# Framing, Information, and Welfare*

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**Abstract**

Consumers often face an overwhelming amount of information when deciding between products, and one of the primary policymaking tools available to improve their informativeness is the framing of this information. We introduce a general theoretical approach that characterizes when one frame is revealed to provide robustly higher welfare than another. Because it is testable, adaptable, and both necessary and sufficient, our condition determines both whether frames are robustly welfare ranked in a particular data set and the overall proportion of data sets in which frames can be so ranked.

**Key words:** Framing effects, behavioral welfare economics, revealed preferences, incomplete information
**JEL codes:** D60, D83, D91

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# 1 Introduction

Given the veritable explosion in data availability and product complexity, consumers often face an overwhelming amount of information when deciding between products. This is true for important decisions such as health care coverage and retirement investments, but also lower-stakes decisions such as choosing cellular plans or entertainment bundles. Not surprisingly, there is increasing evidence that individuals can be poorly informed when making such decisions (for example, see Handel 2013, Handel and Kolstad 2015, and Bhargava, Loewenstein, and Sydnor 2017).[1] One of the primary policymaking tools available to address consumer misinformation is the framing of decision-relevant information. For instance, policymakers can present decision-relevant information, or require firms to present decision-relevant information, in a way that makes it easier for consumers to understand the characteristics of an option (Hastings and Tejeda-Ashton 2008, Choi, Laibson, and Madrian 2009), to navigate the decision problem they face (Abeler and Jager 2015, Esponda and Vespa 2016), or to compare options (Carrera and Villas-Boas 2015, Ericson and Starc 2016, Carpenter, Huet-Vaughn, Matthews, Robbett, Beckett, and Jamison 2019).

These interventions have provided a growing sense that framing can reduce consumer misinformation. However, in the absence of a general theoretical approach for assessing the value of information in each frame, it has been necessary to take a case-by-case approach to figure out which frame is best. In a limited number of cases, there is an objectively correct choice, so whichever frame gets consumers to make that choice more often is clearly best. For example, Handel (2013) and Bhargava, Loewenstein, and Sydnor (2017) leverage settings where some health care plans dominate others. Outside of these special cases, choice options have tradeoffs, and consumers have private knowledge about their tastes and requirements that allow them to negotiate these tradeoffs. To determine the best frame, researchers have built models of how consumers balance these tradeoffs that are specific to each case.

In this paper, we provide a general theoretical approach by characterizing when and how a policymaker can determine the highest welfare frame without ex-ante assumptions about consumer preferences. We introduce an information-theoretic framework and use it to identify two testable conditions that reveal whether one frame has a *robustly* higher value of information than another.[2] If the data satisfy either of these conditions, then one frame has a higher value of information for all utility functions and information structures consistent with the data. If the data do not satisfy one of these conditions, then it is impossible to robustly rank frames based on the value of information.

Our first condition precisely identifies when the policymaker can rank frames without ex-ante assumptions on utility or information. In technical terms, this condition requires

[1] See Handel and Schwartzstein (2018) for an extensive review of this evidence.

[2] Robustness also plays a key role in the information design literature (see Bergemann and Morris 2017).

that a vector representing the difference between the frames in terms of outcomes falls in the cone generated by the restrictions for a utility function to be consistent with the data. This condition corresponds to solving a system of linear equations, so is simple to check, and MATLAB programs that implement it are provided.[3] We first show that this condition is both necessary and sufficient for data to reveal that one frame has a robustly higher value of information. We then show that whenever some options are clearly dominant, this condition can easily be amended to account for the additional restrictions that dominance provides. Because it is testable, adaptable, and both necessary and sufficient, this condition is an effective tool both for determining whether frames are robustly welfare ranked in a particular data set and also for determining the overall proportion of data sets in which frames can be so ranked.[4]

Our second condition for identifying higher welfare frames arises from applying the classic Blackwell approach within our framework. Because the actual information that decision-makers possess is unobservable to the policymaker, we rank frames based on a summary statistic for how well informed decision-makers are when taking each action. The *revealed experiment* for a frame is the distributions of actions in each state. We say a frame has been revealed to have better informed actions if the revealed experiment for the other frame is a garbling of the revealed experiment for that frame. This condition is also simple to check and MATLAB programs are provided that implement it as well.

We show that for any arbitrary decision problem, a frame being revealed to have better informed actions is sufficient to reveal that frame has a robustly higher value of information. However, it is not always necessary. For three standard classes of decision problems, we show there exists data sets where a frame is revealed to have a robustly higher value of information even though neither frame is revealed to have better informed actions. Given this, the condition based on a garbling of revealed experiments is in general only sufficient for determining which frame provides robustly higher welfare.

Finally, we illustrate our approach and testable conditions by considering the framing health care plans. Imagine the choice between a low-premium, high-deductible health care plan ("Value PPO") and high-premium, low-deductible plan ("Premium PPO"). Imagine further that these plans are identical in every way, except that the Value PPO is more cost effective no matter an individual's health outcomes (see Handel 2013 and Bhargava, Loewenstein, and Sydnor 2017 for examples of such dominance in practice). Given this dominance, if plain language framing of deductibles increases the probability that the Value PPO is chosen from 80% to 90%, then this framing is clearly better.

Now imagine that around half of the time, the company offers a version of the Premium

[3] Programs available at https://github.com/danieljosephmartin/framinginformationwelfare.git.

[4] Thus, it could be used to assess if an experimental design has sufficient power to identify the best frame.

PPO that has better doctor availability, but with plain language framing, the Value PPO is still chosen more often (70% of the time). If we assume that Value PPO is still better even when it has worse doctor availability, it would appear that the plain language framing has benefited decision-makers by leading the Value PPO to be chosen more often than the Premium PPO. However, resolving tradeoffs on the behalf of individuals could be misleading in this case.

Imagine that with normal framing, the Premium PPO is chosen more often when it has better doctor availability (80% of the time). This has revealed there is a tradeoff that individuals face between cost effectiveness and doctor availability: the Value PPO is preferred when plans have the same doctor availability, but the Premium PPO is preferred when it has better doctor availability. Because of this tradeoff, it is less clear if plain language framing has improved decision-making. In Section 4.2 we use our second condition to show this particular pattern of choice has revealed that actions are better informed with normal framing, so normal framing is revealed to provide robustly higher welfare in this example.

In fact, it is often possible to determine if one frame provides robustly higher welfare. As noted previously, our first condition is both necessary and sufficient, so it can be used to identify the proportion of data sets in which one frame is robustly welfare ranked. We use this condition to show that two-thirds of all rationalizable data sets in the PPO choice setting allow us to identify that one frame provides a robustly higher value of information. We also show that an additional utility restriction based on the clear dominance of some plans decreases the proportion of data sets that are rationalizable, but does not increase the fraction of rationalizable data sets where a welfare dominant frame can be identified.

Taken together, our approach and two testable conditions show it is possible for a policymaker determine which frame produces the highest welfare, even in which preference tradeoffs are not resolved for decision-makers ex-ante. In additional to its practical benefits, this approach may appeal to policymakers who do not feel comfortable in resolving preference trade-offs on the behalf of individuals because doing so runs the risk of violating norms against paternalism. This is why it is common for policymakers who use frames to provide "nudges" to take a libertarian perspective in which they avoid forcing a particular choice on individuals (Thaler and Sunstein 2008).

The rest of the paper is organized as follows. Section 2 discusses related literature in more detail. Section 3 provides our framework and the first testable condition and shows that this condition is both necessary and sufficient for identifying higher welfare frames. Section 4 provides the second testable condition based on applying the Blackwell approach within our framework and shows that this condition is sufficient for identifying higher welfare frames. Section 5 shows that while sufficient, this condition is not necessary. Section 6 provides the application of our framework to the choice of health care plans. Section 7 concludes.

# 2 Related Literature

## 2.1 Framing and Welfare

Our paper contributes to a growing literature that asks whether a welfare relation can be inferred from choices despite the distortions produced by frames (Salant and Rubinstein 2008, Bernheim and Rangel 2009, Rubinstein and Salant 2011) and whether these approaches can be used to rank frames based on the welfare they generate (Benkert and Netzer 2018).[5] We extend this literature to a large class of additional applications by asking whether a welfare ranking of frames can be inferred from misinformed choices. This includes choices that are distorted by non-rational mental gaps in the processing of information (Handel and Schwartzstein 2018) and rational information frictions such as rational inattention (Sims 2003) and the drift diffusion model (Ratcliff 1978).

Like Salant and Rubinstein (2008) and Bernheim and Rangel (2009), we make one small, but important, addition to the primitives of a decision problem: a frame $f$. However, unlike these papers, we allow decision-makers to have incomplete information about the value of available options, and we use information structures to model the impact that frames have on beliefs about the value of available options. With existing approaches, such as Salant and Rubinstein (2008) and Bernheim and Rangel (2009), there is no uncertainty about the value of available options, so framing has no impact on beliefs.

Another substantive differences between our approach and existing approaches to framing effects is that we allow for stochastic choice within frames,[6] which is particularly useful in the context of framing effects because important effects can have subtle effects, such as swinging the probability of making a choice from 10% to 30%.[7] This feature of our approach complements a growing literature that considers stochastic choice to be an essential data set for studying information and utility (e.g., Manzini and Mariotti 2014, Apesteguia and Ballester 2018). Rehbeck (2019) follows our approach by considering stochastic choice in frames, but he departs from this paper by considering the testable implications for our model when data on states is not available.

[5] This is related to a literature on performing welfare assessments in the presence of behavioral biases (Manzini and Mariotti 2007, Apesteguia and Ballester 2015).

[6] Stochastic choice in frames also appears in a concurrent paper by Bhattacharya, Mukherjee, and Sonal (2017).

[7] This is particularly true when taking the perspective that stochastic choice arises from the choices of many individuals, as many framing effects are measured by the change in the fraction of individuals making each choice.

## 2.2 Framing and Information

Our paper draws motivation from a growing literature on the importance of framing on informativeness.

Early evidence of the connection between framing and informativeness was provided by Tversky and Kahneman (1989), who demonstrated that the framing of a lottery could make it harder or easier to spot that one lottery was dominant. They presented two groups of subjects with the equivalent lottery choice framed in different ways (A or B and C or D):

**A**: 90% white \$0; 6% red win \$45; 1% green win \$30; 1% blue lose \$15; 2% yellow lose \$15

**B**: 90% white \$0; 6% red win \$45; 1% green win \$45; 1% blue lose \$10; 2% yellow lose \$15

**C**: 90% white \$0; 6% red win \$45; 1% green win \$30; 3% yellow lose \$15

**D**: 90% white \$0; 7% red win \$45; 1% green lose \$10; 2% yellow lose \$15

The difficulty in identifying dominance with the second framing of the lotteries had a substantial impact on choice. All 88 participants who were offered A and B chose the dominant alternative B, but 58% of the 124 participants who were offered C and D chose the dominated alternative C. Tversky and Kahneman (1989) did not view this framing effect as reflecting a change in preference, but instead a change in how well informed subjects were about the lotteries. They conclude: "Dominance is masked by a frame in which the inferior option yields a more favorable outcome in an identified state of the world (e.g., drawing a green marble)." Kahneman (2003) generalizes this phenomenon when he explains that framing can impact information because some framings are more "perceptually accessible."

In addition to changing how well options are understood, frames can impact informativeness by changing the number of options considered. Aguiar, Boccardi, Kashaev, and Kim (2018) find evidence in a large online experiment that fewer lottery options are considered when the prizes are made harder to identify by requiring subjects to add and subtract more numbers to determine the size of the prize. In a setting where choice options are just monetary prizes, Caplin, Dean, and Martin (2011) use choice process data from a laboratory experiment to show that making prizes harder to identify in a similar way leads fewer options to be considered.

Not only can framing impact the extent to which experimental subjects understand the value of available choice options, it can also impact how well informed they are more broadly about the choice problem they face. Abeler and Jager (2015) run a lab experiment in which they keep the effective tax rate nearly identical between a simple and complex framing of the tax regime, and they find that with the complex framing, subjects choose the output level further from the payoff-maximizing levels and earn about 23 percent less on average. Ambuehl, Bernheim, and Lusardi (2014) provide experimental subjects with paired valuation tasks in which compound interest payments are presented in either a "simple frame" or a "complex frame," and they find a 10% to 15% difference in valuations within-subject without further educational interventions. Esponda and Vespa (2016) show that the re-framing of classic decision problems appears to improve the extent of contingent thinking. Building on this work and the finding from Cason and Plott (2014) that subjects fail to recognize the game form of the BDM mechanism, Martin and Munoz-Rodriguez (2019) implement a new experimental protocol in which the BDM mechanism is re-framed without changing the underlying extensive form, and this results in twice as many subjects choosing the dominant action.

There is also substantial evidence of the importance of framing on information from the field. Ericson and Starc (2016) show the impact of moving from unregulated health plan descriptions to a standardized format for presenting health care plans. Carrera and Villas-Boas (2015) study alternative ways of framing information about the relative proportion of people buying branded or generic drugs. Choi, Laibson, and Madrian (2009) provide participants in company retirement plans a simplified choice procedure called "Quick Enrollment" in which one option is framed as the default option. This framing is designed to increase how well informed they are by "reducing complexity by allowing employees to focus on evaluating a smaller subset of options (e.g., nonenrollment and the default)." Hastings and Tejeda-Ashton (2008) manipulate the framing of fees in Mexico's privatized social security system. They find that when fees are framed in terms of pesos instead of annual percentage rates, financially illiterate workers focus more on fees, which increases their price sensitivity. Carpenter, Huet-Vaughn, Matthews, Robbett, Beckett, and Jamison (2019) study choice among reloadable prepaid cards under three different framings of the card attributes: the status quo framing, a framing in line with regulatory reforms, and a new proposal designed to more strongly reduce attribute overload. They use structural estimates of individual preferences to conclude that many consumers are misinformed under the status quo framing, are able to identify (and choose) the best card with the regulatory reform framing, and are able to identify (and avoid) the worst card with the proposed framing. Based on these estimates, they find non-trivial welfare gains in the latter frames. Bhargava, Loewenstein, and Sydnor (2017) run an experiment in which the framing of health care plans is varied. Specifically, they introduce a high-clarity condition in which subjects were given a plain-language description of plan deductibles, such as: "You pay the first $500, then the plan covers remaining expenses." They find that in comparison with a condition in which subjects faced the same descriptions as employees did in the field, the fraction of subjects choosing dominated options fell from 48% to 18%.

# 3 Framework

In this section, we first introduce all of the formal objects we will need to determine which frame produces the highest welfare: the elements of each decision problem, the data sets, and the components of our information-theoretic model of framed choice. Next, we determine the information structures, decision rules, and utility functions that are consistent with the data. Finally, we identify a necessary and sufficient condition for all data-consistent utility functions to agree on which frame produces the highest welfare.

## 3.1 Framed Decision Problems and Framed Data

A decision problem contains three parts: states, actions, and prizes. The finite set of states $\Omega$ with cardinally $K > 1$ has arbitrary element $\omega$, $\nu$, or if indexing is useful, $\omega_k$. The finite set of actions $A$ with cardinally $J > 1$ has arbitrary element $a$, $b$, $c$, or $a_j$. The consequences of actions in each state are elements of a finite set of prizes $X$ with cardinally $M > 1$ has arbitrary element $x$ or $x_m$. The decision-maker (DM) receives prize $x(a, \omega) \in X$ when action $a \in A$ is chosen in state $\omega \in \Omega$.

The combination of actions $A$, states $\Omega$, and prizes $X$ can capture a wide array of economic settings. Labor market decisions can be modeled with the number of hours to work as actions, the possible income tax regimes as states, and the take-home pay received and leisure hours as prizes. Consumption decisions can be modeled with the possible bundle choices as actions, the possible prices of goods as states, and the resulting goods received and expenditure lost as prizes.

We generate framed versions of a decision problem by adding frames $f$ and $g$ to the decision problem.[8] We take frames to be elements of the choice environment that are varied by the policymaker in an effort to influence choice, but that do not restrict the prizes available to the DM or impact the DM's preferences for those prizes. Because the DM's preferences over prizes determines their ex-post welfare, the frame does not alter the DM's ex-post welfare. For example, normal framing can cause individuals to sometimes pick lower welfare health care coverage, which produces lower ex-ante welfare, but the framing does not impact the ex-post welfare obtained from the coverage itself.

In many examples, the frame is the way that decision-relevant information is presented. In the case of Hastings and Tejeda-Ashton (2008), the framing is whether fees are presented in terms of pesos or APR. In the case Bhargava, Loewenstein, and Sydnor (2017), the framing

[8] While it is possible to extend our framework to settings with more than two frames, in virtually all of the examples cited in this paper, the policy question involves a choice between one of two frames, so we restrict our attention two frames in order to simplify the exposition. In section 7, we discuss the implications of our framework for settings with more than two frames.

is whether deductibles are presented with plain language or not. In the case of Beshears, Choi, Laibson, and Madrian (2009), the framing is whether information about the mutual fund options is presented with a summary or not.

In practice, there is a fair amount of latitude in defining the actions and frames for a given economic setting. For example, imagine the choice between a low-premium, high-deductible health care plan ("Value PPO") and high-premium, low-deductible plan ("Premium PPO"). In this setting, an action is selecting a health care plan. Let action $a_P$ be choosing the Premium PPO and action $a_V$ be choosing the Value PPO. The labeling of these actions ("Value" and "Premium") might instead be considered to be part of the frame if they were varied across decision problems in a way that impacted choice.

We imagine a policymaker who has access to framed data sets $P_f(a,\omega)$ and $P_g(a,\omega)$, which specify for frames $f$ and $g$ the probability of choosing action $a$ and being in state $\omega$.[9] As with other stochastic data sets, this data set can be interpreted as watching the decision-maker face a decision infinitely often. In practice, one might estimate it from repeated but finite choice data or by looking at a population rather than an individual, as in the literature on discrete choice following McFadden (1973). For notational simplicity, we assume that all prizes can be obtained by taking some action in some state, and that in each frame, each action is chosen in some state and an action is chosen in each state.[10]

## 3.2 Information and Welfare

Frames can impact informativeness by making it easier or harder to internalize readily-available information about the state or by directing information gathering through focusing attention on certain actions or states, either rationally or due to mental gaps (Handel and Schwartzstein 2018). To capture these possibilities, we use an information-theoretic approach to model the choices made under different frames.

First, we assume that the DM starts with a strictly positive prior over states given by $\mu \in \Gamma = \Delta(\Omega)$. Given the signal structure associated with a frame, the DM receives a signal realization and forms posterior $\gamma \in \Gamma = \Delta(\Omega)$, and this process is summarized in each frame $h \in \{f, g\}$ by the information structure $\pi_h : \Omega \to \Delta(\Gamma)$, where $\Delta(\Gamma)$ is the set of probability distributions over $\Gamma$ with finite support. Given their posterior belief, the DM implements in each frame $h \in \{f, g\}$ the decision rule $\sigma_h : \Gamma \to \Delta(A)$. These decision rules maximize expected utility based on utility function $u : X \to \mathbb{R}$, which does not vary across frames.

[9] This extends the state-dependent stochastic choice (SDSC) data set introduced by Caplin and Martin (2015) to a setting with frames.

[10] Our main results would still go through without these assumptions, but doing so would require carefully specifying the support of the data set for each frame and adding technical regularity conditions, which would necessitate several pieces of additional notation while adding little additional economic insight.

With these model objects in hand, we can formally define our primary welfare criterion for evaluating frames. The *value of information* in frame $h \in \{f, g\}$ is the average ex-post utility in that frame:

$$\sum_{\omega\in\Omega} \mu(\omega) \sum_{\gamma\in\Gamma(\pi_h)} \pi_h(\gamma|\omega) \sum_{a\in A} \sigma_h(a|\gamma) u(x(a, \omega))$$

where $\mu(\omega)$ is the prior probability of state $\omega$, $\pi_h(\gamma|\omega)$ is the probability in frame $h$ of reaching posterior $\gamma$ in state $\omega$, $\Gamma(\pi_h)$ is the set of posteriors where $\pi_h(\gamma|\omega) > 0$ for some $\omega \in \Omega$, $\sigma_h(a|\gamma)$ is the probability in frame $h$ of the DM taking action $a$ when holding posterior belief $\gamma$, and $u(x(a, \omega))$ is the DM's utility from the prize received when choosing action $a$ in state $\omega$.[11]

In this setup, the prior $\mu$ does not depend on the framing of the decision problem. We make this assumption so that the starting level of informativeness is equal across frames, which closely aligns the value of information in a frame with the information structure for that frame. As a result, we can cleanly apply the Blackwell approach within our framework.[12] However, the assumption of a fixed prior is not necessary for our other results. For instance, the testable necessary and sufficient condition we derive with a fixed prior is identical to the testable necessary and sufficient condition for a framework where the prior is allowed to change with the frame.

The assumption of a fixed prior across frames is suitable for settings where the policymaker cannot condition the choice of frame on the realization of the state for technical, legal, or political reasons. For example, the policymaker may not be able to adjust the frame to account for variation in the state across time if altering the presentation of decision-relevant information is too costly. With large-scale choice settings, such as the choice of Medicare prescription plans, the financial outlay of printing and mailing new information packets to all Medicare participants could make regular changes in those packets prohibitively expense. In addition, the policymaker may be restricted to providing the same frame for all individuals regardless of the state if variation in frames (outside of limited A/B testing) is viewed as discriminatory or unfair. However, by fixing the prior, our framework does not capture cases where the policymaker's choice of frame directly signals to the decision maker something about the distribution of states. For example, it is conceivable that a policymaker could use the frame to signal that one action is more likely to provide a high utility prize, just as information can be communicated by firms through their choice of product lines (Kamenica 2008).

[11] An alternative way to define the value of information is as the improvement over the utility from taking actions at prior beliefs (see Lara and Gossner 2020 and Frankel and Kamenica 2018). Since the prior is fixed across frames in our framework, this definition would provide the same relative welfare assessments.

[12] A fixed prior across experiments is also assumed in simplified proofs of Blackwell's theorem (see Crémer 1982; Leshno and Spector 1992; Perez-Richet 2017; Oliveira 2018).

## 3.3 Data Consistency

The policymaker wants to rank frames $f$ and $g$ based on the value of information, but many important components of the value of information are unobservable to the policymaker: the prior $\mu$, the information structures $\pi_f$ and $\pi_g$, the decision rules $\sigma_f$ and $\sigma_g$, and the utility function $u$.

To bridge the gap between these unobservables and the data, we define and characterize a representation that places three restrictions on the unobservable components of the value of information. First, the prior $\mu$, information structures $\pi_f$ and $\pi_g$, and decision rules $\sigma_f$ and $\sigma_g$ generate specific predictions about the probability of choosing action $a$ in state $\omega$ in the corresponding frames, and these predictions must match the observed data. Second, the information structures $\pi_f$ and $\pi_g$ must be Bayes plausible. Third, expected utility is maximized given the information structures $\pi_f$ and $\pi_g$, decision rules $\sigma_f$ and $\sigma_g$, and utility function $u$.

**Definition 1.** *Utility function $u$, prior $\mu$, information structures $\pi_f$ and $\pi_g$, and decision rules $\sigma_f$ and $\sigma_g$ provide a Bayesian Expected Utility representation (**BEU**) of the data sets $P_f$ and $P_g$ if they satisfy:*

1. ***Data Matching**: For all $h \in \{f, g\}$, $a \in A$, and $\omega \in \Omega$,*

$$P_h(a, \omega) = \mu(\omega) \sum_{\gamma \in \Gamma(\pi_h)} \pi_h(\gamma|\omega)\sigma_h(a|\gamma)$$

2. ***Bayesian Updating**: For all $\omega \in \Omega$,*

$$\gamma(\omega) = \frac{\mu(\omega)\pi_h(\gamma|\omega)}{\sum_{\nu \in \Omega} \mu(\nu)\pi_h(\gamma|\nu)} \quad \text{for all } \gamma \in \Gamma(\pi_h)$$

   *where $\gamma(\omega)$ is the probability of state $\omega$ in posterior $\gamma$.*

3. ***Maximization**: For all $h \in \{f, g\}$, $a, b \in A$, and $\gamma \in \Gamma(\pi_h)$ s.t. $\sigma_h(a|\gamma) > 0$,*

$$\sum_{\omega \in \Omega} \gamma(\omega)u(x(a, \omega)) \geq \sum_{\omega \in \Omega} \gamma(\omega)u(x(b, \omega))$$

   *with the inequality strict for some $h \in \{f, g\}$, $a, b \in A$, and $\gamma \in \Gamma(\pi_h)$ s.t. $\sigma_h(a|\gamma) > 0$.*

We say that utility function $u$ *rationalizes* data sets $P_f$ and $P_g$ if there exists a prior $\mu$, Bayes plausible information structures $\pi_f$ and $\pi_g$, and decision rules $\sigma_f$ and $\sigma_g$ that together provide a BEU of the data, and we say that $P_f$ and $P_g$ are *rationalizable* if there exists a $u$ that rationalizes them. In a BEU, one Maximization inequality is required to hold strictly to prevent $P_f$ and $P_g$ from being trivially rationalized by a utility function that gives the same utility to all prizes.

To identify the set of all rationalizing utility functions, we extend the approach of Caplin and Martin (2015), who identify a necessary and sufficient condition for choosing optimally given private information.

**Condition 1** (No Improving Action Switches in any Frame (**NIAS-F**))**.** *Utility function $u$ satisfies NIAS-F if*

$$\sum_{\omega \in \Omega} P_h(a, \omega) u(x(a, \omega)) \geq \sum_{\omega \in \Omega} P_h(a, \omega) u(x(b, \omega))$$

*for all $h \in \{f, g\}$ and $a, b \in A$, and the inequality is strict for some $h \in \{f, g\}$ and $a, b \in A$.*

Each NIAS-F inequality can be interpreted as it being better not to make a "wholesale" switch from taking action $a$ to taking action $b$. As the following theorem indicates, the set of utility functions that satisfy NIAS-F is the set of all rationalizing utility functions.

**Theorem 1.** *Utility function $u$ rationalizes data sets $P_f$ and $P_g$ if and only if it satisfies NIAS-F.*

*Proof.* See Appendix. □

The necessity of NIAS-F is immediate. To show the sufficiency of NIAS-F, we construct Bayes plausible information structures that the utility function $u$ rationalizes and that could have predicted the data. These information structures put positive weight on a single posterior for each action: the *revealed posterior* for that action. Formally, the revealed posterior $\gamma_h^a$ for action $a$ in frame $h$ is defined as the distribution of states when that action is taken:

$$\gamma_h^a(\omega) = \frac{P_h(a, \omega)}{\sum_{\nu \in \Omega} P_h(a, \nu)} = \frac{P_h(a, \omega)}{P_h(a)}$$

for each $\omega \in \Omega$. For a Bayesian DM, the revealed posterior $\gamma_h^a$ represents the average posterior belief held when taking action $a$ in frame $h$. Because all revealed posteriors fall in the convex hull of the actual posteriors held by the DM, the Bayes plausible information structure we construct using revealed posteriors is weakly less informative than any information structure consistent with the data. As a result, the information structure based on revealed posteriors represents the minimal level of informativeness the DM can have given the data. Thus, revealed posteriors provide a lower bound on how well informed the DM is in a frame, which we will leverage later when applying the Blackwell approach within our framework.

NIAS-F plays two roles in the analysis that follows. First, when there is no utility function that satisfies NIAS-F, then the data sets $P_f$ and $P_g$ cannot be rationalized, which means the value of information for DMs cannot be assessed using our model. Thus, we will require NIAS-F to hold as a precondition for making a welfare comparison between frames. If the policymaker wants to use our framework to rank frames, but the data does not satisfy NIAS-F, then one possible way forward is to determine the maximal portion of the data that satisfies NIAS-F and perform welfare analysis using only that portion of the data, as proposed by Apesteguia and Ballester (2020).

Second, because NIAS-F provides the set of all rationalizing utility functions, it will play a key role in identifying when the value of information is higher in one frame for all rationalizing utility functions.

## 3.4 Identifying Higher Welfare Frames

We define the *revealed value of information* for frame $h \in \{f, g\}$ as the average ex-post utility in that frame, which is given by

$$\sum_{a \in A} \sum_{\omega \in \Omega} P_h(a, \omega) u(x(a, \omega))$$

The revealed value of information in a frame is tightly connected to the actual value of information in that frame. Because a BEU must satisfy Data Matching, if the revealed value of information is higher in frame $h$ for a given utility function $u$, then the value of information is higher in frame $h$ for every BEU that includes $u$.

To provide an ordering of frames based on the revealed value of information that holds for all rationalizing utility functions, we say that frame $f$ has been *revealed to have a robustly higher value of information* than frame $g$ (denoted as $f \succsim_W g$) if the revealed value of information is at least as high in frame $f$ than frame $g$ for every utility function $u$ that satisfies NIAS-F. Formally, this means that $f \succsim_W g$ if

$$\sum_{a \in A} \sum_{\omega \in \Omega} P_f(a, \omega) u(x(a, \omega)) \geq \sum_{a \in A} \sum_{\omega \in \Omega} P_g(a, \omega) u(x(a, \omega))$$

for every $u$ such that for every $a, b \in A$,

$$\sum_{\omega \in \Omega} P_f(a, \omega) u(x(a, \omega)) \geq \sum_{\omega \in \Omega} P_f(a, \omega) u(x(b, \omega))$$

and

$$\sum_{\omega \in \Omega} P_g(a, \omega) u(x(a, \omega)) \geq \sum_{\omega \in \Omega} P_g(a, \omega) u(x(b, \omega))$$

with one inequality strict.

We produce a testable necessary and sufficient condition for $f \succsim_W g$ by showing that these systems of linear inequalities have a simple geometric structure in the space of all

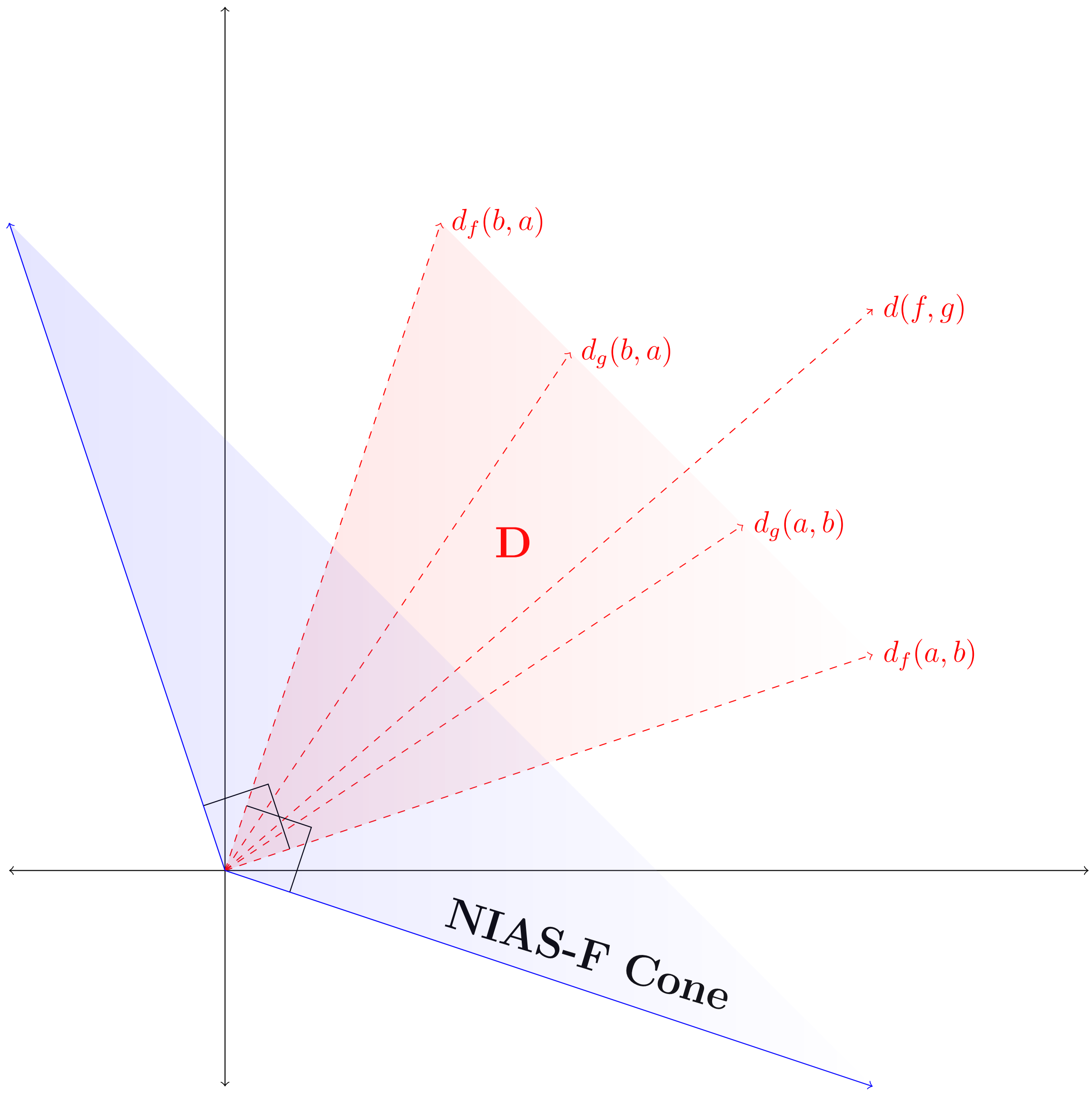


Figure 1: Illustration of the geometric structure of ranking frames for a decision problem with 2 actions and 2 prizes.

possible prize lotteries ($\mathbb{R}^M$), which is illustrated for a two prize decision problem in Figure 1. Because of this structure, we can reduce $f \succsim_W g$ to a single system of linear equations.

First, each NIAS-F inequality can be represented as an M-dimensional vector $\vec{d}_h(a, b)$ that gives the prize lottery gained from not making a wholesale switch from action $a$ to action $b$ in frame $h$. In other words, the additional probability of receiving each prize from not making this wholesale switch. The convex cone $D$ formed by all NIAS-F inequalities is

$$D = \{\alpha_1 \vec{d}_{h_1}(a_1, b_1) + ... + \alpha_N \vec{d}_{h_N}(a_N, b_N) | \alpha_n \in \mathbb{R}_+, h_n \in \{f, g\}, a_n, b_n \in A\}$$

A utility function can be represented as an M-dimensional vector $\vec{u}$ where element $m$ gives the utility of prize $x_m$. A utility vector $\vec{u}$ satisfies NIAS-F if $\vec{d} \bullet \vec{u} \geq 0$ for every vector $\vec{d} \in D$. We call the convex cone formed by all $\vec{u}$ that satisfy NIAS-F the *NIAS-F Cone*.

Let $\vec{d}(f, g)$ be an M-dimensional vector that gives the prize lottery gained from being in frame $f$ instead of frame $g$. In other words, the additional probability of receiving each prize from being in frame $f$. Frame $f$ being revealed to have a robustly higher value of information ($f \succsim_W g$) is equivalent to $\vec{d}(f, g) \bullet \vec{u} \geq 0$ for all $\vec{u}$ in the NIAS-F Cone. Thus, $f \succsim_W g$ if and only if the vector $\vec{d}(f, g)$ is in $D$ because a vector is in $D$ if and only if it has a non-negative dot product with all $\vec{u}$ in the NIAS-F Cone.

Finally, because $\vec{d}(f, g)$ is in $D$ if and only if it is a non-negative weighted average of vectors in $D$, a necessary and sufficient condition for $f \succsim_W g$ corresponds to the prize lottery gained from being in frame $f$ being a non-negative weighted average of the prize lotteries gained from not making wholesale switches from any action in either frame. This condition, which we call *Frame Improvement without Action Switches (FIAS)* for frame $f$, is defined for a weighting function $t_h : A \times A \to \mathbb{R}_+$, which provides these non-negative weights.

**Condition 2** (Frame Improvement without Actions Switches (**FIAS**))**.** *Weighting function $t_h$ satisfies FIAS for frame $f$ if*

$$\sum_{h \in \{f,g\}} \sum_{a \in A} \sum_{b \in A} \sum_{\omega \in \Omega} P_h(a, \omega)(\boldsymbol{1}_{\{x(a,\omega)=x\}} - \boldsymbol{1}_{\{x(b,\omega)=x\}}) t_h(a, b)$$
$$= \sum_{a \in A} \sum_{\omega \in \Omega} (P_f(a, \omega) - P_g(a, \omega)) \boldsymbol{1}_{\{x(a,\omega)=x\}}$$

*for every $x \in X$.*

The following lemma formally shows that FIAS provides a necessary and sufficient condition for rationalizable data sets to reveal that one frame has a robustly higher value of information. After restating NIAS-F and FIAS in terms of matrix multiplication, the proof of this lemma follows as a direct consequence of Farkas lemma (Farkas 1902).

**Lemma 1.** *Given that NIAS-F is satisfied, $f \succsim_W g$ if and only if there exists a weighting function $t_h$ that satisfies FIAS for frame $f$.*

*Proof.* See Appendix. $\square$

This lemma has an economic interpretation in terms of preferences over prize lotteries. FIAS states that the difference in the prize lotteries offered by the frames can be represented as the difference in two compound lotteries: one composed of the prize lotteries from taking each action $a$ in frame $h$ and the other composed of prize lotteries from taking each action $b$ with the same probability as $a$. These compound lotteries have the same weights for all $h \in \{f, g\}$ and $a, b \in A$, which are given by a normalized version of $t_h$. Because NIAS-F is satisfied, there exists a preference relation over lotteries such that every element of one compound lottery is weakly preferred to every element in the other compound lottery and strictly preferred for at least one element. Since all elements of the two compound lotteries are preference ordered, the compound lotteries are also preference ordered, which means the prize lotteries given by each frame are as well.

### 3.5 Testability

Determining whether or not there exists a $t_h$ that satisfies FIAS corresponds to determining whether there is a solution to a system of linear equations, so it is simple to check if frames are welfare ranked. We provide MATLAB computer programs that determine whether a solution to this linear system exists for a given set of data.[13]

Because NIAS-F must be satisfied as a precondition for ranking frames, it must be checked as well. Fortunately, determining whether there exists a $u$ that satisfies NIAS-F corresponds to determining whether there is a solution to a system of linear inequalities, which is also simple to check in practice. A potential wrinkle is that one inequality must hold strictly for NIAS-F to be satisfied, and in principle, any inequality could hold strictly.

However, the following lemma indicates which NIAS-F inequalities must hold with equality and which must hold strictly. In words, it states that an NIAS-F inequality in frame $h$ for $a, b \in A$ holds with equality if and only if the prize lottery representing that NIAS-F inequality can be expressed as a non-positive combination of the prize lotteries representing other NIAS-F inequalities.

**Lemma 2.** *For every $u$ that satisfies NIAS-F,*

$$\sum_{\omega \in \Omega} P_h(a, \omega) u(x(a, \omega)) = \sum_{\omega \in \Omega} P_h(a, \omega) u(x(b, \omega))$$

[13] Programs available at https://github.com/danieljosephmartin/framinginformationwelfare.git.

*for $a, b \in A$ if and only if there exists a collection of $N$ triples with generic element $(h_n, a_n, b_n)$ having $h_1, ..., h_N \in \{f, g\}$, $a_1, ..., a_N \in A$, $b_1, ..., b_N \in A$, and $(h_n, a_n, b_n) \neq (h, a, b)$ and non-positive weights $w_1, ..., w_N$ such that for every $x \in X$*

$$\begin{aligned} &\sum_{\omega\in\Omega} P_h(a,\omega)(\boldsymbol{1}_{\{x(a,\omega)=x\}} - \boldsymbol{1}_{\{x(b,\omega)=x\}}) \\ &= \sum_{n=1}^{N} w_n * \left( \sum_{\omega\in\Omega} P_{h_n}(a_n,\omega)(\boldsymbol{1}_{\{x(a_n,\omega)=x\}} - \boldsymbol{1}_{\{x(b_n,\omega)=x\}}) \right) \end{aligned} \tag{1}$$

*Proof.* See Appendix. □

Since this can be expressed as a linear system of equations, NIAS-F can easily be restated as a system of linear equations and strict linear inequalities. We also provide MATLAB computer programs that determine whether a solution exists to these linear systems.

Finally, as mentioned previously, there are settings where some options are clearly dominant, and NIAS-F and FIAS can be easily amended to account for these additional restrictions. Say for example, that prize $x_1$ clearly dominates prize $x_2$. This restriction on utility can be incorporated into NIAS-F by generating an additional linear inequality given by

$$\sum_{x\in X} (\mathbf{1}_{x=x_1} - \mathbf{1}_{x=x_2})\, u(x) \geq 0$$

Clearly, this restriction on the set of admissible utility functions can only reduce the proportion of data sets where there exists a $u$ that satisfies NIAS-F.

Although FIAS in not expressed in terms of utility, the dominance of prize $x_1$ over prize $x_2$ can be incorporated into FIAS for frame $f$ by requiring that, in addition to the weighting function $t_h$, there is exists a non-negative $t$ that solves

$$\begin{aligned} & \sum_{h\in\{f,g\}} \sum_{a\in A} \sum_{b\in A} \sum_{\omega\in\Omega} P_h(a,\omega)(\mathbf{1}_{\{x(a,\omega)=x\}} - \mathbf{1}_{\{x(b,\omega)=x\}}) t_h(a,b) \\ + \quad & (\mathbf{1}_{x=x_1} - \mathbf{1}_{x=x_2})\, t \\ = \quad & \sum_{a\in A} \sum_{\omega\in\Omega} (P_f(a,\omega) - P_g(a,\omega)) \mathbf{1}_{\{x(a,\omega)=x\}} \end{aligned}$$

for every $x \in X$. If $t$ is equal zero, this reduces to the requirement for FIAS, so this addition can only increase the proportion of data sets where there exists a weighting function $t_h$ that satisfies FIAS. Sensibly, knowledge about dominance improves our ability to rank frames according to their welfare.

Since accounting for the clear dominance of one prize over another can only reduce the proportion of data sets that satisfy NIAS-F and only increase the proportion of data sets that satisfy FIAS, the net effect of adding such restrictions on the proportion of data sets where frames can be welfare ranked is ambiguous.

# 4 Blackwell Within Our Framework

In this section, we generate an informativeness order by applying the Blackwell approach within our framework, and this provides a second testable condition for a frame to be revealed to have a robustly higher value of information.

## 4.1 Revealed Experiments and Better Informed Actions

Informativeness orders are typically generated using the information structures and decision rules of DMs, but since these are unobservable to the policymaker, we instead generate an informativeness order using the distribution of actions in each state, which we call the *revealed experiment* for frame $h$.[14] Revealed experiments can be interpreted as the outcome of a signal structure that generates a signal at which each action is optimal, as in the "action recommendation" approach used in the information design literature (see Bergemann and Morris 2017). Thus, the revealed experiment reflects the minimal level of informativeness that the DM can have given the distribution of actions in each state. Though the DM's actual level of informativeness is indeterminate, we show that frames being ordered according to this summary statistic of their informativeness is sufficient for frames to be welfare ordered.

We say frame $f$ is *revealed to have better informed actions* than frame $g$ (denoted $f \succsim_B g$) if the revealed experiment for frame $g$ is a garbling of the revealed experiment for frame $f$. Formally, this means there exists a function $S : A \times A \to R_+$ with $\sum_{a \in A} S(c,a) = 1$ for all $c \in A$ such that for all $a \in A$ and $\omega \in \Omega$,

$$\frac{P_g(a,\omega)}{P_g(\omega)} = \sum_{c \in A} \frac{P_f(c,\omega)}{P_f(\omega)} S(c,a)$$

The following lemma establishes that for any arbitrary decision problem, when NIAS-F is satisfied, a frame being revealed to have better informed actions is sufficient for it to be revealed to have a robustly higher value of information.

**Lemma 3.** *Given that NIAS-F is satisfied, $f \succsim_B g$ implies $f \succsim_W g$.*

*Proof.* See Appendix. □

Blackwell and Girshick (1954) show that if one experiment is a garbling of another experiment then the posteriors generated by one experiment are in the convex hull of the posteriors

[14] We know of no other papers that measure informativeness using the distribution of actions in each state. If we interpret the DM's action as a signal of their signal, then a related paper is Gossner and Tomala (2006), who study the evolution of beliefs about the DM's predictions of the next signal for an observer who only observes signals of the DM's signals.

generated by the other experiment. The following lemma establishes the same for revealed experiments and revealed posteriors.

**Lemma 4.** *Given that NIAS-F is satisfied, if $f \succsim_B g$, then there exists a function $T : A \times A \to R_+$ with $\sum_{c \in A} T(c,a) = 1$ for all $a \in A$ such that for all $a \in A$ and $\omega \in \Omega$,*

$$\gamma_g^a(\omega) = \sum_{c \in A} \gamma_f^c(\omega) T(c,a)$$

*Proof.* See Appendix. □

From this lemma, it follows if the revealed posteriors in frame $g$ are not in the convex hull of the revealed posteriors in frame $f$, then frame $f$ cannot be revealed to have better informed actions. In Section 5, we will use this to identify cases where neither frame is revealed to have better informed actions.

## 4.2 An Example of Better Informed Actions

Consider again the choice between a low-premium, high-deductible health care plan ("Value PPO") and high-premium, low-deductible plan ("Premium PPO") that can be framed using plain language descriptions of deductibles. We will now provide example data that reveal one frame has better informed actions.[15]

In this example, when the Premium PPO has the same doctor availability, the Value PPO is chosen 80% of the time with normal framing and 90% of the time with plain language framing. When the Premium PPO has better doctor availability, the Value PPO is chosen 20% of the time with normal framing and 70% of the time with plain language framing. We represent the corresponding data sets for normal framing ($P_N$) and plain language framing ($P_{PL}$) as matrices where the actions of choosing the Premium PPO ($a_P$) or Value PPO ($a_V$) are given in the rows and states of same doctor availability ($\omega_S$) and better doctor availability ($\omega_B$) are given in the columns:

$$P_N = \begin{array}{cc} \omega_S & \omega_B \\ \end{array} \begin{pmatrix} \frac{10}{100} & \frac{40}{100} \\ \frac{40}{100} & \frac{10}{100} \end{pmatrix} \begin{array}{c} a_P \\ a_V \end{array} \quad \& \quad P_{PL} = \begin{array}{cc} \omega_S & \omega_B \\ \end{array} \begin{pmatrix} \frac{5}{100} & \frac{15}{100} \\ \frac{45}{100} & \frac{35}{100} \end{pmatrix} \begin{array}{c} a_P \\ a_V \end{array}$$

The NIAS-F inequalities for frame $N$ reveal there is a tradeoff that individuals face between cost effectiveness and doctor availability: the Value PPO is preferred when plans

[15] We analyze this choice setting in more detail and with more generality in Section 6.

have the same doctor availability, but the Premium PPO is preferred when it has better doctor availability. Formally,

$$\frac{10}{100}u(x_{PS}) + \frac{40}{100}u(x_{PB}) \;\; \geq \;\; \frac{50}{100}u(x_V)$$

and

$$\frac{50}{100}u(x_V) \;\; \geq \;\; \frac{40}{100}u(x_{PS}) + \frac{10}{100}u(x_{PB})$$

Together, these directly imply $u(x_{PB}) \geq u(x_{PS})$. We also know that $u(x_V) \geq u(x_{PS})$ because if one plan was always preferred, there would be no reason to ever choose $a_P$ if there was any uncertainty about the state.

Because of this tradeoff, it is less clear which framing produces higher welfare. However, by examining the revealed posteriors, we can see why normal framing is revealed to have better informed actions, and hence a robustly higher value of information. The revealed posteriors in frame $N$ are

$$\gamma_N^{a_P} = \begin{pmatrix} \frac{10}{50} \\ \frac{40}{50} \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix} = \begin{pmatrix} 20\% \\ 80\% \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix} \quad \& \quad \gamma_N^{a_V} = \begin{pmatrix} \frac{40}{50} \\ \frac{10}{50} \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix} = \begin{pmatrix} 80\% \\ 20\% \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix}$$

and the revealed posteriors in frame $PL$ are

$$\gamma_{PL}^{a_P} = \begin{pmatrix} \frac{5}{20} \\ \frac{15}{20} \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix} = \begin{pmatrix} 25\% \\ 75\% \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix} \quad \& \quad \gamma_{PL}^{a_V} = \begin{pmatrix} \frac{45}{80} \\ \frac{35}{80} \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix} = \begin{pmatrix} 56.25\% \\ 43.75\% \end{pmatrix} \begin{matrix} \omega_S \\ \omega_B \end{matrix}$$

These revealed posteriors reveal that on average, there is better information about the state when actions are taken in frame $N$.

What kind of utility function and signal structure could have produced this data and these revealed posteriors? Assume that the DM's utility function is

$$u(x_{PS}) = \frac{3}{2}u(x_V) = 2u(x_{PB})$$

so is indifferent between choosing either plan if a fully uninformative signal is received. Now imagine that the decision maker pays no attention and chooses randomly 40% of the time in each state with normal framing. Also imagine that the plain language framing reduces full inattention by 50%, but makes the deductibles dimension more salient, so 60% of the time the decision maker only pays attention the high deductible dimension with the plain language framing. In combination with the utility of each prize, this produces the data and revealed posteriors given above.

# 5 Welfare Ranking Versus Informativeness Ranking

In this section, we show that a frame need not be revealed to have better informed actions for it to be revealed to have a robustly higher value of information. In other words, there is a gap between the welfare order ($\succsim_W$) and the informativeness order ($\succsim_B$). This is easy to show in trivial cases such as when there are just two possible prizes or all actions give the same prize in some state. However, it can also occur outside of these trivial cases.

For three standard classes of decision problems, we show there exist data sets where a frame is revealed to have a robustly higher value of information even though neither frame is revealed to have better informed actions. This includes decision problems where state-dependent prizes are obtained when the action matches the state, where every action yields a distinct prize in every state, and where each action has two types of outcomes. We also assess whether such data sets are "rare" or "common" by estimating the proportion of data sets where a frame is revealed to have a robustly higher value of information even though neither frame is revealed to have better informed actions.[16] In addition, for each decision problem we examine one such data set in detail.

## 5.1 Tracking Problems

We first consider "tracking" decision problems in which the decision maker receives a state-specific prize $x_k$ if their action matches state $\omega_k$ and prize $x_B$ if they fail to match the action to the state. For this class of decision problems, the map $x(a, \omega)$ between actions, states, and prizes is given by

$$x(a_j, \omega_k) = \begin{cases} x_k & j = k \\ x_B & j \neq k \end{cases}$$

For the 3 action and 3 state version of this tracking problem, the map between actions, states, and prizes can be represented as a matrix where actions $a_1$ to $a_3$ are given in the rows and states $\omega_1$ to $\omega_3$ are given in the columns:

$$\begin{array}{cc} \begin{matrix} \omega_1 & \omega_2 & \omega_3 \end{matrix} & \\ \begin{pmatrix} x_1 & x_B & x_B \\ x_B & x_2 & x_B \\ x_B & x_B & x_3 \end{pmatrix} & \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} \end{array}$$

Using the MATLAB programs provided, we estimate that for this 3 action and 3 state tracking problem, approximately 65% of rationalizable data sets $P_f$ and $P_g$ reveal that one

[16]We sample over the space of all possible data sets by taking 100,000 uniform random draws of the prior over states and 100,000 uniform random draws for the distribution of actions in each frame.

frame has a robustly higher value of information even though neither frame is revealed to have better informed actions.

In this section, we describe such a $P_f$ and $P_g$ in detail.[17] We represent these data sets as matrices where actions $a_1$ to $a_3$ are given in the rows and states $\omega_1$ to $\omega_3$ are given in the columns:

$$P_f = \begin{array}{c} \begin{array}{ccc} \omega_1 & \omega_2 & \omega_3 \end{array} \\ \begin{pmatrix} \frac{20}{100} & 0 & 0 \\ 0 & \frac{22}{100} & \frac{18}{100} \\ 0 & \frac{18}{100} & \frac{22}{100} \end{pmatrix} \end{array} \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} \quad \& \quad P_g = \begin{array}{c} \begin{array}{ccc} \omega_1 & \omega_2 & \omega_3 \end{array} \\ \begin{pmatrix} \frac{10}{100} & \frac{20}{100} & \frac{20}{100} \\ \frac{5}{100} & \frac{20}{100} & 0 \\ \frac{5}{100} & 0 & \frac{20}{100} \end{pmatrix} \end{array} \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array}$$

In the analysis that follows, we will show that these data sets reveal the prizes from matching actions to states are "good" and the prize from not matching actions is "bad". Formally, this means that for all utility functions that rationalize $P_f$ and $P_g$, $u(x_k) \geq u(x_B)$ for all $k \in \{1, 2, 3\}$. Given this, frame $f$ will be revealed to have a robustly higher value of information because in that frame the DM matches actions to states more often in every state.

However, while the DM will be revealed to be perfectly informed when taking action $a_1$ in frame $f$, the DM will be revealed to be somewhat better informed about the matching state when taking actions $a_2$ and $a_3$ in frame $g$. As a result, the DM will not be revealed to have better informed actions in either frame. In terms of signal structures, it is as if the DM has a signal structure in frame $f$ that is perfectly informative about whether the state is $\omega_1$, but is not as informative about the other states as the signal structure in frame $g$.

Without loss of generality we set $u(x_B) = 0$, so we can compute the revealed value of information in frame $f$ as

$$\frac{20}{100}u(x_0) + \frac{22}{100}u(x_1) + \frac{22}{100}u(x_2)$$

and in frame $g$ as

$$\frac{10}{100}u(x_0) + \frac{20}{100}u(x_1) + \frac{20}{100}u(x_2)$$

Clearly, if $u(x_1)$, $u(x_2)$, or $u(x_3)$ are revealed to be non-negative for all rationalizing utility functions, then frame $f$ is revealed to have a robustly higher value of information.

The fact that these utilities are non-negative can established through the NIAS-F inequalities in frame $f$. The NIAS-F inequality in frame $f$ for $a_1$ chosen over action $a_2$ gives

[17] This example can be generalized to arbitrarily many actions with at least as many states as actions.

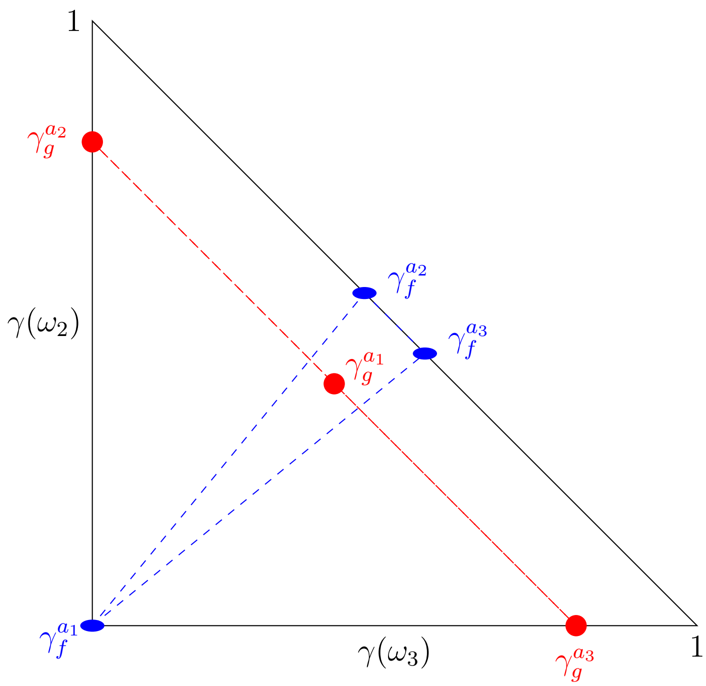


Figure 2: Revealed posteriors for tracking example. Blue ovals give revealed posteriors for frame $f$, red dots give revealed posteriors for frame $g$, and dashed lines provided the convex hull of the revealed posteriors for each frame.

$u(x_1) \geq 0$ because

$$\begin{aligned} \sum_{\omega \in \Omega} P_f(a_1, \omega) u(x(a_1, \omega)) &\geq \sum_{\omega \in \Omega} P_f(a_1, \omega) u(x(a_2, \omega)) \\ \frac{20}{100} u(x_1) &\geq \frac{20}{100} u(x_B) = 0 \end{aligned}$$

Likewise, the NIAS-F inequality in frame $f$ for $a_2$ chosen over action $a_1$ gives $u(x_2) \geq 0$, and the NIAS-F inequality for $a_3$ chosen over action $a_1$ gives $u(x_3) \geq 0$.

While the DM is revealed to have a robustly higher value of information in frame $f$, the DM is also revealed to be less informed about the matching state when taking actions $a_2$ and $a_3$ in frame $f$. The revealed posteriors in frame $f$ are

$$\gamma_f^{a_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix} \quad \& \quad \gamma_f^{a_2} = \begin{pmatrix} 0 \\ \frac{22}{40} \\ \frac{18}{40} \end{pmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix} \quad \& \quad \gamma_f^{a_3} = \begin{pmatrix} 0 \\ \frac{18}{40} \\ \frac{22}{40} \end{pmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix}$$

and the revealed posteriors in frame $g$ are

$$\gamma_g^{a_1} = \begin{array}{c} \begin{pmatrix} \frac{10}{50} \\ \frac{20}{50} \\ \frac{20}{50} \end{pmatrix} \end{array} \begin{array}{c} \omega_1 \\ \omega_2 \\ \omega_3 \end{array} \quad \& \quad \gamma_g^{a_2} = \begin{pmatrix} \frac{5}{25} \\ \frac{20}{25} \\ 0 \end{pmatrix} \begin{array}{c} \omega_1 \\ \omega_2 \\ \omega_3 \end{array} \quad \& \quad \gamma_g^{a_3} = \begin{pmatrix} \frac{5}{25} \\ 0 \\ \frac{20}{25} \end{pmatrix} \begin{array}{c} \omega_1 \\ \omega_2 \\ \omega_3 \end{array}$$

These revealed posteriors are presented in Figure 2, which shows the probability of just two states ($\omega_2$ and $\omega_3$) as the third is constrained to be the remainder. For example, the revealed posterior for action $a_1$ in frame $f$ gives a likelihood of zero to these states, so it puts all likelihood on state $\omega_1$.

As a consequence of Lemma 4, if the revealed posteriors for neither frame fall within the convex hull of the revealed posteriors of the other frame, then neither frame can be revealed to have better informed actions. Clearly, this is true for this example. The revealed posterior for action $a_1$ in frame $f$, which is perfectly informed, cannot be expressed as a convex combination of the less than fully informed revealed posteriors in frame $g$ and the revealed posteriors for actions $a_2$ and $a_3$ in frame $g$ cannot be expressed as a convex combination of revealed posteriors in frame $f$ as they are better informed about the matching states.

## 5.2 Problems with Distinct Prizes

Next, we consider a common class of decision problems in which every action yields a distinct prize in every state, so that $x(a, \omega) \neq x(b, \nu)$ if $a \neq b$ or $\omega \neq \nu$. We estimate that for the 3 action and 3 state version of this decision problem, roughly 6.5% of rationalizable data sets $P_f$ and $P_g$ reveal that one frame has a robustly higher value of information even though neither frame is revealed to have better informed actions.

One example of this is given by

$$P_f = \begin{array}{c} \begin{array}{ccc} \omega_1 & \omega_2 & \omega_3 \end{array} \\ \begin{pmatrix} \frac{24}{72} & 0 & 0 \\ 0 & \frac{16}{72} & \frac{8}{72} \\ 0 & \frac{8}{72} & \frac{16}{72} \end{pmatrix} \end{array} \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \quad \& \quad P_g = \begin{array}{c} \begin{array}{ccc} \omega_1 & \omega_2 & \omega_3 \end{array} \\ \begin{pmatrix} \frac{12}{72} & \frac{6}{72} & \frac{6}{72} \\ \frac{6}{72} & \frac{5}{72} & \frac{13}{72} \\ \frac{6}{72} & \frac{13}{72} & \frac{5}{72} \end{pmatrix} \end{array} \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array}$$

Like the tracking example, these data sets will reveal that the DM prefers the prize obtained when choosing action $a_1$ when the state is $\omega_1$, prefers the prizes obtained when choosing $a_2$ and $a_3$ in the other states, and is perfectly informed when taking action $a_1$. Once again, it is as if in frame $f$ the DM gets a signal realization that is perfectly informative of whether the state is $\omega_1$, so knows to take action $a_1$ if the state is $\omega_1$ and not to choose action $a_1$ otherwise.

However, unlike the tracking example, this $P_f$ and $P_g$ reveal that the utility obtained from taking actions $a_2$ and $a_3$ is the same in every state.[18] This follows from the fact that the NIAS-F inequalities for $a_2$ chosen over $a_3$ and $a_3$ chosen over $a_2$ hold with equality in both frames, which is a consequence of Lemma 2. Lemma 2 states that an NIAS-F inequality is equal to zero if and only if the prize lottery representing that NIAS-F inequality can be expressed as a non-positive combination of the prize lotteries representing other NIAS-F inequalities.[19] For example, the prize lottery representing the NIAS-F inequality for $a_2$ chosen over $a_3$ in frame $f$ is $\frac{16}{72}(u(a_2, \omega_2) - u(a_3, \omega_2))$ and $\frac{8}{72}(u(a_2, \omega_3) - u(a_3, \omega_3))$.[20] The negative of this can be obtained by simply adding together the prize lotteries from the NIAS-F inequalities for $a_3$ chosen over $a_2$ in frame $f$, for $a_2$ chosen over $a_3$ in frame $g$, and for $a_3$ chosen over $a_2$ in frame $g$.

Given that the NIAS-F inequalities for $a_2$ chosen over $a_3$ and $a_3$ chosen over $a_2$ hold with equality in frame $f$, the utility differences between $a_2$ and $a_3$ in $\omega_2$ and the utility differences between $a_2$ and $a_3$ in $\omega_3$ are both equal to 0 because those NIAS-F inequalities say

$$\frac{16}{72}\left(u(a_2, \omega_2) - u(a_3, \omega_2)\right) + \frac{8}{72}\left(u(a_2, \omega_3) - u(a_3, \omega_3)\right) = 0$$

and

$$-\frac{8}{72}\left(u(a_2, \omega_2) - u(a_3, \omega_2)\right) - \frac{16}{72}\left(u(a_2, \omega_3) - u(a_3, \omega_3)\right) = 0$$

which is only possible if $u(a_2, \omega_2) - u(a_3, \omega_2) = 0$ and $u(a_2, \omega_3) - u(a_3, \omega_3) = 0$. Likewise, given that the NIAS-F inequalities for $a_2$ chosen over $a_3$ and $a_3$ chosen over $a_2$ hold with equality in frame $g$, the utility difference between $a_2$ and $a_3$ in $\omega_1$ is also equal to 0. Thus, the utility from taking $a_2$ is the same as the utility from taking $a_3$ in every state.

Given this, the revealed value of information is higher in frame $f$ if

$$\begin{aligned} & \frac{12}{72}(u(a_1, \omega_1) - u(a_2, \omega_1)) \\ + \quad & \frac{6}{72}(u(a_2, \omega_2) - u(a_1, \omega_2) + u(a_2, \omega_3) - u(a_1, \omega_3)) \geq 0 \end{aligned}$$

To show that this holds, we first note that $u(a_1, \omega_1) \geq u(a_2, \omega_1)$ (the DM preferring to take action $a_1$ in state $\omega_1$) follows directly from the NIAS-F inequality for $a_1$ chosen over $a_2$ in frame $f$. Second, because $a_2$ and $a_3$ give the same utility in every state, the NIAS-F

[18] This example can also be generalized to any version of this problem with arbitrarily many actions and at least as many states as actions.

[19] The equality of NIAS-F inequalities is a fairly common occurrence in this class of decision problems. We estimate that for the 3 action and 3 state version of this decision problem, at least one NIAS-F inequality holds with equality for roughly 25% of all rationalizable data sets.

[20] Given that there are no common prizes across states and actions in this decision problem, we will shorten $u(x(a, \omega))$ to $u(a, \omega)$.

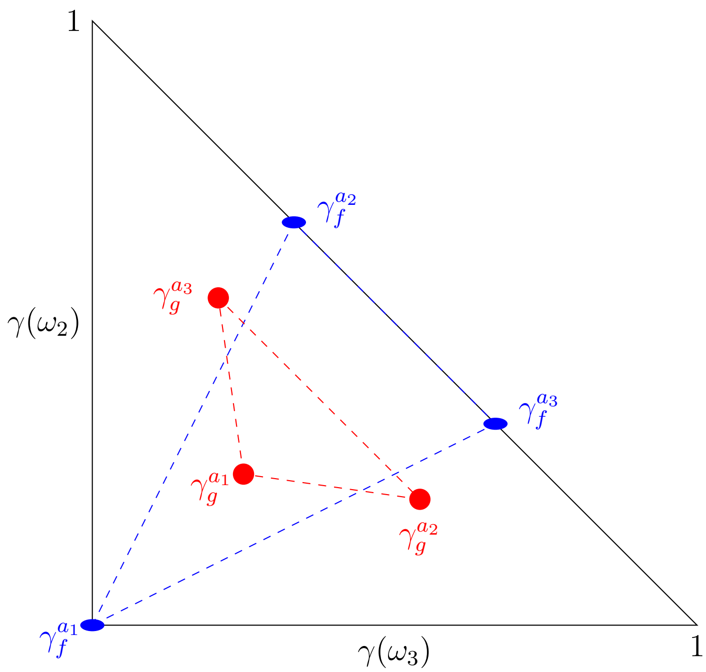


Figure 3: Revealed posteriors for distinct prize example. Blue ovals give revealed posteriors for frame $f$, red dots give revealed posteriors for frame $g$, and dashed lines provided the convex hull of the revealed posteriors for each frame.

inequalities for $a_2$ chosen over $a_1$ and $a_3$ chosen over $a_1$ in frame $f$ yield

$$\frac{16}{72}(u(a_2,\omega_2)-u(a_1,\omega_2))+\frac{8}{72}(u(a_2,\omega_3)-u(a_1,\omega_3))\geq 0$$

and

$$\frac{8}{72}(u(a_2,\omega_2)-u(a_1,\omega_2))+\frac{16}{72}(u(a_2,\omega_3)-u(a_1,\omega_3))\geq 0$$

Adding these together gives

$$u(a_2,\omega_2)-u(a_1,\omega_2)+u(a_2,\omega_3)-u(a_1,\omega_3)\geq 0$$

With this, we have that frame $f$ is revealed to have a robustly higher value of information.

However, frame $f$ is not revealed to have better informed actions. The revealed posteriors in frame $f$ are

$$\gamma_f^{a_1}=\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{matrix}\omega_1\\\omega_2\\\omega_3\end{matrix} \quad \& \quad \gamma_f^{a_2}=\begin{pmatrix}0\\\frac{16}{24}\\\frac{8}{24}\end{pmatrix}\begin{matrix}\omega_1\\\omega_2\\\omega_3\end{matrix} \quad \& \quad \gamma_f^{a_3}=\begin{pmatrix}0\\\frac{8}{24}\\\frac{16}{24}\end{pmatrix}\begin{matrix}\omega_1\\\omega_2\\\omega_3\end{matrix}$$

and the revealed posteriors in frame $g$ are

$$\gamma_g^{a_1} = \begin{pmatrix} \frac{12}{24} \\ \frac{6}{24} \\ \frac{6}{24} \end{pmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix} \quad \& \quad \gamma_g^{a_2} = \begin{pmatrix} \frac{6}{24} \\ \frac{5}{24} \\ \frac{13}{24} \end{pmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix} \; \gamma_g^{a_3} = \begin{pmatrix} \frac{6}{24} \\ \frac{13}{24} \\ \frac{5}{24} \end{pmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix}$$

These revealed posteriors are presented in Figure 3. Once again, actions cannot be revealed to be better informed in either frame because the revealed posteriors in neither frame fall within the convex hull of the revealed posteriors in the other frame.

Also, the revealed posteriors for actions $a_2$ and $a_3$ can be better informed about different states in frames $f$ and $g$ because the DM does not care if action $a_2$ or $a_3$ is taken if the state is not $\omega_1$, as both actions yield the same higher expected utility. This is also why being relatively better informed in frame $g$ when taking actions $a_2$ and $a_3$ (conditional on the state not being $\omega_1$) does not alter the welfare order over frames.

## 5.3 Problems with Two Types

Finally, we consider a third class of decision problems in which every action yields two types of outcomes. Formally, there are two possible prizes associated with each action (as in the tracking problem), so that

$$|\{x(a,\omega)|\omega \in \Omega\}| = 2 \text{ for all } a \in A$$

and in which no actions yield the same prize (as in the distinct prize problem), so that

$$x(a,\omega) \neq x(b,\nu) \text{ if } a \neq b \text{ or } \omega \neq \nu$$

An example of an economic setting that can have this structure is one in which each option is distinct but of uncertain quality, such as choosing between a Value PPO and a Premium PPO when both plans can have better doctor availability or worse.

The 3 action version of this decision problem can be represented as a matrix where actions $a_1$ to $a_3$ are given in the rows and states $\omega_1$ to $\omega_8$ are given in the columns:

$$\begin{array}{c c} \begin{matrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 & \omega_6 & \omega_7 & \omega_8 \end{matrix} & \\ \begin{pmatrix} x_1 & x_1 & x_1 & x_1 & x_2 & x_2 & x_2 & x_2 \\ x_4 & x_3 & x_4 & x_3 & x_3 & x_4 & x_3 & x_4 \\ x_6 & x_6 & x_5 & x_5 & x_6 & x_5 & x_5 & x_6 \end{pmatrix} & \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} \end{array}$$

This matrix shows that action $a_1$ yields two types of outcomes ($x_1$ and $x_2$), action $a_2$ yields two other types of outcomes ($x_3$ and $x_4$), action $a_3$ yields yet two other types of outcomes ($x_5$ and $x_6$). It also shows that any combination of outcomes is possible in some state.

We estimate that for this decision problem, roughly 43% of rationalizable data sets $P_f$ and $P_g$ reveal that one frame has a robustly higher value of information even though neither frame is revealed to have better informed actions. Again we describe in detail such a $P_f$ and $P_g$, which are given by

$$P_f = \begin{array}{c} \begin{array}{cccccccc} \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 & \omega_6 & \omega_7 & \omega_8 \end{array} \\ \begin{pmatrix} \frac{6}{48} & \frac{6}{48} & \frac{6}{48} & \frac{6}{48} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{48} & \frac{2}{48} & \frac{3}{48} & \frac{3}{48} \\ 0 & 0 & 0 & 0 & \frac{2}{48} & \frac{4}{48} & \frac{3}{48} & \frac{3}{48} \end{pmatrix} \end{array} \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array}$$

$$P_g = \begin{array}{c} \begin{array}{cccccccc} \omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5 & \omega_6 & \omega_7 & \omega_8 \end{array} \\ \begin{pmatrix} \frac{4}{48} & \frac{4}{48} & \frac{4}{48} & \frac{4}{48} & \frac{2}{48} & \frac{2}{48} & \frac{2}{48} & \frac{2}{48} \\ \frac{1}{48} & \frac{1}{48} & \frac{1}{48} & \frac{1}{48} & 0 & \frac{4}{48} & \frac{2}{48} & \frac{2}{48} \\ \frac{1}{48} & \frac{1}{48} & \frac{1}{48} & \frac{1}{48} & \frac{4}{48} & 0 & \frac{2}{48} & \frac{2}{48} \end{pmatrix} \end{array} \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array}$$

These data sets will reveal that for action $a_1$, odd-numbered prizes are "good" and even-numbered prizes are "bad" in the sense that odd-numbered prizes give higher utility. They will also reveal that the utility difference between odd-numbered prizes and even numbered prizes for actions $a_2$ and $a_3$ sum to zero, so that the choice probabilities of these actions do not factor into the welfare ranking of frames.

Frame $f$ is revealed to have a robustly higher value of information if

$$\frac{8}{48}\left(u(x_1) - u(x_2)\right) - \frac{3}{48}\left(u(x_3) - u(x_4)\right) - \frac{3}{48}\left(u(x_5) - u(x_6)\right) \geq 0$$

for all $u$ that rationalize $P_f$ and $P_g$. As before, this can be substantially simplified by showing that the NIAS-F inequalities for $a_2$ chosen over $a_3$ and $a_3$ chosen over $a_2$ hold with equality in both frames as a consequence of Lemma 2.[21]

Given that the NIAS-F inequalities for $a_2$ chosen over $a_3$ and $a_3$ chosen over $a_2$ hold with equality in frame $f$, we get

$$\frac{4}{48}u(x_3) + \frac{8}{48}u(x_4) = \frac{8}{48}u(x_5) + \frac{4}{48}u(x_6) \tag{2}$$

and

$$\frac{8}{48}u(x_3) + \frac{8}{48}u(x_4) = \frac{4}{48}u(x_5) + \frac{8}{48}u(x_6)$$

Adding these together, we get

$$u(x_3) - u(x_6) = u(x_5) - u(x_4)$$

[21]We estimate that for the 3 action and 8 state version of this decision problem, at least one NIAS-F inequality holds with equality for roughly 8% of all rationalizable data sets.

Substituting this into 2 gives

$$\frac{4}{48}\left(u(x_3)-u(x_6)\right)=\frac{8}{48}\left(u(x_3)-u(x_6)\right)$$

Hence, $u(x_3)-u(x_6)=0$ and $u(x_5)-u(x_4)=0$.

Thus, frame $f$ is revealed to have a robustly higher value of information if

$$\frac{8}{48}\left(u(x_1)-u(x_2)\right)\geq 0$$

From the NIAS-F inequalities for $a_1$ chosen over $a_2$ and $a_1$ chosen over $a_3$ in frame $f$, we know that

$$\frac{24}{48}u(x_1)\geq\frac{12}{48}\left(u(x_3)+u(x_5)\right)$$

and from the NIAS-F inequalities for $a_3$ chosen over $a_1$ and $a_2$ chosen over $a_1$ in frame $f$, we know that

$$\frac{12}{48}\left(u(x_3)+u(x_5)\right)\geq\frac{24}{48}u(x_2)$$

Together, these inequalities confirm that $u(x_1)\geq u(x_2)$, so frame $f$ is revealed to have a robustly higher value of information.

However, frame $f$ is not revealed to have better informed actions. The revealed posteriors in frame $f$ are

$$\gamma_f^{a_1}=\begin{array}{cl}\begin{pmatrix}\frac{6}{24}\\\frac{6}{24}\\\frac{6}{24}\\\frac{6}{24}\\0\\0\\0\\0\end{pmatrix} & \begin{array}{l}\omega_1\\\omega_2\\\omega_3\\\omega_4\\\omega_5\\\omega_6\\\omega_7\\\omega_8\end{array}\end{array}\quad\&\quad\gamma_f^{a_2}=\begin{array}{cl}\begin{pmatrix}0\\0\\0\\0\\\frac{4}{12}\\\frac{2}{12}\\\frac{3}{12}\\\frac{3}{12}\end{pmatrix} & \begin{array}{l}\omega_1\\\omega_2\\\omega_3\\\omega_4\\\omega_5\\\omega_6\\\omega_7\\\omega_8\end{array}\end{array}\quad\&\quad\gamma_f^{a_3}=\begin{array}{cl}\begin{pmatrix}0\\0\\0\\0\\\frac{2}{12}\\\frac{4}{12}\\\frac{3}{12}\\\frac{3}{12}\end{pmatrix} & \begin{array}{l}\omega_1\\\omega_2\\\omega_3\\\omega_4\\\omega_5\\\omega_6\\\omega_7\\\omega_8\end{array}\end{array}$$

and the revealed posteriors in frame $g$ are

$$\gamma_g^{a_1}=\begin{array}{cl}\begin{pmatrix}\frac{4}{24}\\\frac{4}{24}\\\frac{4}{24}\\\frac{4}{24}\\\frac{2}{24}\\\frac{2}{24}\\\frac{2}{24}\\\frac{2}{24}\end{pmatrix} & \begin{array}{l}\omega_1\\\omega_2\\\omega_3\\\omega_4\\\omega_5\\\omega_6\\\omega_7\\\omega_8\end{array}\end{array}\quad\&\quad\gamma_g^{a_2}=\begin{array}{cl}\begin{pmatrix}\frac{1}{12}\\\frac{1}{12}\\\frac{1}{12}\\\frac{1}{12}\\0\\\frac{4}{12}\\\frac{2}{12}\\\frac{2}{12}\end{pmatrix} & \begin{array}{l}\omega_1\\\omega_2\\\omega_3\\\omega_4\\\omega_5\\\omega_6\\\omega_7\\\omega_8\end{array}\end{array}\quad\&\quad\gamma_g^{a_3}=\begin{array}{cl}\begin{pmatrix}\frac{1}{12}\\\frac{1}{12}\\\frac{1}{12}\\\frac{1}{12}\\\frac{4}{12}\\0\\\frac{2}{12}\\\frac{2}{12}\end{pmatrix} & \begin{array}{l}\omega_1\\\omega_2\\\omega_3\\\omega_4\\\omega_5\\\omega_6\\\omega_7\\\omega_8\end{array}\end{array}$$

The revealed posteriors for actions are not revealed to be better informed in either frame because the revealed posteriors for neither frame fall within the convex hull of the revealed posteriors for the other frame. The revealed posterior for action $a_1$ in frame $f$, which is certain of the state being $\omega_1$, $\omega_2$, $\omega_3$, or $\omega_4$ (perfectly informed about action $a_1$ yielding the good prize), cannot be expressed as a convex combination of the less than fully informed revealed posteriors in frame $g$ and the revealed posteriors for actions $a_2$ and $a_3$ in frame $g$ cannot be expressed as a convex combination of revealed posteriors in frame $f$. This occurs because while the revealed posterior for action $a_2$ is better informed about state $\omega_5$ in frame $f$, but it is relatively better informed about state $\omega_6$ in frame $g$, conditioning on the state not being $\omega_1$, $\omega_2$, $\omega_3$, or $\omega_4$.

# 6 Application to Health Care Plan Choice

In this section, we revisit the choice setting discussed in the introduction in which individuals choose between a low-premium, high-deductible health care plan ("Value PPO") and high-premium, low-deductible plan ("Premium PPO").

## 6.1 Mapping Into Our Framework

Around half the time, the company offers a version of the Premium PPO that has better doctor availability than the Value PPO, but the plans are otherwise identical. They are presented in two different ways. One framing uses a standard description of options, and the other framing uses a plain language description of deductibles.

To map this into our framework, we need to define actions, states, prizes, and frames. In this choice setting, an action is selecting a health care plan. Let action $a_P$ be choosing the Premium PPO and action $a_V$ be choosing the Value PPO. As discussed previously, the labeling of these actions ("Value" and "Premium") might be considered to be part of the frame if they were varied across decision problems in an effort to impact choice.

We define the states as the characteristics of plans that are varied. Let $\omega_S$ be the state where the Premium PPO has the same doctor availability, and $\omega_B$ be the state where it has better doctor availability. The prizes are the actual health care coverage received from selecting a health care plan when it has certain characteristics. Let prize $x_{PS}$ be the Premium PPO coverage when it has the same doctor availability, prize $x_{PB}$ be the Premium PPO coverage when it has better doctor availability, and prize $x_V$ be the Value PPO coverage.

The map between actions, states, and prizes for this decision problem can be represented as a matrix where actions $a_P$ and $a_V$ are given in the rows and states $\omega_S$ and $\omega_B$ are given in the columns:

$$\begin{array}{cc}
\begin{array}{cc} \omega_S & \omega_B \end{array} & \\
\begin{pmatrix} x_{PS} & x_{PB} \\ x_V & x_V \end{pmatrix} & \begin{array}{c} a_P \\ a_V \end{array}
\end{array}$$

Finally, we define frames as the features of the presentation that are varied. Let $N$ be the normal framing and $PL$ be the plain language framing. Together, these framed decision problems are given by $A = \{a_P, a_V\}$, $\Omega = \{\omega_S, \omega_B\}$, $X = \{x_{PS}, x_{PB}, x_V\}$, and a frame $h \in \{N, PL\}$.

For these framed decision problems, we use data sets $P_N$ and $P_{PL}$ to rank frames $N$ and $PL$ in terms of their welfare and informativeness. In this choice setting, the ranking of frames based on welfare and the ranking of frames based on informativeness are identical. It can be shown that for *simple decision problems*, which have just two actions and states,[22] a frame having been revealed to have better informed actions is both necessary and sufficient for that frame to have been revealed to have a robustly higher value of information whenever there are more than two prizes. Thus, we can rank frames $N$ and $PL$ either if FIAS is satisfied or if there is a garbling of revealed experiments.

However, as a precondition for ranking frames, we need first that the data sets $P_N$ and $P_{PL}$ are rationalizable, which is true if NIAS-F is satisfied. Using the MATLAB programs provided, we estimate that approximately 50% of the possible data sets $P_N$ and $P_{PL}$ are rationalizable. This means that for all such $P_N$ and $P_{PL}$, there exists a utility function $u$ where

$$P_h(a_P, \omega_S)u(x_{PS}) + P_h(a_P, \omega_B)u(x_{PB}) \quad \geq \quad P_h(a_P)u(x_V)$$

and

$$P_h(a_V)u(x_V) \quad \geq \quad P_h(a_V, \omega_S)u(x_{PS}) + P_h(a_V, \omega_B)u(x_{PB})$$

for $h \in \{N, PL\}$ and with one inequality strict. Given Lemma 2, determining whether such a utility function exists corresponds to finding the solution to a system of linear inequalities and linear equations, so it is simple to check in practice.

Of the data sets $P_N$ and $P_{PL}$ that are rationalizable, we estimate that approximately 66% satisfy FIAS, which means that the value of information is revealed to be robustly higher for one of the frames. For FIAS to be satisfied, there must exist a weighting function $t_h$ such that the prize lottery gained from being in one frame instead of the other can be represented

[22] Such decision problems play a key role in Frankel and Kamenica (2018), who show any decision problem with a binary state space induces the same measure of uncertainty as a collection of simple decision problems.

as a non-negative combination of the prize lotteries gained by not switching away from any action in either frame. For instance, the prize lottery gained from choosing in frame $PL$ instead of frame $N$ is simply:

$$\begin{gathered} P_{PL}(a_P, \omega_S) - P_N(a_P, \omega_S) \text{ chance of } x_{PS} \\ P_{PL}(a_P, \omega_B) - P_N(a_P, \omega_B) \text{ chance of } x_{PB} \\ P_{PL}(a_V) - P_N(a_V) \text{ chance of } x_V \end{gathered}$$

Thus, FIAS for frame $PL$ requires that this is equal to:

$$\begin{gathered} \textstyle\sum_{h\in\{N,PL\}} P_h(a_P, \omega_S)t_h(a_P, a_V) - P_h(a_V, \omega_S)t_h(a_V, a_P) \text{ chance of } x_{PS} \\ \textstyle\sum_{h\in\{N,PL\}} P_h(a_P, \omega_B)t_h(a_P, a_V) - P_h(a_V, \omega_S)t_h(a_V, a_P) \text{ chance of } x_{PB} \\ \textstyle\sum_{h\in\{N,PL\}} P_h(a_P)t_h(a_P, a_V) - P_h(a_V)t_h(a_V, a_P) \text{ chance of } x_V \end{gathered}$$

Whether a $t_h$ exists that satisfies FIAS corresponds to finding whether there is a solution to a system of linear equations, so it is also simple to check in practice.

When applying our framework and testing these conditions more broadly, two natural issues arise: how to generate the stochastic choice data sets and how to account for sampling error. In regards to the first issue, stochastic choice data sets are widely employed across the social sciences and are typically estimated by pooling together the decisions of multiple individuals or one individual over time. Either way, this process introduces the possibility of both observable heterogeneity and unobservable heterogeneity in information and utility. When implementing our framework, it would be natural to use stochastic choice data sets that are separately estimated for different groups based on observable characteristics. This opens up the potentially interesting and important possibility that the welfare orderings over frames might differ across groups. In other words, a frame that makes one group better off might not make another group better off. In addition, the testable content of our model can be brought to bear on this issue. If the NIAS-F inequalities are not satisfied for a group, then that level of aggregation is not suitable for our model.

The second issue is highly related to the first. When estimating a stochastic choice data set using many individual observations, there is the possibility of sampling error. One way to account for this possibility is to use bootstrapping to estimate many possible stochastic choice data sets and see if the welfare ordering over frames remains consistent across each draw.

## 6.2 Accounting for Clear Dominance

In this choice setting, we assumed that the Value PPO was more cost effective no matter an individual's health outcomes, so when the Premium PPO has the same doctor availability, the

Value PPO is clearly dominant. Formally, we know that $u(x_V) \geq u(x_{PB})$ for all rationalizing utility functions. As discussed previously, this extra restriction can easily be incorporated into NIAS-F and FIAS.

First, this restriction on utility can be incorporated into NIAS-F by adding the following extra linear inequality to the system of NIAS-F inequalities:

$$u(x_V) - u(x_{PB}) \geq 0$$

This extra inequality does not impact the ease of testing NIAS-F, but it does reduce the set of admissible utility functions. For this decision problem, we estimate that this additional restriction reduces the proportion of possible data sets $P_N$ and $P_{PL}$ that satisfy NIAS-F by half.

Second, this restriction can be incorporated into FIAS by requiring that, in addition to the weighting function $t_h$, there is exists a non-negative $t$ that accounts for the prize lottery given by this extra restriction. For example, FIAS for frame $PL$ would require that the prize lottery gained from choosing in frame $PL$ instead of frame $N$ is equal to:

$$\begin{gathered}
-t + \textstyle\sum_{h \in \{N,PL\}} P_h(a_P, \omega_S)t_h(a_P, a_V) - P_h(a_V, \omega_S)t_h(a_V, a_P) \text{ chance of } x_{PS} \\
\textstyle\sum_{h \in \{N,PL\}} P_h(a_P, \omega_B)t_h(a_P, a_V) - P_h(a_V, \omega_S)t_h(a_V, a_P) \text{ chance of } x_{PB} \\
t + \textstyle\sum_{h \in \{N,PL\}} P_h(a_P)t_h(a_P, a_V) - P_h(a_V)t_h(a_V, a_P) \text{ chance of } x_V
\end{gathered}$$

The only change to the prize lottery given previously is to subtract a $t$ chance of $x_{PS}$ and add a $t$ chance of $x_V$. Because $t$ can equal zero, this can increase the proportion of data sets that satisfy FIAS. However, for this decision problem, this additional restriction does not noticeably improve our ability to rank frames. Once again, we estimate that approximately 66% of rationalizable data sets satisfy FIAS.

# 7 Conclusion and Discussion

In this paper, we provide a framework for determining whether one of two frames gives higher welfare. While in practice policymakers often select between one of two frames – for instance, the choice between sticking with the current framing or moving to a new one – our framework can be readily extended to provide a ranking over many frames. In a setting with many frames, the statement of NIAS-F would be the same, but with $h \in \{f_1, f_2, ..., f_n\}$ instead of $h \in \{f, g\}$. Likewise, when assessing the relative ranking of two frames, the statement of FIAS would remain the same beyond the same change to the set of frames that the weighting function must cover. To have a complete ranking of frames, there would need to exist a non-negative weighting function that satisfies FIAS for every pair of frames. One potential advantage to considering rich variation in frames is that it could provide tighter bounds on the value of information in each frame.

Another natural extension of our framework is to account for information costs in the welfare assessment of frames. In an experiment by Beshears, Choi, Laibson, and Madrian (2009), subjects chose between investment portfolios with just the raw information about stocks or a short summary of this information in the form of a Summary Prospectus. They found that accounting for the costs of information was essential because "the principal welfare gain from the Summary Prospectus comes from allowing investors to spend less time and effort to arrive at the same portfolio decision they would have come to after reading only the statutory prospectus." The presence of information costs is likely to reduce the gap between the welfare order and the informativeness order because if there is little value to learning about states when taking some actions (as in our examples provided in Section 5.2), then that information is unlikely to be obtained when information is costly.

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# 8 Appendix

**Proof of Theorem 1:**

*Proof.* 1) Exists a BEU $(u, \mu, \pi_f, \pi_g, \sigma_f, \sigma_g) \Rightarrow u$ that satisfies NIAS-F. For any $h \in \{f, g\}$, $a, b \in A$, and $\gamma \in \Gamma(\pi_h)$ s.t. $\sigma_h(a|\gamma) > 0$, substituting the Bayesian restriction into the Maximization restriction gives

$$\sum_{\omega \in \Omega} \mu(\omega)\pi_h(\gamma|\omega)u(x(a,\omega)) \geq \sum_{\omega \in \Omega} \mu(\omega)\pi_h(\gamma|\omega)u(x(b,\omega))$$

Multiplying both sides by the probability of choosing action $a$ for posterior $\gamma$ given decision rule $\sigma_h$ yields

$$\sigma_h(a|\gamma)\sum_{\omega \in \Omega} \mu(\omega)\pi_h(\gamma|\omega)u(x(a,\omega)) \geq \sigma_h(a|\gamma)\sum_{\omega \in \Omega} \mu(\omega)\pi_h(\gamma|\omega)u(x(b,\omega))$$

Since this holds for all $\gamma \in \Gamma(\pi_h)$ s.t. $\sigma_h(a|\gamma) > 0$, summing across all $\gamma \in \Gamma(\pi_h)$ and rearranging summations yields

$$\sum_{\omega \in \Omega} \mu(\omega) \sum_{\gamma \in \Gamma(\pi_h)} \pi_h(\gamma|\omega)\sigma_h(a|\gamma)u(x(a,\omega)) \geq \sum_{\omega \in \Omega} \mu(\omega) \sum_{\gamma \in \Gamma(\pi_h)} \pi_h(\gamma|\omega)\sigma_h(a|\gamma)u(x(b,\omega))$$

Finally, substituting the Data Matching restriction into this inequality yields the NIAS-F inequality for $h \in \{f, g\}$ and $a, b \in A$. Also, by the steps above, we can directly show that the NIAS-F inequality must hold strictly for the $h \in \{f, g\}$ and $a, b \in A$ in which the Maximization restriction holds strictly.

2) Exists a $u$ that satisfies NIAS-F $\Rightarrow$ Exists a BEU $(u, \mu, \pi_f, \pi_g, \sigma_f, \sigma_g)$. In addition to the $u$ that satisfies NIAS-F, we construct $\mu$, $\pi_f$, $\pi_g$, $\sigma_f$, and $\sigma_g$ from the data in such a way that together they form a BEU.

First, we define $\mu(\omega) = \sum_{a \in A} P_f(a,\omega) = P_f(\omega)$. Next, we construct an information structure for frame $f$ that puts positive probability on the revealed posterior $\gamma_f^a$ for each action $a \in A$. This posterior corresponds to the distribution of states when action $a$ is taken according to $P_f$, so is given by

$$\gamma_f^a(\omega) = \frac{P_f(a,\omega)}{\sum_{\nu \in \Omega} P_f(a,\nu)} = \frac{P_f(a,\omega)}{P_f(a)}$$

Because the same distribution of states can occur when more than one action is taken, we partition the set of possible actions into $E_f \leq J$ sets $A_f^e$ for $e \in \{1, ..., E_f\}$ with identical $\gamma_f^e$ within each such set. Using this, we define

$$\pi_f(\gamma|\omega) = \sum_{b \in A_f^e} \frac{P_f(b,\omega)}{P_f(\omega)} \quad \text{if } \gamma = \gamma_f^e \text{ for some } e \in \{1, ..., E_f\}$$

and equal to 0 otherwise.

To complete the construction, we define $\sigma_f(a|\gamma)$ as

$$\sigma_f(a|\gamma) = \frac{P_f(a)}{\sum_{b\in A_f^e} P_f(b)} \quad \text{if } \gamma = \gamma_f^e \text{ for some } e \in \{1, ..., E_f\}$$

and equal to 0 otherwise. We define $\gamma_g^a(\omega)$, $E_g$ $A_g^e$, $\pi_g(\gamma|\omega)$, and $\sigma_g(a|\gamma)$ analogously.

First, we show that this constructed BEU satisfies the Data Matching restriction for $P_f$. Because $\pi_f(\gamma|\omega)$ is only non-zero for posteriors s.t. $\gamma = \gamma_f^e$ for some $e \in \{1, ..., E_f\}$, the constructed BEU gives

$$\begin{aligned}\mu(\omega) \sum_{\gamma\in\Gamma(\pi_f)} \pi_f(\gamma|\omega)\sigma_f(a|\gamma) =& P_f(\omega) \sum_{e=1}^{E_f} \sum_{b\in A_f^e} \frac{P_f(b,\omega)}{P_f(\omega)} \frac{P_f(a)}{\sum_{b\in A_f^e} P_f(b)} \\ =& \sum_{e=1}^{E_f} \frac{\sum_{b\in A_f^e} P_f(b,\omega)P_f(a)}{\sum_{b\in A_f^e} P_f(b)}\end{aligned} \tag{3}$$

Because $a, b \in A_f^e$ if and only if $\gamma_f^a(\omega) = \gamma_f^b(\omega)$ for all $\omega \in \Omega$, for any $a \in A_f^e$ and $\omega \in \Omega$,

$$\sum_{b\in A_f^e} P_f(b) = \sum_{b\in A_f^e} P_f(b,\omega) \frac{P_f(a)}{P_f(a,\omega)} \tag{4}$$

Substitution of equation (4) into equation (3) yields,

$$\begin{aligned}\mu(\omega) \sum_{\gamma\in\Gamma(\pi_f)} \pi_f(\gamma|\omega)\sigma_f(a|\gamma) &= \sum_{e=1}^{E_f} \frac{\sum_{b\in A_f^e} P_f(b,\omega)P_f(a)}{\sum_{b\in A_f^e} P_f(b,\omega)\frac{P_f(a)}{P_f(a,\omega)}} \\ &= P_f(a,\omega) \sum_{e=1}^{E_f} \frac{\sum_{b\in A_f^e} P_f(b,\omega)P_f(a)}{\sum_{b\in A_f^e} P_f(b,\omega)P_f(a)} = P_f(a,\omega)\end{aligned}$$

Thus, the Data Matching restriction holds for frame $f$, and the same steps can be used to show that it holds for frame $g$.

Next, we show that the constructed BEU satisfies the Bayesian Updating restriction for all $\gamma \in \Gamma(\pi_f)$. For all $e \in \{1, ..., Q_f\}$, $a \in A_f^e$, and $\omega \in \Omega$, by Data Matching (and given that action $a$ is only chosen with positive probability at posterior $\gamma_f^e$) we have that

$$\gamma_f^e(\omega) = \frac{P_f(a,\omega)}{\sum_{\nu\in\Omega} P_f(a,\nu)} = \frac{\mu(\omega)\pi_f(\gamma_f^e|\omega)\sigma_f(a|\gamma_f^e)}{\sum_{\nu\in\Omega} \mu(\nu)\pi_f(\gamma_f^e|\nu)\sigma_f(a|\gamma_f^e)} = \frac{\mu(\omega)\pi_f(\gamma_f^e|\omega)}{\sum_{\nu\in\Omega} \mu(\nu)\pi_f(\gamma_f^e|\nu)}$$

Thus, all posteriors in $\Gamma(\pi_f)$ satisfy the Bayesian Updating restriction, and the same steps can be used to show that it holds for all posteriors in $\Gamma(\pi_g)$.

Finally, we show that the constructed BEU satisfies the Maximization restriction for all $a, b \in A$ and $\gamma \in \Gamma(\pi_f)$ s.t. $\sigma_f(a|\gamma) > 0$. For any $e \in \{1, ..., E_f\}$, $a \in A_f^e$, and $b \in A$, dividing both sides of the NIAS-F inequality for frame $f$ and $a, b \in A$ by $P_f(a) > 0$ and then directly substituting for $\gamma_f^e(\omega)$ gives

$$\sum_{\omega \in \Omega} \gamma_f^e(\omega) u(x(a,\omega)) \geq \sum_{\omega \in \Omega} \gamma_f^e(\omega) u(x(b,\omega))$$

Thus, for any $e \in \{1, ..., E_f\}$, $a \in A_f^e$, and $b \in A$, the Maximization restriction holds because by definition $\gamma_f^e(\omega) \in \Gamma(\pi_f)$ and $\sigma(a|\gamma_f^e(\omega)) > 0$. Also, because all $a \in A$ belong to $A_f^e$ for some $e \in \{1, ..., E_f\}$, this holds for all $a, b \in A$. The same logic holds for any $e \in \{1, ..., E_g\}$, $a \in A_g^e$, and $b \in A$. $\square$

**Proof of Lemma 1:**

*Proof.* The NIAS-F inequality for frame $h \in \{f, g\}$ and actions $a, b \in A$ can be expressed as a 1 by $M$ row vector $\vec{d_h}(a, b)$ where element $m$ of this vector gives the difference in the probability of receiving prize $x_m$ from taking action $a$ and from taking action $b$ with the same probability. This is given by

$$\sum_{\omega \in \Omega} P_h(a,\omega)(\mathbf{1}_{\{x(a,\omega)=x_m\}} - \mathbf{1}_{\{x(b,\omega)=x_m\}})$$

where $1_{\{x(a,\omega)=x_m\}}$ is an indicator function that takes a value of 1 when the prize from taking action $a$ in state $\omega$ yields prize $x_m$.

Stacking the row vectors for all NIAS-F inequalities for frame $h \in \{f, g\}$ produces a $J^2$ by $M$ matrix $D_h$ where

$$D_h = \begin{bmatrix} \vec{d_h}(a_1, a_1) \\ \vec{d_h}(a_1, a_2) \\ ... \\ \vec{d_h}(a_J, a_{J-1}) \\ \vec{d_h}(a_J, a_J) \end{bmatrix}$$

and stacking the matrix of NIAS-F inequalities for both frames produces a $2 * J^2$ by $M$ matrix $D$ where

$$D = \begin{bmatrix} D_f \\ D_g \end{bmatrix}$$

Based on this matrix $D$, NIAS-F can be restated as the $M$ by 1 column vector $u \in \mathbb{R}^M$ satisfying $Du \geq 0$ with $Du(m) > 0$ for some $m \in \{1, ..., M\}$.

In addition, the requirement for frame $f$ to be revealed to have a robustly higher value of information than frame $g$ can be expressed as a 1 by $M$ row vector $\vec{d}$ where element $m$ gives the expected gain in prize $x_m$ from being in frame $f$ instead of frame $g$, which is given by

$$\sum_{a \in A} \sum_{\omega \in \Omega} (P_f(a,\omega) - P_g(a,\omega)) \mathbf{1}_{\{x(a,\omega)=x_m\}}$$

Given that NIAS-F is satisfied, $f \succsim_W g$ can be restated as $\vec{d}u \geq 0$ for all $u \in \mathbb{R}^M$ that satisfy NIAS-F. With this notation, both directions of the lemma follow from Farkas lemma.
1) Exists $t \in \mathbb{R}_+^{2*J^2}$ s.t. $D^T t = (\vec{d})^T \Rightarrow$ For all $u \in \mathbb{R}^M$ satisfying NIAS-F, $\vec{d}u \geq 0$. Assume not. Take $u \in \mathbb{R}^M$ such that NIAS-F is satisfied, so that $Du \geq 0$, but $\vec{d}u < 0$. By Farkas lemma, there cannot exist a $t \in \mathbb{R}_+^{2*J^2}$ s.t. $D^T t = (\vec{d})^T$, which is a contradiction.
2) For all $u \in \mathbb{R}^M$ satisfying NIAS-F, $\vec{d}u \geq 0 \Rightarrow$ Exists $t \in \mathbb{R}_+^{2*J^2}$ s.t. $D^T t = (\vec{d})^T$.
Assume there does not exist $t \in \mathbb{R}_{++}^{2*J^2}$ such that $D^T t = (\vec{d})^T$. By Farkas lemma, there must exist a $u \in \mathbb{R}^M$ satisfying NIAS-F and with $\vec{d}u < 0$, which is a contradiction. $\square$

**Proof of Lemma 2:**

*Proof.* First, for any $u$ that satisfies NIAS-F,

$$\sum_{x \in X} \sum_{n=1}^{N} w_n * \left( \sum_{\omega \in \Omega} P_{h_n}(a_n, \omega)(\mathbf{1}_{\{x(a_n,\omega)=x\}} - \mathbf{1}_{\{x(b_n,\omega)=x\}} \right) \geq 0$$

for any collection of triples $(h, a_1, b_1)$ for $h_1, ..., h_N \in \{f, g\}$, $a_1, ..., a_N \in A$, and $b_1, ..., b_N \in A$ with $(h_n, a_n, b_n) \neq (h, a, b)$ and non-negative weights $w_1, ..., w_N$. Assuming that equation (1) holds, for any $u$ that satisfies NIAS-F,

$$-1 * \sum_{x \in X} \left( \sum_{\omega \in \Omega} P_h(a, \omega)(\mathbf{1}_{\{x(a,\omega)=x\}} - \mathbf{1}_{\{x(b,\omega)=x\}} \right) u(x) \geq 0$$

For any $u$ that satisfies NIAS-F,

$$\sum_{x \in X} \left( \sum_{\omega \in \Omega} P_h(a, \omega)(\mathbf{1}_{\{x(a,\omega)=x\}} - \mathbf{1}_{\{x(b,\omega)=x\}} \right) u(x) \geq 0$$

as well, so it must equal 0. Second, if it equals 0 for all $u$ that satisfy NIAS-F, then it must be that a collection of other NIAS-F inequalities imply

$$-1 * \sum_{x \in X} \left( \sum_{\omega \in \Omega} P_h(a, \omega)(\mathbf{1}_{\{x(a,\omega)=x\}} - \mathbf{1}_{\{x(b,\omega)=x\}} \right) u(x) \geq 0$$

for all $u$ that satisfy NIAS-F, so there does not exist a $u$ that satisfies NIAS-F and

$$-1 * \sum_{x \in X} \left( \sum_{\omega \in \Omega} P_h(a, \omega)(\mathbf{1}_{\{x(a,\omega)=x\}} - \mathbf{1}_{\{x(b,\omega)=x\}} \right) u(x) < 0$$

By Farkas lemma, there must exist non-negative weights on that collection of NIAS-F inequalities that give equation (1), completing the proof. $\square$

**Proof of Lemma 3:**

*Proof.* Take any $S$ that gives $f \succsim_B$. This allows us to express $\sum_{\omega\in\Omega} P_g(a,\omega)\mathbf{1}_{\{x(a,\omega)=x\}}$ in terms of $P_f$:

$$\begin{aligned}\sum_{\omega\in\Omega} P_g(a,\omega)\mathbf{1}_{\{x(a,\omega)=x\}} &= \sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_g(a,\omega) = \sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\frac{P_g(a,\omega)}{P_g(\omega)}P_g(\omega)\\ &= \sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\sum_{c\in A}\frac{P_f(c,\omega)}{P_f(\omega)}S(c,a)P_g(\omega) = \sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\sum_{c\in A}P_f(c,\omega)S(c,a)\end{aligned}$$

Starting from the NIAS-F inequalities, we have

$$\sum_{x\in X}\left(\sum_{\omega\in\Omega}P_h(a,\omega)(\mathbf{1}_{\{x(a,\omega)=x\}}-\mathbf{1}_{\{x(b,\omega)=x\}}\right)u(x)\geq 0$$

and

$$\sum_{x\in X}\left(\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\sum_{c\in A}P_f(c,\omega)S(c,a)-\sum_{\omega\in\Omega}\mathbf{1}_{\{x(b,\omega)=x\}}\sum_{c\in A}P_f(c,\omega)S(c,a)\right)u(x)\geq 0$$

for every $a,b\in A$ and one strict in some frame.

For any $a,b\in A$, multiplying both sides of the NIAS-F inequality for frame $f$ by $S(a,b)$ gives

$$\sum_{x\in X}\left(\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)S(a,b)-\sum_{\omega\in\Omega}\mathbf{1}_{\{x(b,\omega)=x\}}P_f(a,\omega)S(a,b)\right)u(x)\geq 0$$

This holds for all $a,b\in A$, so we can sum over both, which gives

$$\sum_{x\in X}\left(\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)S(a,b)-\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(b,\omega)=x\}}P_f(a,\omega)S(a,b)\right)u(x)\geq 0 \quad (5)$$

The difference in the expectation of prize $x$ from switching to frame $f$ from frame $g$ is given by

$$\sum_{a\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)-\sum_{a\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\sum_{c\in A}P_f(c,\omega)S(c,a)$$

Hence, we want to show that for every utility function $u$ that satisfies NIAS-F,

$$\sum_{x\in X}\left(\sum_{a\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)-\sum_{a\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\sum_{c\in A}P_f(c,\omega)S(c,a)\right)u(x)\geq 0 \qquad (6)$$

Because $\sum_{b\in A} S(a,b)=1$, $P_f(a,\omega)$ is equivalent to $\sum_{b\in A} P_f(a,\omega)S(a,b)$, so 6 is equal to

$$\sum_{x\in X}\left(\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)S(a,b)-\sum_{a\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}\sum_{c\in A}P_f(c,\omega)S(c,a)\right)u(x)\geq 0$$

By relabeling $c$ and $b$, it becomes

$$\sum_{x\in X}\left(\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)S(a,b)-\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(b,\omega)S(b,a)\right)u(x)\geq 0$$

Relabeling $b$ and $a$ in a similar way gives

$$\sum_{x\in X}\left(\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(a,\omega)=x\}}P_f(a,\omega)S(a,b)-\sum_{a\in A}\sum_{b\in A}\sum_{\omega\in\Omega}\mathbf{1}_{\{x(b,\omega)=x\}}P_f(a,\omega)S(a,b)\right)u(x)\geq 0$$

So equation (5) implies equation (6) as desired. □

**Proof of Lemma 4:**

*Proof.* Take any $S$ with $S(c,a)\in\mathbb{R}_+$ and $\sum_{c\in A} S(c,a)=1$ such that for all $a\in A$ and $\omega\in\Omega$,

$$\frac{P_g(a,\omega)}{P_g(\omega)}=\sum_{c\in A}\frac{P_f(c,\omega)}{P_f(\omega)}S(c,a)$$

Multiplying both sides by $P_g(\omega)$ or $P_f(\omega)$ gives

$$P_g(a,\omega)=\sum_{c\in A}P_f(c,\omega)S(c,a)$$

Dividing both sides by $P_g(a)$ gives

$$\frac{P_g(a,\omega)}{P_g(a)}=\sum_{c\in A}P_f(c,\omega)\frac{1}{P_g(a)}S(c,a)$$

or

$$\frac{P_g(a,\omega)}{P_g(a)}=\sum_{c\in A}\frac{P_f(c,\omega)}{P_f(c)}\frac{P_f(c)}{P_g(a)}S(c,a)$$

Define

$$T(c,a)=\frac{P_f(c)}{P_g(a)}S(c,a)$$

So that

$$S(c,a)=\frac{P_g(a)}{P_f(c)}T(c,a)$$

Thus,

$$\begin{aligned}
\frac{P_g(a,\omega)}{P_g(a)} &= \sum_{c\in A} \frac{P_f(c,\omega)}{P_f(c)} \frac{P_f(c)}{P_g(a)} S(c,a) \\
&= \sum_{c\in A} \frac{P_f(c,\omega)}{P_f(c)} \frac{P_f(c)}{P_g(a)} \frac{P_g(a)}{P_f(c)} T(c,a) \\
&= \sum_{c\in A} \frac{P_f(c,\omega)}{P_f(c)} T(c,a) = \sum_{c\in A} \gamma_f^c(\omega) T(c,a)
\end{aligned}$$

as desired. It remains to show that $\sum_{c\in A} T(c,a) = 1$ for all $a \in A$. Summing up the definition of $T(c,a)$ gives

$$\sum_{c\in A} T(c,a) = \sum_{c\in A} \frac{P_f(c)}{P_g(a)} S(c,a)$$

This is equivalent to showing that

$$P_g(a) = \sum_{c\in A} P_f(c) S(c,a)$$

This is true because $f \succsim_B g$ means for each $a \in A$ and $\omega \in \Omega$

$$P_g(a,\omega) = \sum_{c\in A} P_f(c,\omega) S(c,a)$$

and summing over states provides the desired relationship. $\square$