(G_{M}(s)) \]

PROPERTIES:

1. A grade markup function that mimics a Quota

2. Simple implementation: need only knowledge of grades, race

3. A self-adjusting grade markup rule
   - Equivalent to Color-Blind in a symmetric competition
Markup Function Comparison:
Color-Blind vs. Admission Preference vs. Quota
Conclusion

1. Alternative college admission policies have very different effects on investment, inequality, and welfare.

2. AA alters both the competitive channel and productive channel of HC investment incentives.

3. US Admission Preference significantly improves market outcomes for minorities, reduces inequality.

4. A simple alternative Admission Preference rule exists that is Kaldor-Hicks superior to current system, induces lower inequality, but involves significant shifts of wealth away from non-minority college applicants.
THE END
With a grade markup function $\tilde{S}(s)$, the relation between minority and non-minority achievement is summarized by

$$C' \left[ s; \gamma^{-1}_M(s) \right] = C' \left[ \tilde{S}(s); \gamma^{-1}_N(\tilde{S}(s)) \right] \tilde{S}'(s).$$

With linear costs, this reduces to

$$\gamma^{-1}_M(s) = \gamma^{-1}_N(\tilde{S}(s))\tilde{S}'(s),$$

and with additive markup, it becomes

$$\gamma^{-1}_M(s) = \gamma^{-1}_N(s + \Delta),$$
**DEMOGRAPHIC PARAMETER:**

\[
\hat{\mu} = \frac{\sum_{u=1}^{U} M_u}{\sum_{u=1}^{U} (M_u + N_u)} \text{ using FTF enrollment}
\]