Competition, Entry, and the Duration of Contracts:  
Bundling Over Time

(PRELIMINARY: NOT FOR DISTRIBUTION)

Alexander MacKay*

May 28, 2015

Abstract

What determines the length of a contract? A contract that is too short bears the burden of excessive transaction costs. On the other hand, contracts that are too long eliminate the option of switching among the lowest-cost providers. Equilibrium contracts balance these two forces.

Fundamentally, the tradeoff of transaction costs and supply costs is a question of bundling. The optimal bundle size (contract duration) generates the lowest expected cost per unit (period) while accounting for the additional costs of separate transactions. In this paper, I focus on the application to bundling across time.

Within this theoretical framework, I estimate a model of procurement auctions where 1) contract length is determined endogenously, 2) the distribution of private costs depends on the length of the contract, and 3) there is unobserved auction-specific heterogeneity. I demonstrate nonparametric identification when only the winning bids are observed and entry is exogenous. The nonparametric identification of private values and auction-level heterogeneity holds for models without endogenous cost-shifters (contract length). In markets with imperfect competition, the efficient contract is generally not equal to the equilibrium (procurer-optimal) contract. Buyers or sellers may use the duration of a contract to exercise ex ante market power.

In a dataset on U.S. federal labor contracts for janitors, I find that about three-quarters of the contracts are too short and one-quarter are too long. Counter-intuitively, I find that the efficiency loss is increasing in the number of bidders.

The nature of buyer-seller relationships varies across markets. In some markets, such as the market for computer processors, a single seller (Intel) supplies the needs of the buyer

*Department of Economics, University of Chicago (mackay@uchicago.edu).
(Apple) over a long period. In others, such as commodity spot markets, the buyer-seller relationships are transient and perhaps anonymous. Across many markets, these relationships are governed by fixed-term contracts. In a classic survey of industrial prices, Stigler and Kindahl (1970) found that over a quarter of business-to-business supply transactions were under fixed-term contracts. In electricity markets, fixed-term contracts are common between purchasers and producers, as well as between producers and suppliers of raw inputs like coal (Joskow (1985)) and natural gas (Crocker and Masten (1985)). In labor markets, the prevalence of such contracts varies from country to country; on average, 14 percent of labor contracts in the European Union are of limited duration.

This paper provides a theory of buyer-seller relationships governed by fixed-term contracts: why these contracts vary in length, and how market-determined contracts may differ from efficient contracts. Contracts that are determined by market participants (buyers and sellers) may be too long or too short, resulting in wasteful social costs. Counterintuitively, these extra costs may increase as a market become more competitive. Therefore, from a policy standpoint, highly competitive markets may be of more concern for regulators than those that are more concentrated. This result occurs because market participants care about price rather than cost, and the price responds more quickly to a change in contract length than the cost when the number of bidders is large.

What determines the equilibrium duration of a contract? For concreteness, suppose a seller (the government) is issuing contracts of a set duration. With no transaction costs and a moderate degree of competition, the seller would like the contracts to be as short as possible, to ensure that the bidder with the lower cost over each time increment is the winning bidder. When the contracts are long, averaging across many periods increases the expected costs for the winning bidders.

With transaction costs, the procurer increases the duration of the contract to reduce the frequency of these costs. In equilibrium, the marginal cost of the transaction is balanced against the cost-reducing benefit of switching more frequently. This tradeoff is mediated by the degree of competition; for many cost distributions, the optimal contract length that is U-shaped in the number of suppliers. For moderate levels of competition, having more bidders increases the return to shorter contracts. When competition is sufficiently intense,
each additional supplier reduces the gap between the lowest instantaneous cost and the lowest average cost, which leads to longer contracts in equilibrium.

Imperfect competition drives a wedge between the revenue-optimal contracts and the contract size that maximizes social surplus. I demonstrate that the direction of the wedge is tied to whether the buyer surplus is increasing or decreasing with the length of the contract. For U-shaped models, the efficient length is shorter than the optimal length for low levels of competition. For high levels of competition, the pattern reverses. This paper provide a framework for analyzing optimal contracts with transaction costs and imperfect competition, which are two essential characteristics of real-world contracts. Insights from this problem can be applied to bundling more generally.

Building on this framework, I develop and estimate an empirical model of contract length with imperfect competition and transaction costs. Using procurement data from the U.S. government for janitorial labor contracts, I estimate the distribution of costs suppliers face, recover the transaction costs, and estimate the efficient contract. In this draft, I present preliminary results. I find that about three-quarters of the contracts are too short and one-quarter are too long. In this market, wasteful social costs are increasing with the number of competitors - the counter-intuitive result alluded to above.

In this application, I develop results for nonparametric identification of auctions with private costs and unobservable, auction-specific common costs when only the winning bids are observed. As many datasets are only available with winning bids, these results should prove valuable to many future studies. The standard independent private values framework is not flexible enough to explain many real-word bids.

My work on the tradeoff between transaction costs and price is closely related to the models of contract length of Dye (1985) and Gray (1978), who take the stochastic price process as given. The contribution of this paper is to use standard tools of industrial organization to model this price process and explore its implications.

The economic literature on contracts is deep and far-ranging. For clarity, I abstract away from features of ex post market power that have been studied previously. Examples include risk-sharing, principal-agent relationships, the holdup problem, and incomplete contracting. My work is complementary to these existing models and may have a substantial impact on any policy-relevant estimates.

This model is equivalent to a simultaneous bundling problem. There is a strong literature on the bundling problem, yet most of the theoretical work deals with a price-setting monopolist. The closest model to the one developed in this paper is given by Palfrey (1983). Palfrey looks a at seller choosing between independent auctions and a single, bundled auction for all goods. I advance his analysis by allowing for intermediate degrees of bundling and by introducing transaction costs. Salinger (1995) and Bakos and Brynjolfsson (1999)
contribute an insight that is fundamental to this paper: bundling reduces the variance of average valuations. In the setting of Bakos and Brynjolfsson (1999), this is an information advantage to the seller. However, as I demonstrate in this paper, bundling reduces total surplus and revenue when there are no transaction costs. The analysis of optimal mechanisms for bundling goods is rare. Hart and Nisan (2014) explore the revenue efficiency of simple mechanisms to sell multiple goods.\footnote{Though at first it may seem related, the work on multi-unit auctions is quite distinct. In that strand of literature, the good is homogeneous, and buyer demand is not independent across units. In this paper, on the other hand, downward-sloping demand occurs a result of heterogeneous valuations of units.}

In Section 1, I use a simple example to demonstrate the divergence between efficient and optimal contracts, as well as show the characteristic U shape in the number of bidders. In Section 2, I provide the general theoretical framework. There is a simple relationship between the buyer surplus and the gap between the efficient and the revenue-optimal contracts. In Sections 3 and 4, I turn to an application in procurement where the bundle is the length of the contract. In Section 3, I demonstrate the identification results for the auction model used in the application. Section 4 provides empirical results for U.S. federal service contracts for a particular industry. Section 5 concludes.

1 A Simple Example

Suppose a buyer wishes to purchase a good for two periods. The buyer decides to implement a first-price auction, in which the highest bid from \( N \) sellers wins. Each contract the buyer signs has a transaction cost of \( \delta \). This transaction cost may be thought of as a search cost or a contracting cost. Should the buyer issue a single two-period contract, or two one-period contracts?

Suppose that sellers’ costs are drawn from a standard uniform distribution, and suppose that the draws are independent across sellers and periods. When the buyer issues single-period contracts, the auctions are independent. When the buyer issues a bundled contract, the seller’s cost for the contract is the sum of the two per-period costs. In this case, the bundled cost follows a triangular distribution. With two contracts, the buyer must pay an additional \( \delta \) compared to the bundled contract. Table 1 gives the total expected price to the buyer under the symmetric auction equilibrium, including the transaction cost \( \delta = 0.2 \), for different values of \( N \).

Notice that the buyer-optimal contract is U-shaped in the number of sellers. When the market is not very competitive, the bundled contract is optimal. Bundling periods together reduces the variance in costs to sellers. This has two effects: 1) a reduction in information rents, or margins, and 2) an increase in the minimum cost draw. When competition is low, the reduction in information rents outweighs the increased in expected cost to the winner. For middle values of \( N \) (e.g. 10 and 30 in this example), the increased variance of costs of
Table 1: Buyer-Optimal and Efficient Contract Length for $\delta = 0.2$

<table>
<thead>
<tr>
<th>$N$</th>
<th>One-Year</th>
<th>Two-Year</th>
<th>Optimal</th>
<th>One-Year</th>
<th>Two-Year</th>
<th>Efficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.73</td>
<td>1.43</td>
<td>2</td>
<td>1.07</td>
<td>0.97</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1.40</td>
<td>1.20</td>
<td>2</td>
<td>0.90</td>
<td>0.85</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1.20</td>
<td>1.07</td>
<td>2</td>
<td>0.80</td>
<td>0.78</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1.07</td>
<td>0.99</td>
<td>2</td>
<td>0.73</td>
<td>0.72</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0.97</td>
<td>0.93</td>
<td>2</td>
<td>0.69</td>
<td>0.68</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>0.90</td>
<td>0.88</td>
<td>2</td>
<td>0.65</td>
<td>0.65</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.84</td>
<td>0.84</td>
<td>1</td>
<td>0.62</td>
<td>0.62</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0.80</td>
<td>0.80</td>
<td>1</td>
<td>0.60</td>
<td>0.60</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.76</td>
<td>0.77</td>
<td>1</td>
<td>0.58</td>
<td>0.58</td>
<td>1</td>
</tr>
<tr>
<td>20</td>
<td>0.59</td>
<td>0.61</td>
<td>1</td>
<td>0.50</td>
<td>0.47</td>
<td>2</td>
</tr>
<tr>
<td>30</td>
<td>0.53</td>
<td>0.54</td>
<td>1</td>
<td>0.46</td>
<td>0.43</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>0.50</td>
<td>0.49</td>
<td>2</td>
<td>0.45</td>
<td>0.40</td>
<td>2</td>
</tr>
</tbody>
</table>

separate contracts is an advantage to the buyer. As $N$ gets large, this advantage is mitigated by two factors: 1) the minimum draw of the bundled contract converges to the minimum for separate contracts, and 2) the transaction costs become relatively large. At some value of $N$, the bundled contract becomes optimal.\(^7\)

Table 1 also displays the total cost for each mechanism. The mechanism with the lowest total cost is the efficient mechanism. Notice that the efficient contract also exhibits a U-shape. When markets are insufficiently competitive (for example, $N = 7$) the efficient contract is smaller than the buyer-optimal bundle. However, for certain larger values of $N$, the efficient contract is larger than the buyer-optimal contract.

The qualitative features shown in this simple example generalize to a much broader class of models. In the next section, I explore the basic theoretical framework. For implementations of specific models, including details on the U-shape class of models and a microfounded model, see Appendix A.

## 2 A General Framework of Optimal and Efficient Bundle Size

Suppose that a buyer needs the supply of a good for a length of time $S$. The buyer can issue contracts of arbitrary duration $T$. For each contract, the buyer faces a transaction cost of $\delta$. This transaction cost may represent the cost of dealing with a seller, search costs, or the cost of implementing a mechanism. For clarity, I rule out endogenous asymmetries, such

\(^7\)In the limit, the expected per-period cost approaches zero (the minimum) for both one-year and two-year contracts. The bundled contract is optimal as it minimizes transaction costs.
as learning by doing, capacity constraints, and relationship-specific investments. In some markets, such as the application I study, these will not be important features. In markets where they are important, these features will complement this model.

The buyer selects a supplier via a mechanism with a per-period stochastic price \( P(N, T) \), where the price distribution may depend on the length of the contract and the number of sellers, \( N \).\(^8\) The buyer’s problem is

\[
\min_{J, \{T_j\}} \sum_{j=1}^{J} (E[P(N, T_j)] + \delta) \text{ s.t. } \sum_{j=1}^{J} T_j = S.
\]

If the distribution of \( E[P(N, T_j)] \), the expected per-period price, is stationary, an optimal solution will have \( T_j = T \forall j \). (Let \( S \) be sufficiently large to ignore the leftovers). Then the problem reduces to minimizing the average per-unit price including of transaction costs.

\[
\min_T E[P(N, T)] + \frac{\delta}{T} \quad (1)
\]

For a given mechanism, the procurer selects the optimal \( T \) satisfying the first order condition

\[
\frac{dE[P(N, T)]}{dT} \bigg|_{T=T^*} = \frac{\delta}{T^2} \quad (2)
\]

For any finite equilibrium \( T^* \), \( \frac{dE[P(N, T)]}{dT} \bigg|_{T=T^*} > 0 \). For the rest of this section, I assume that such interior solutions exist. In general, \( \frac{dE[P(N, T)]}{dT} \) will be positive when the per-period cost distribution is stable and the market is sufficiently competitive. As \( T \) increases, suppliers average cost draws across multiple units. This shrinks the variance of the bundled cost distribution, which increases the expected minimum cost when markets are competitive. I call this feature stochastic diseconomies of scale. In the limit, all suppliers’ costs are equal to the long-run average. The increase in expected minimum cost drives up the expected price.

What about efficiency? The social planner’s concern is expected costs, rather than expected price.\(^9\) Thus, the efficient contract solves

\[
\min_T E[C(N, T)] + \frac{\delta}{T} \quad (3)
\]

with the first-order condition

\[
\frac{dE[C(N, T)]}{dT} \bigg|_{T=T^*} = \frac{\delta}{T^2}
\]

\(^8\)\( N \) may depend on \( T \).

\(^9\)In this setting, I assume the social planner is limited by information constraints; in this setting the social planner cannot observe the private information about sellers’ costs. This reflects the idea that the mechanism (and the associated transaction costs) are important to the truthful revelation of information. A third party with full information would solve a different problem, awarding the contract to the lowest-cost seller at every instant and switching when the net savings outweigh the transaction cost.
In general, \( E[P(N,T)] \neq E[C(N,T)] \), which will result in an inefficiency when the contract is determined by the buyer. As long as interior solutions exist (see Proposition (2)), this result gives the simple relationship that the efficient bundle size \( \tilde{T} \) will be larger than the equilibrium bundle size when \( \frac{dE[P(N,T)]}{dT} > \frac{dE[C(N,T)]}{dT} \). Defining the expected seller surplus as \( E[\pi(N,T)] = E[P(N,T)] - E[C(N,T)] \), we have the following result:

**Proposition 1.** When interior solutions exist, the efficient contract will be longer than the equilibrium contract if and only if the expected seller surplus is increasing at \( \dot{T} \):\[ \tilde{T} > \dot{T} \iff \left( \frac{dE[P(N,T)]}{dT} - \frac{dE[C(N,T)]}{dT} \right) \big|_{T=\dot{T}} > 0 \]

\[ \iff \frac{dE[\pi(N,T)]}{dT} \big|_{T=\dot{T}} > 0 \]

The existence of interior solutions depends on the concavity of the expected price function.

**Proposition 2.** Interior solutions to the buyer’s problem and social planner’s problem exist as long as the first-order conditions (2) and (3) can be satisfied and \( E[P(N,T)] \) and \( E[C(N,T)] \) are not too concave. In particular, \( \frac{d^2E[P(N,T)]}{dT^2} \big|_{T=\dot{T}} > -2\frac{dE[P(N,T)]}{dT} \big|_{T=\dot{T}} \) and \( \frac{d^2E[C(N,T)]}{dT^2} \big|_{T=\ddot{T}} > -2\frac{dE[C(N,T)]}{dT} \big|_{T=\ddot{T}} \). These are the second-order conditions to ensure that first-order conditions above achieve a maximum.

### 2.1 Allocation of Term-Setting Rights

Given the general model, we can identify settings in which inefficiency arising from market power over contract length may be of first-order importance. In this section, I provide some intuition and a heuristic guide to the assignment of term-setting rights to limit such inefficiencies.

The buyer’s problem can be written in the following form:

\[
\min_T E[P(N,T) - C(N,T)] + E[C(N,T)] + \frac{\delta}{T} = \min_T E[\pi(N,T)] + E[C(N,T)] + \frac{\delta}{T}
\]

Notice that when \( \frac{dE[\pi(N,T)]}{dT} = 0 \), this problem is equivalent to the social planner problem. Therefore, when the buyer sets the duration of the contract, these contracts will be efficient when the seller surplus does not change with the length of the contract. The more sensitive buyer surplus is to the duration of the contract, the greater the potential for inefficiency.

What about assigning contract term-setting power to the sellers? Sellers solve the prob-
lem:
\[
\max_T E[P(N, T) - C(N, T)] - \frac{\delta}{T} = \min_T -E[P(N, T)] + E[C(N, T)] + \frac{\delta}{T}
\]

Sellers solve the social planner problem when \( \frac{dE[P(N, T)]}{dT} = 0 \). Therefore, if price is not sensitive to contract duration, it is efficient to let the sellers determine the length of the contract.\(^\text{10}\)

If either price or buyer surplus changes with the duration of the contract, there is potential for inefficiency arising from market power. A simple heuristic to mitigate efficiency loss is to let sellers determine contract duration when the duration affects price more than buyer surplus, and to let buyers determine contract duration otherwise.

These heuristics, combined with Proposition 1, provide insight into which settings may allow for substantive inefficiencies and whether the efficient contract is longer or shorter. Below, I provide a simple example to illustrate how changing the allocation of rights over duration may lead to vastly different outcomes.

**Example: Markup Pricing** Suppose sellers in equilibrium follow a simple markup pricing rule, \( P = \mu C \). Then the buyer’s problem is
\[
\min_T \mu E[C(N, T)] + \frac{\delta}{T}
\]
and the seller’s problem is
\[
\min_T (1 - \mu) E[C(N, T)] + \frac{\delta}{T}
\]
As \( \mu \geq 1 \) in equilibrium, the seller’s problem reverses the sign that expected costs enter in the objective function. By increasing costs, sellers increase total profits. In this setting, the buyer should determine the duration. The greater the markup, the more that the equilibrium contract will diverge from the efficient contract.

### 2.2 Achieving Efficiency with a Tax

The efficient contract can be achieved with a per-transaction tax (or subsidy) when either side of the transaction holds the term-setting rights. When the buyer determines the length

\(^\text{10}\)Sellers have an equivalent rule to Proposition 1: \( \dot{T}_S > \ddot{T} \iff \frac{\partial E[P(N, T)]}{\partial T} \big|_{T=\ddot{T}} > 0 \). This means that either 1) \( \dot{T}_S > \dddot{T} > \dddot{B} \), 2) \( \dddot{B} > \dddot{T} > \dddot{S} \), or 3) \( \dot{T}_S > \dddot{T} \cap \dddot{B} > \dddot{S} \). The case where both the buyer-optimal and seller-optimal contract are shorter than the efficient contract is ruled out by the fact that per-period costs must be increasing at the efficient contract for an interior solution.
of the contract, the efficient per-transaction tax $\tau_B$ solves

$$\tau_B = T^2 \frac{dE[\pi(N,T)]}{dT} \bigg|_{T=\bar{T}}$$

This tax equates the buyer’s problem with the social planner’s problem. Note below how the tax causes the externality on the seller to drop out at the efficient contract.

$$\bar{T} = \arg \min_T E[\pi(N,T)] + E[C(N,T)] + \frac{\delta + \tau_B}{T}$$

Analogously, the efficient tax on the seller (when the seller has term-setting rights) is given by

$$\tau_S = -T^2 \frac{dE[P(N,T)]}{dT} \bigg|_{T=\bar{T}}$$

In general, $\tau_S \neq \tau_B$. A policymaker has a choice between two efficient taxes, with different effects on tax revenue.

### 3 An Application with Procurement Auctions

For this section and the following, I turn to an application with procurement auctions, where the procurer (the buyer) determines the duration of the contract. The procurer solves equation (1), where the stochastic price mechanism $P(N,T)$ is the price that would result from a first-price auction.

This theory suggests an empirical approach that I pursue in Section 4.3. First, I use observed prices to estimate the functions $E[P(N,T)]$ and $E[C(N,T)]$. Second, using the procurer’s first-order condition, I recover $\delta$ for each auction. Finally, using standard industrial organization tools to find the relationship between price and cost, I find the efficient contract length and calculate the counterfactual welfare change.

In the auction model, I allow for auction-specific unobserved heterogeneity, which is important for explaining common features of real-world data. Below, I provide versions of the model with exogenous and endogenous entry. With exogenous entry, the model is nonparametrically identified when only the winning bid is observed. With endogenous entry, nonparametric identification is only obtained when multiple bids are observed. In the empirical section, I use a parametric version of the endogenous entry model, as reduced-form evidence suggests that entry is endogenous.
3.1 The Model: Exogenous Entry

The game is in two stages. In the first stage, the procurer observes the size of the project, \( x \), the number of bidders, \( N \), and a proportional transaction cost to re-contract, \( \delta \). The procurer sets the length of the contract optimally to minimize per-period costs.

An exogenous set of bidders observe the size of the auction, \( x \), per-period private cost \( c \), and the length of the contract, \( T \). They submit a sealed proportional bid, \( b \), in a first-price auction. The final per-period price \( (y = b_{1:N} \cdot x) \) and the number of bidders is observed.

Bidders maximize

\[
\max_b (b \cdot x \cdot T - c \cdot x \cdot T) \cdot \Pr(b \text{ wins}|N)
\]

Given that \( x \) is common to all bidders, standard auction results give the solution that

\[
E[b_{1:N}|N] = E[c_{2:N}]
\]

The procurer’s problem is then

\[
\min_T x \cdot E[P(N,T)] + \frac{\delta \cdot x}{T} = \min_T x \cdot E[b_{1:N}|N,T] + \frac{\delta \cdot x}{T} = \min_T E[c_{2:N}|T] + \frac{\delta}{T}
\]

The preliminary results in Section 4 implement a parametric version of this model. The exact specification is described in the section.

3.2 Identification

For each auction, the econometrician observes \( N, T \), and the per-period price \( y_N = b_{1:N} \cdot x \). When \( x|T \perp c|T \), this model is identified when there is sufficient variation in \( N \). The strategy is to use variation in \( N \) that affects bids in a known way. The identification proof holds irrespective of \( T \) and any exogenous variables that may be in the conditioning set.

\textbf{Proof:}

For \( N \) and \( N + 1 \) bids, we have

\[
\ln y_N|T = \ln b_{1:N}|T + \ln x|T \\
\ln y_{N+1}|T = \ln b_{1:N+1}|T + \ln x|T
\]

This gives the relation of characteristic functions

\[
\frac{\varphi_{\ln y_N|T}(z)}{\varphi_{\ln y_{N+1}|T}(z)} = \frac{\varphi_{\ln b_{1:N}|T}(z) \cdot \varphi_{x|T}(z)}{\varphi_{\ln b_{1:N+1}|T}(z) \cdot \varphi_{x|T}(z)}
\]
The left-hand side can be nonparametrically estimated given sufficient data. On the right-hand side, the heterogeneity drops out. Since \( \varphi \ln y \mid T(z) \) is data, once we have identified \( \varphi \ln b_{1,N} | T(z) \), we also have identified \( \varphi \ln b_{x} | T(z) \). Recall that

\[
b = \beta(c; N) = c + \int_{c}^{\infty} \frac{(1 - F(\xi))^{N-1} d\xi}{(1 - F(\xi))^{N-1}}.
\]

Let \( f \) denote the density of private costs. The density of bids is given by

\[
g(b; N) = f(\beta^{-1}(b; N)) \frac{d\beta^{-1}(b; N)}{db}.
\]

From the formula for order statistics, \( Ng(e^v; N)(1 - G(e^v; N))^{N-1}e^v \) is the density of the (log) winning bid for \( N \) bidders. Expressing the characteristic functions in terms of the density for bids, we have

\[
\frac{\varphi_{\ln b_{1,N}} | T(z)}{\varphi_{\ln b_{1,N+1}} | T(z)} = \frac{\int e^{izv} Ng(e^v; N)(1 - G(e^v; N))^{N-1}e^v dv}{\int e^{izv}(N + 1)g(e^v; N + 1)(1 - G(e^v; N + 1))^{N}e^v dv}
\]

where \( G \) (and \( g \)) is a known functional of \( F \). [Point identification is in progress]. However, note that this relation holds at every value of \( z \), and this is for only two values of \( N \).

Many datasets only have the winning bids, and unobserved heterogeneity is common. This method for nonparametric identification with only the winning bid allow economists to have a richer understanding of many more datasets.

### 3.3 The Model: Endogenous Entry

More generally, the procurer will use the length of the contract to induce entry in addition to shifting the cost to each supplier. In this setting, I model the game in three stages: 1) the procurer sets the duration of the contract based on transaction costs, the size of the project, and market concentration, 2) suppliers decide to enter based on the size of the project, the length of the contract (via profits), and entry costs, and 3) suppliers observe private costs and submit a bid to the first-price auction.

I model this as follows: the procurer knows the size of the project, \( x \), but he does not know the entry costs to suppliers, \( k \). The procurer may observe market characteristics \( z \) that are informative about entry costs. \( T \) is set endogenously by the procurer, who solves the a cost minimization problem when \( k \) is not known. The procurer sets \( T \) to minimize

\[
T^* = \arg\min_T x \sum_{n=1}^{N} E[c_{2,n} | T] \cdot P(N = n | T, x, z) + \frac{\delta}{T}
\]
The first-order condition gives

\[
\bar{x} \sum_{n=1}^{N} \left( \frac{dE[c_{2,n}|T]}{dT} \cdot P(N = n|T, x, z) + E[c_{2,n}|T] \cdot \frac{dP(N = n|T, x, z)}{dT} \right) - \delta \frac{T^2}{T^2} = 0
\]

The last line decomposes the procurer’s incentives into cost-change effect and an entry effect. When the suppliers’ expected profits are increasing with \( T \), the entry effect will be negative, as \( T \) induces entry via higher profits. More entry causes a decrease in expected price, as the expected price is decreasing in \( N \). In equilibrium, the cost-change effect must be positive, as the whole term will be positive. Compare this to the first-order condition for the social planner:

\[
\bar{x} \sum_{n=1}^{N} \left( \frac{dE[c_{1,n}|T]}{dT} \cdot P(N = n|T, x, z) + E[c_{1,n}|T] \cdot \frac{dP(N = n|T, x, z)}{dT} \right) - \delta \frac{T^2}{T^2} = 0
\]

In this model, as \( E[c_{1,n}|N, T] < E[c_{2,n}|N, T] \) at \( T \) for all \( N \), the entry effect will be less negative for the social planner. Analogously, the cost-change effect will be smaller (less positive) for the social planner. The combined effect of these two forces determines whether the efficient contract length will be larger than the optimal contract length in equilibrium.

Suppliers observe the length of the contract and a per-period entry cost \( k(z) \) before deciding to enter, and they enter only if expected profits are positive. Suppliers perfectly predict the number of entrants, as in McAfee and McMillan (1987), so that bidding is efficient conditional on \( T \). The entry condition is given by

\[
\frac{1}{N} (E[b_{1,n}|N, T] - E[c_{1,n}|T]) \cdot x \cdot T - k(z) \cdot T > 0
\]

\[
\frac{1}{N} (E[c_{2,n}|T] - E[c_{1,n}|T]) \cdot x - k(z) > 0
\]

After entry, suppliers observe private per-period costs \( c \) and bid according to

\[
\max_b (b \cdot x \cdot T - c \cdot x \cdot T) \Pr(b \text{ wins}|N).
\]

[Empirical results using this model are in progress. Endogenous entry appears to be a key feature of the data, as the winning price is increasing with \( N \).]

\[11\]In the author’s experience, versions of this model in which suppliers pay one-time entry costs instead of per-period entry costs are poorly identified empirically. This is because changing the length of the contract affects entry both via the expected price and directly via the duration of the contract; it can be difficult to disentangle these two effects.
4 Empirical Results

4.1 Data

The contracts studied in this paper are 3,505 competitive contracts for custodial and janitorial services for the United States federal government. Janitorial services were chosen from all federal contracting goods and services because they are numerous, the product is homogenous, and there is a lot of variation in contract duration. That is, this market is a relatively clean setting to analyze the determinants of transaction costs. Further, the market is sizable, totaling $5.1 billion from 2004 to 2013.

The data were obtained from USASpending.gov, which regularly imports and organizes data from the Federal Procurement Data System (FPDS). By law, the FPDS keeps public records of all federal contracts. The full set of 29,636 U.S. janitor contracts from 2000 to 2014 were further cleaned using a system of filters described in Section 4.1.2. Data on other products in the FPDS were used to construct a measure of transaction cost and control variables for the size of the office. These variables are detailed in Section 4.1.4.

In addition, I merge these data with data on local labor market conditions: panel data on county-level unemployment and labor force, and the number of registered janitors in the same 5-digit ZIP code and the surrounding 3-digit ZIP code.

4.1.1 Janitor Contracts: Institutional Details

For each contract, I observe key variables including start and end dates, the total value of the project, the 9-digit ZIP code where the project is performed, the number of offers received for the contract, and variables indicating whether or not the contract is competitive. In addition, there are detailed variables on the winning bidder, including the DUNS number, which allows me to track whether an incumbent wins a follow-on contract.

Competitive contracts are contracts that are posted publicly and allow open competition from registered vendors.\textsuperscript{12} Many of these contracts are posted on the centralized web portal FedBizOpps.gov.\textsuperscript{13} On the website, a prospective supplier can view the contract details, including contract duration and the square footage of the building, requirements for the job, and a list of interested suppliers. From the portal, a supplier submits a bid to the contracting office that includes the total price over the duration of the contract. The contracting office

\textsuperscript{12}These contracts fall under three categories: Full and Open Competition, Full and Open Competition after the Exclusion of Sources, and Competed Under Simplified Acquisition. 86 percent of the contracts deemed Full and Open Competition after the Exclusion of Sources are listed as a small business set-aside. As 96 percent of the contracts are won by small businesses (as determined by the contracting officer), I ignore this distinction for the purposes of analysis. See Federal Acquisition Regulation (FAR) Part 5.

\textsuperscript{13}35 percent of the 3,505 are posted on FedBizOpps.gov, another 52 percent fall under the $25,000 threshold required for centralized posting, but are still required to be available to “promote competition to the maximum extent practicable.” (FAR Part 13.104). Another 13 percent claim an exception. See FAR Part 6.
determines the winning supplier primarily based on the lowest price. By law, the contracting office must justify selecting other than the lowest-price offer.\textsuperscript{14}

Janitorial services are provided by a mix of firms and individuals: 39 percent of the winning suppliers in the contracts analyzed have 2 or fewer employees. In 3.4 percent of the contracts, firms have over 250 employees, yet three-quarters of these are classified as small businesses by the contracting office.

Importantly, contract duration is determined locally by the local contracting officer. As several of these officers described to the author, contract duration depends on how costly it is to re-contract and how competitive is the local market for janitors. This motivates using this market as a case study for the model developed in this paper. Transaction costs and competition are key motivating factors for the procuring agencies.

4.1.2 Data Cleaning

Location, local employment, and local business data were available for 21,151 contracts from 2000 to 2014. I apply a set of sequential filters to clean the data, which results in a pool of 3,505 competitive contracts. Table 2 provides summary statistics for each set of observations removed by a filter.

First, I use text analysis to remove contracts that do not include the key words “janitor-” or “custodia-”, and from this group I remove those contain key words for specific cleaning services: “window”, “duct”, “carpet”, and “floor” (A). This filter creates a more homogeneous product of general janitorial services.

From these contracts, I remove contracts that were not available to competition (B). I then remove the 61 contacts prior to 2004 and all contracts that start after 2012, as many of the latter contracts are still in progress (C). To control for entry error by the contracting officer, I construct a measure of actual contract length based on modifications to the contract in the data. I remove any contracts that have more than a one-year gap between the listed duration and the measure I construct (D). Finally, I restrict my analysis to contracts that receive multiple bids (E, F).

4.1.3 Estimation Data

Summary statistics for the set of 3,505 competitive contracts are displayed in Table 3.

Figure 1 displays a scatterplot of the logged values of the winning bids on the y-axis against the number of bidders on the x-axis. The pattern observed in the scatterplot motivates the more flexible model developed in this paper. Both the standard IPV assumptions and single-dimensional unobserved heterogeneity are too restrictive. IPV would result in a strict negative relationship between price and \( N \), whereas endogenous entry based on

\textsuperscript{14}Based on the guidelines established by FAR and conversations with local contracting offices, the contracting office will prefer suppliers that have an established history.
Table 2: Summary Stats: Filtered by Data Cleaning Steps

<table>
<thead>
<tr>
<th>Variable</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bids</td>
<td>1.9</td>
<td>1.0</td>
<td>4.1</td>
<td>4.7</td>
<td>0.9</td>
<td>4.1</td>
</tr>
<tr>
<td>(2.4)</td>
<td>(0.6)</td>
<td>(3.4)</td>
<td>(4.0)</td>
<td>(0.3)</td>
<td>(3.1)</td>
<td></td>
</tr>
<tr>
<td>Price per Month</td>
<td>23576.1</td>
<td>29639.2</td>
<td>29008.8</td>
<td>5418.8</td>
<td>25205.2</td>
<td>29566.7</td>
</tr>
<tr>
<td>(247400.1)</td>
<td>(192318.0)</td>
<td>(516916.0)</td>
<td>(24888.9)</td>
<td>(394231.3)</td>
<td>(571239.7)</td>
<td></td>
</tr>
<tr>
<td>log10(Price per Month)</td>
<td>3.4</td>
<td>3.7</td>
<td>3.0</td>
<td>3.0</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>(0.8)</td>
<td>(0.8)</td>
<td>(0.6)</td>
<td>(0.6)</td>
<td>(0.7)</td>
<td>(0.6)</td>
<td></td>
</tr>
<tr>
<td>Contract Length</td>
<td>13.8</td>
<td>19.8</td>
<td>19.9</td>
<td>33.5</td>
<td>14.8</td>
<td>24.5</td>
</tr>
<tr>
<td>(18.0)</td>
<td>(21.0)</td>
<td>(15.7)</td>
<td>(17.0)</td>
<td>(14.4)</td>
<td>(20.8)</td>
<td></td>
</tr>
<tr>
<td>Num. Employees</td>
<td>191411.7</td>
<td>4700.8</td>
<td>779.4</td>
<td>110.9</td>
<td>217.8</td>
<td>154.4</td>
</tr>
<tr>
<td>(1.7e+07)</td>
<td>(139975.3)</td>
<td>(14128.2)</td>
<td>(2112.5)</td>
<td>(1876.3)</td>
<td>(2599.9)</td>
<td></td>
</tr>
<tr>
<td>log10(Num. Employees)</td>
<td>1.3</td>
<td>1.7</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>(1.0)</td>
<td>(1.0)</td>
<td>(0.9)</td>
<td>(0.8)</td>
<td>(1.0)</td>
<td>(0.8)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>7938</td>
<td>4601</td>
<td>1508</td>
<td>1918</td>
<td>1681</td>
<td>3505</td>
</tr>
</tbody>
</table>

This table displays summary statistics for the full pool of 21,151 federal government contracts for janitor services in the United States from 2000 through 2014. The data were cleaned by applying sequential filters; the five filters are described below. The remaining 3,507 contracts are in column F. These contracts are competitive contracts from 2004 to 2012 that match key words in the description, receive multiple bids, and have a calculated contract duration within one year of the listed initial contract duration.

A: Contracts without key words (e.g., “janitor-”, “custodia-”)
B: Non-competitive contracts
C: Contracts signed before 2004 or after 2012
D: Contracts with large prediction error in contract duration
E: Contracts receiving one bid or less
F: Estimation data

Table 3: Summary Statistics for Competitive Janitorial Contracts

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-Period Price ($)</td>
<td>29,500</td>
<td>570,000</td>
<td>23</td>
<td>933</td>
<td>16,000,000</td>
</tr>
<tr>
<td>Length (months)</td>
<td>24.0</td>
<td>20.6</td>
<td>0.03</td>
<td>12.2</td>
<td>133.2</td>
</tr>
<tr>
<td>Bidders</td>
<td>4.1</td>
<td>3.1</td>
<td>2</td>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>Transaction Cost Measure</td>
<td>148.8</td>
<td>1,586.8</td>
<td>0.065</td>
<td>17.9</td>
<td>52,500</td>
</tr>
</tbody>
</table>
project size would result in a positive relationship. The cloud, with high bids for large and small $N$, suggest two-dimensional unobserved heterogeneity in which both project size and entry costs vary.

These contracts have a great deal of variation in length. Figure 2 plots the contract length against the number of bidders. Contracts tend to cluster at yearly increments, but the support from 0 to 60 months is well-covered. There is an increasing relationship between the number of bidders and the length of the contract. I examine this empirical relationship further in the reduced-form regressions of Section 4.2.

### 4.1.4 Transaction Costs

To understand how transaction costs enter into the decision of the procurer, I generate a measure of transaction costs using data for other services at the same location (9-digit zip code) of the janitor contracts. My measure of transaction costs is the total expenditures (in thousands of dollars) divided by the total number of contract modifications for all non-service products excluding Furniture and Office Equipment ($\frac{\text{Expenditures}}{\text{Modifications}}$). The economic intuition behind this measure is that offices that have higher transaction costs will resort to fewer contract modifications for a given level of spending. I use non-service contracts for the same location to mitigate any overlap with unobservables that would directly affect the price of janitor contracts. Spearman's correlation of this measure with an equivalent one for janitor-related categories is 0.24.
In all of the regressions in Section 4.2, I control for spending and the number of modifications for other housekeeping services, furniture, and office equipment at the same location as the contract. These variables are used to proxy for the size of the building.

4.1.5 Dynamic Considerations

In a dynamic setting, the procurement process might result in asymmetries between bidders that would invalidate the assumptions of symmetry that have been maintained thus far in the paper. Winning bidders may be at an advantage in subsequent auctions due to learning by doing or lowered transaction costs of retaining the same supplier. Additionally, competing bidders may retain some information about competitors if costs are correlated over time.\footnote{Saini (2012) discusses the literature on endogenous asymmetries and evaluates a model in which capacity constraints hurt the winning bidder.}

In the janitorial data, dynamic considerations appear to be meaningful. For this reason, I develop a parametric model that allows for computationally tractable bidder asymmetries (see Appendix B). [Results are in progress.]

Within the set of 7,115 competitive contracts matching key words between 2004 and 2012, I identify 1,785 contracts as follow-on contracts with multiple bids. Follow-on contracts are those that have another janitorial services contract in the same location within the past year. Of these, 45 percent are won by the same vendor as the preceding contract. Under symmetry, we would expect incumbents to win these auctions 34 percent of the time.\footnote{As I only observe winning bidders, I am unable to adjust for when a supplier does not bid on a follow-on to}
In this section, I present reduced-form regressions to document the following empirical facts:

1. Positive correlation between contract length and the number of bidders,
2. Positive correlation between contract value and the number of bidders, and
3. Positive correlation between a measure of transaction costs and contract length.

The positive correlation between contract value and the number of bidders (Fact #2) is robust to controlling for the location, indicating that there is selective entry based on the size of the project. In Table 4, I demonstrate these results with reduced-form results using only 5-digit ZIP codes with multiple observations. In the exogenous entry case, we would expect competition to drive down the winning bid. Though I correct for the endogeneity in the structural model in the next section, I present suggestive evidence that the coefficient on the number of bidders is negative with instrumental variable estimates in Table 5. Here, local labor market conditions of unemployment, labor force participation, and the number of janitor establishments in the surrounding 3-digit ZIP serve as supply-side instruments for the number of bidders. As expected, the sign flips to negative. The presence of the positive coefficients in Table 4 is consistent with entry the supplier’s current contract. However, I do observe that incumbents win 78 percent of the time for follow-on contracts that received a single bid.

This is to use the within-ZIP variation to identify the effect of bidders on price.
Table 5: IV Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Baseline</th>
<th>(3) Zip FE</th>
<th>(4) Zip FE</th>
<th>(5) Zip+4 FE</th>
<th>(6) Zip+4 FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bids</td>
<td>0.785∗</td>
<td>0.827∗</td>
<td>0.026</td>
<td>-0.015</td>
<td>0.017</td>
<td>-0.054</td>
</tr>
<tr>
<td></td>
<td>(0.213)</td>
<td>(0.269)</td>
<td>(0.060)</td>
<td>(0.114)</td>
<td>(0.056)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>Contract Length</td>
<td>-0.028∗</td>
<td>-0.030∗</td>
<td>-0.014∗</td>
<td>-0.015∗</td>
<td>-0.014∗</td>
<td>-0.015∗</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Transaction Costs</td>
<td>0.383∗</td>
<td>0.375∗</td>
<td>0.089</td>
<td>0.063</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.056)</td>
<td>(0.081)</td>
<td>(0.090)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spending Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Month-Year FEs</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
+ p < .1, ∗ p < .05

conditional on the size of the project. [The preliminary structural results assume exogenous entry; endogenous entry results are in progress.]

In Table 6, I report the results for the first stage of the above IV regressions. These regressions highlight the positive correlation between the length of the contract and the number of bidders. As the F-stat on the first-stage indicates, these instruments in the current form are weak.

Finally, Table 7 provides results for regressions of contract length on transaction costs and an index of competitiveness. As we would expect, contracts duration is increasing with the transaction costs.

4.3 Parametric Model

NB: Results using the endogenous entry model of Section 3.3 are in progress. Currently, the results employ a version of the exogenous entry model of 3.1.

For the empirical results in this paper, I employ a parametric approach. For this application, the parametric assumptions make computation of the maximum likelihood estimates feasible. For simplicity, I assume the following:

- $c \sim \text{Weibull}$, with mean $\mu(T) = \exp(\mu_0 + \mu_1 T)$ and shape $\alpha(T) = \exp(\alpha_0 + \alpha_1 T)$
- $\delta \sim U(0, \bar{\delta})$
- $x \sim \text{Exp}(1)$ [more flexible model in progress]
Table 6: First-Stage Regressions: Number of Bidders

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Baseline</th>
<th>(3) Zip FEs</th>
<th>(4) Zip FEs</th>
<th>(5) Zip+4 FEs</th>
<th>(6) Zip+4 FEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(Unemployment)</td>
<td>0.261</td>
<td>0.299</td>
<td>1.125*</td>
<td>0.942*</td>
<td>1.221*</td>
<td>1.027*</td>
</tr>
<tr>
<td></td>
<td>(0.155)</td>
<td>(0.207)</td>
<td>(0.214)</td>
<td>(0.500)</td>
<td>(0.232)</td>
<td>(0.536)</td>
</tr>
<tr>
<td>ln(Labor Force)</td>
<td>-0.090</td>
<td>-0.160</td>
<td>2.392</td>
<td>2.468</td>
<td>3.007*</td>
<td>3.060</td>
</tr>
<tr>
<td></td>
<td>(0.161)</td>
<td>(0.211)</td>
<td>(1.615)</td>
<td>(1.761)</td>
<td>(1.753)</td>
<td>(1.912)</td>
</tr>
<tr>
<td>Outside Establishments</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.027*</td>
<td>-0.025*</td>
<td>-0.028*</td>
<td>-0.028*</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Contract Length</td>
<td>0.021*</td>
<td>0.021*</td>
<td>0.012*</td>
<td>0.009</td>
<td>0.009</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Transaction Costs</td>
<td>-0.139*</td>
<td>-0.135*</td>
<td>0.601*</td>
<td>0.424*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.059)</td>
<td>(0.223)</td>
<td>(0.233)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spending Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Month-Year FEs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Observations</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
<td>1515</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.038</td>
<td>0.124</td>
<td>0.610</td>
<td>0.665</td>
<td>0.625</td>
<td>0.680</td>
</tr>
<tr>
<td>$F$</td>
<td>8.5</td>
<td>1.9</td>
<td>6.5</td>
<td>1.9</td>
<td>9.7</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
+ $p < .1$, * $p < .05$

Table 7: Reduced-Form Regressions: Contract Length

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Zip FEs</th>
<th>(3) Zip + Time FEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction Costs</td>
<td>1.513*</td>
<td>2.113*</td>
<td>2.175*</td>
</tr>
<tr>
<td></td>
<td>(0.244)</td>
<td>(0.640)</td>
<td>(0.647)</td>
</tr>
<tr>
<td>Competitiveness Index</td>
<td>1.373*</td>
<td>2.839*</td>
<td>0.607</td>
</tr>
<tr>
<td></td>
<td>(0.195)</td>
<td>(1.079)</td>
<td>(2.284)</td>
</tr>
<tr>
<td>Spending Controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Month-Year FEs</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>3489</td>
<td>3489</td>
<td>3489</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.033</td>
<td>0.773</td>
<td>0.800</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
+ $p < .1$, * $p < .05$
Without taking a stance on the underlying per-period cost process, I allow the private costs to depend on the length of the contract in a flexible manner.\textsuperscript{18} Using the procurer and bidder problems below, I estimate the parameters \( (\mu_0, \mu_1, \alpha_0, \alpha_1, \text{ and } \bar{\delta}) \).

\[
\min_{T} E[c_{2:N}|T] + \frac{\delta}{T} \\
\max_b (b - c)(1 - F(c))^{N-1}
\]

\textbf{4.4 Estimation and [Preliminary] Results}

\textit{NB: Results using the endogenous entry model of Section 3.3 are in progress. Currently, the results employ a version of the exogenous entry model of 3.1.}

Estimation proceeds in three steps. In the first step, I use maximum likelihood to estimate the parameters of the cost distributions. In the second step, I use the estimated parameters and the procurer’s objective function to recover \( \delta \). In the third step, I find the efficient contract length and calculate the welfare change of implementing efficient contracts.

The estimated cost distribution is displayed in Figure 3. The mean of the cost distribution is displayed, as well as key percentiles of the cost distribution (not the distribution of the estimates). The variance of the distribution declines slightly with the length of the contract. The key change over time is the increase in the mean. Parameter estimates are given in Table 8. [Bootstrapped standard errors for the full model are to be included here with the revised empirical approach].

In Figure 4, I display the expected transaction costs as a proportion of the per-period price. The median is close to one, meaning that the transaction costs are about equivalent to one month’s price for the service. The maximum is 36 months. Figure 5 displays the raw expected costs.

In Figure 6, I present a simple comparison of the procurer-optimal contracts versus those that lower social cost. The efficient contracts are shorter when \( N = 2 \) and longer when \( N > 2 \). The mean difference of 2.9 months is greater than ten percent of the mean contract length.

\textsuperscript{18}For a microfounded model, see Appendix A.
Figure 3: Estimated Cost Distribution

![Cost Distribution Graph]

Figure 4: Expected Transaction Costs as a Proportion of Price

![Transaction Costs Graph]
The efficiency loss at the equilibrium contract length is shown in 7. The loss ranges from 0 to 2 percent, and it is increasing in the number of bidders. This is quite counter-intuitive. The reason? The procurer has greater market power over the price when the number of bidders is large. Though the winning supplier margins are decreasing with the number of bidders, the ratio of the slopes $\frac{dE[P(N,T)]/dT}{dE[C(N,T)]/dT}$ is increasing with the number of bidders. In this case, the ratio is increasing because the expected cost (the first-order statistic) approaches the lower bound more quickly than the expected price (the second-order statistic). In the estimated model, the procurer over-values the costs of increasing the length of the contract, as the price is increasing at a faster rate than social costs.
Figure 6: Difference Between Equilibrium Length and Efficient Length

Figure 7: Efficiency Loss by Equilibrium Length and Number of Bidders
5 Conclusion

In this paper, I provide a framework for the analysis of optimal and efficient fixed-term in the presence of imperfect competition and transaction costs. One of the main insights of this paper is that contract duration affects costs and entry, and that it may be used as a lever to exercise market power.

When markets have few competitors, the socially optimal contract is shorter than the procurer-optimal contract. When markets are competitive and the cost distribution is bounded, the efficient contract is longer than the procurer-optimal contract.

Using U.S. federal procurement data, I estimate a parametric model under the framework to show that this wedge can be economically significant. Counter-intuitively, I find that efficiency loss is increasing in the number of bidders.

Additionally, as a key step in developing the model, I provide nonparametric identification results for auctions with private values and a common unobserved component, when only the winning bid is observed. The model deals more flexibly with particular distributions of data in the real world.

References


A Benchmark Models

In this section, I present two benchmark models: a U-shape model using the Weibull distribution, and a microfounded model using stochastic processes. In the Weibull model, the underlying value distribution is a parametric function of the size of the bundle. In the microfounded Ornstein-Uhlenbeck model, I show an example of an underlying cost process that may justify such a parameterization. I outline the particulars of the setup and provide some intuition behind the main results. For this section, I frame the problem in terms of a seller auctioning off an item to a set of buyers.

For concreteness, suppose the seller parcels out the good using $J$ first-price auctions to $N$ symmetric bidders. Each bidder receives valuation draws $v_j$ from $F(\cdot; \theta(T))$, where the parameters of the distribution, $\theta$, depend on the size of the bundle. The draws are independent across bidders and auctions. The bidder has linear utility over his surplus. The bidder’s problem is therefore:

$$\max_{\{b_j\}} \sum_{j=1}^{J} (v_j - b_j) F_{b_j \theta}(b_j)^{N-1}$$
By independence, this problem reduces to a set of \( J \) independent auctions, with the well known result that expected winning bid is the second-order statistic of the valuations: 

\[
E[b_{N,N}] = E[v_{(N-1):N}].
\]

The seller’s problem is

\[
\max_T E[v_{(N-1):N}(\theta(T))] - \frac{\delta}{T}.
\]

I will focus on cases where the mean is fixed and the variance of the valuation distribution is decreasing with \( T \). There are three reasons for limiting the analysis to decreasing variance. (1) When the per-unit valuation distribution is stable, averaging across multiple draws results in smaller variance. (2) With \( T \) large enough, it must be true. Every buyer has the same expected valuation, which the average will converge to by the law of large numbers. (3) Higher variance in costs benefits the seller when markets are competitive. If the variance is increasing in \( T \), then the combination of this feature and the incentive to lower transaction costs leads to bundling everything together.

In particular, let us examine a class of models for which \( \theta \) maps \( T \) into the standard deviation of \( v \). Further, assume that \( \theta'(T) < 0 \) and that \( E[v|T] = E[v] \). Equilibria are defined by the solution to the seller’s problem, and they fall into the classes below:

**Proposition 3.** Under the conditions above,

i. Revenue-Optimal Size:

- For \( N = 2 \): \( \frac{\partial E[v_{1:2}(\theta(T))]}{\partial \sigma} < 0 \). Therefore, the revenue-optimal mechanism sets \( T = \tau \).
- For \( N = 3 \): \( \frac{\partial E[v_{2:3}(\theta(T))]}{\partial \sigma} \) may be greater than or less than zero. When the valuation distribution is symmetric, \( \frac{\partial E[v_{2:3}(\theta(T))]}{\partial \sigma} = 0 \), which results in \( T = \tau \).
- For \( N \geq 4 \): \( \frac{\partial E[v_{(N-1):N}(\theta(T))]}{\partial \sigma} > 0 \). For any \( \tau \), there exists a \( \delta \) such that an interior solution exists. This solution sets

\[
- \frac{\partial E[v_{(N-1):N}(N,\xi(T))]}{\partial \sigma} \frac{d\theta(T)}{dT} \cdot T^2 = \delta.
\]

ii. Efficient Size:

- For \( N \geq 2 \): \( \frac{\partial E[v_{N,N}(\theta(T))]}{\partial \sigma} > 0 \). For any \( \tau \), there exists a \( \delta \) such that an interior solution exists.

**A.1 U-Shape Models**

**Proposition 4.** There exists a class of valuation distributions for which the efficient bundle size is U-shaped in the number of bidders. These distributions have the following characteristics:
1. **Holding the mean constant, the largest order statistic is monotonically increasing with the variance.**

2. **As the size of the bundle increases, the mean remains fixed but the variance declines.**
   *Further, the function mapping bundle size to variance is not too convex.*

3. **The support of the valuation distribution is bounded from above.**

Properties 1 and 2 above give rise to stochastic diseconomies of scale: averaging across larger bundles results in lower expected valuations. The third condition gives rise to the U-shape, and is quite natural, especially in procurement.\(^9\) Without providing sufficient conditions over the class, I argue that it is rather general. Members of this class include the Weibull, the gamma, and the beta distributions. I explore the Weibull distribution below, but the intuition as to why the U shape occurs is straightforward, and I will provide it here.

The planner’s role is to set the increase in the first order statistic equal to the marginal transaction cost:

\[
\frac{\partial E[v_{N:N}(N, \xi(T))]}{\partial \sigma} \left( - \frac{d\xi(T)}{dT} \right) = \frac{\delta}{T^2}
\]

Examine Figure 8 above. In it, I have plotted the expected revenue for the Weibull model explored later in the section, for \(N = 4\) and \(N = 10\). At \(\sigma = 0\), the expected revenue is

\(^{19}\)In which case the cost distribution is be bounded from below.
zero regardless of $N$. At first, as the variance increases, the expected revenue rises more quickly for larger $N$. However, the expected revenue reaches the boundary more quickly, so the slope of the plotted curve at a particular value for $\sigma$ decreases with $N$ after a certain threshold. This behavior is what governs the U shape in the number of bidders. For $N$ small enough, an increase in the number of bidders causes the marginal value of parceling to increase (toward the left-hand side of the graph in Figure 8). For $N$ large enough, an increase in the number of bidders causes a decrease in the marginal value of parceling.

Below, I demonstrate the characteristic U shape with the Weibull distribution.

**The Weibull distribution** Suppose that bidders draw valuations $v_i = \bar{v} - \eta_i$, where $\eta_i$ follows a Weibull distribution with CDF

$$F(x) = 1 - \exp \left( - \left( \frac{x}{\lambda} \right)^k \right), \quad x \in [0, \infty).$$

I allow the parameters to change with $T$, the size of the bundle. However, to restrict attention to cases where the mean does not change, I restrict the mean cost draw to be constant and equal to $\mu$. That gives the restriction

$$\lambda = \frac{\mu}{\Gamma(1 + \frac{1}{k})}.$$

Then the variance of the distribution is decreasing in $k$.\(^\text{20}\) We can write $k = \alpha(T)$, where $\alpha(T)$ is an increasing function that maps the size of the bundle to the parameter of the distribution. Here, we use an increasing function so that the variance is declining in $T$. The first-order statistic for $N$ draws from the Weibull distribution is

$$E[\eta_{1:N}] = \mu N^{-1/k}$$

The seller’s problem is then

$$\max_T E[v_{N:N}(\alpha(T))] - \delta = \max_T \left( \bar{v} - \mu N^{-1/\alpha(T)} \right) - \delta T$$

**Proposition 5.** For the Weibull model, the efficient bundle size is U-shaped in the number of bidders, as long as $\alpha(T)$ is not too concave.\(^\text{21}\)

\(^{20}\)The variance is given by $\lambda^2 \left[ \Gamma(1 + \frac{2}{k}) - (\Gamma(1 + \frac{1}{k}))^2 \right] = \mu^2 \left( \frac{\Gamma(1 + \frac{2}{k})}{(\Gamma(1 + \frac{1}{k}))^2} - 1 \right)$.

\(^{21}\)This is the converse of the convexity assumption used earlier, as $\alpha(T)$ is increasing in this case. The condition is $\alpha''(T) > \frac{-2\alpha'(T)}{\alpha'(T)^2}$.
Proof Normalize $\mu = 1$. This gives the first-order condition

$$\frac{1}{\alpha(T)^2} \ln N \cdot N^{-1/\alpha(T)} = \frac{1}{\alpha'(T)} \frac{\delta}{T^2}$$

Given the concavity assumption, the right-hand side will be decreasing in $T$. The derivative of the left-hand side with respect to $N$ is

$$\frac{1}{\alpha(T)^2} N^{-1/\alpha(T)-1} - \frac{1}{\alpha(T)^3} \ln N \cdot N^{-1/\alpha(T)-1}$$

Therefore

$$\frac{d}{dN} LHS > 0 \iff 1 > \frac{1}{\alpha(T)} \ln N \quad (4)$$

Suppose this relation holds for some $N$, and $N$ increases. The LHS increases; therefore $T$ must fall. Both $1/\alpha(T)$ and $\ln N$ increase. Eventually, this causes the relation in (4) to switch. At this point, an increase in $N$ decreases the LHS, which causes $T$ to increase.

Proposition 6. For the Weibull model, the optimal bundle size is U-shaped in the number of bidders, as long as $\alpha''(T) > -2\alpha'(T)$. Further, the minimum size is obtained at a higher value of $N$ compared to the efficient bundle size.

Proof The second-order statistic for the Weibull distribution, with $\mu = 1$, is $N(N-1)^{-1/k} - (N-1)N^{-1/k}$. Therefore the first-order condition is given by

$$\frac{1}{\alpha(T)^2} \left[ \ln(N - 1)N \cdot (N - 1)^{-1/\alpha(T)} - \ln N(N - 1) \cdot N^{-1/\alpha(T)} \right] = \frac{1}{\alpha'(T)} \frac{\delta}{T^2}$$

The left-hand side is positive, increasing and then decreasing in $N$. Following the same logic as the proof for the efficient bundle, the optimal bundle is U-shaped in $N$.

A.2 A Model with Microfoundations

The Weibull model of the previous section took as an assumption that the valuations could be parameterized as a function of $T$. Here, I will provide a model of underlying valuations that generates both the distribution of valuations and how the size of the bundle shapes the distribution. Suppose that valuations are distributed over a spectrum $\tau$ according to an Ornstein-Uhlenbeck diffusion process. The valuation process $X_t$ is governed by the differential equation
\[ dx_t = \theta (\mu - x_t) + \sigma dW_t \]

where \( W_t \) is a Wiener process. This process is stationary over \( t \). That is, any bundle of size \( T \) will have the same unconditional distribution as any other bundle of size \( T \). Define the average valuation over time \( T \) as

\[ v_T = \frac{1}{T} \int X_t dt \]

Then \( v_T \) is Gaussian with mean \( \mu \) and variance \( \frac{1}{T^2} \sigma^2 (\theta T + e^{-\theta T} - 1) \). When valuations are Gaussian, \( E[v_{N,N}(\sigma)] = E[z_{N,N}] \sigma + \mu \), where \( z \) is a standard normal. Let \( \xi : T \rightarrow \sigma \). The efficient bundle size \( T \) solves

\[ \max_T E[z_{N,N}] \xi(T) + \mu - \frac{\delta}{T} \]

This results in the first-order condition

\[ E[z_{N,N}] \xi'(T) + \frac{\delta}{T^2} = 0 \]

\[ -\xi'(T) T^2 = \frac{\delta}{E[z_{N,N}]} \]

As shown in the appendix, \( \frac{d}{dT} (-\xi'(T) T^2) > 0 \). Therefore,

**Proposition 7.** The efficient bundle size is decreasing in the number of bidders.

Unlike the U-shape models, the microfounded model here does not have an upper bound on the valuation. Further, as the analysis above holds for the \( N - 1 \) order statistic when \( N > 3 \):

**Proposition 8.** The optimal bundle size is decreasing in the number of bidders. It is optimal to sell everything in a single bundle for \( N \in \{2, 3\} \).

Additionally, in we have that \( E[z_{(N-1):N}] < E[z_{N:N}] \). Therefore,

**Proposition 9.** The efficient bundle size is smaller than the revenue-optimal bundle.

### B A Parametric Auction Model with Asymmetry

Suppose that there are two types of bidders in an auction, with \( n_1 \) and \( n_2 \) bidders of each type. The equilibrium bid function for the first type is given by:

\[ \beta_1(c) = c + \int_c^{\infty} \frac{[1 - F_1(u)]^{n_1-1} [1 - F_2(u)]^{n_2}}{[1 - F_1(c)]^{n_1-1} [1 - F_2(c)]^{n_2}} du \]
The density of the winning bid, \( w \), has the following form:

\[
    f(w) = \frac{n_1 f_1(c)}{(n_1 - 1) \frac{f_1(c)}{1 - F_1(c)}} + \frac{n_2 f_2(c)}{n_1 (n_1 - 1) \frac{f_1(c)}{1 - F_1(c)} + (n_2 - 1) \frac{f_2(c)}{1 - F_2(c)}}
    \int_c^\infty \left[ \frac{1 - F_1(c)}{1 - F_1(u)} \right]^{n_1 - 1} \left[ \frac{1 - F_2(c)}{1 - F_2(u)} \right]^{n_2 - 1} du
\]

This density is computationally tractable when the costs distributions for both types are Weibull and the shape parameter is held constant across types. In practice, this model allows for tractable location-shifting asymmetries in auction models. These results can be extended to incorporate more than two types.