The Pervasive Importance of Tightness in Labor-Market Volatility *

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Abstract

A distinctive contribution of the unemployment model of Diamond, Mortensen, and Pissarides is the creation of an economically coherent concept of labor-market tightness. In a tighter market, jobseekers find jobs more quickly and employers take longer to fill jobs. The evidence in this paper on individual job-finding rates and on job-filling rates of employers shows that a single index factor describes tightness. Jobfinding rates and jobfilling rates move in parallel as functions of the single index. We account for heterogeneity in both rates. For the jobfinding rate, we use success rates for jobseekers over one-month and 12-month periods of potential search in data from the Current Population Survey, and we distinguish 16 categories of jobseekers, based on their activities and experiences leading up to the measurement date. For the jobfilling rate, we use the ratio of hires to job openings in the Job Openings and Labor Turnover Survey, distinguishing between 16 industries. We conclude that the DMP model’s concept of tightness is central to the understanding of fluctuations in unemployment and thus in employment and output. We show that our index of labor-market tightness is highly correlated with movements of total hours of work and substantially correlated with movements of aggregate output, although noncyclical influences—total factor productivity and labor-force participation—are important sources of output volatility as well.

JEL E24, J64

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The modern theory of labor-market volatility rests on the central concept of market tightness. In a tighter market, jobseekers find jobs more quickly and employers fill jobs more slowly. Diamond, Mortensen, and Pissarides’s model of unemployment (1) provides a rigorous framework for understanding tightness, a previously murky concept. Tightness varies in response to changes in the incentives facing employers to create jobs. We show that tightness is at the heart of fluctuations in unemployment and employment, and that variations in tightness have effects that pervade the entire labor market. These variations have a large role, along with stochastic variations in productivity growth, in the volatility of output. No model of business-cycle fluctuations can claim realism without embodying an account of tightness. We also show that tightness moves in unison across the entire U.S. labor market—a single latent factor characterizes the movements of the jobfinding process for many types of workers and recruiting success in many private industries.

A key object in the DMP model is the matching function, which shows how the labor market maps stocks of jobseekers and recruiting efforts into flows of hires. We develop a matching-function specification that recognizes heterogeneity among types of jobseekers and among employers in different industries. Jobseeking success rates respond positively to tightness and recruiting success rates respond negatively. From data from the Current Population Survey, we measure coefficients relating jobfinding rates to tightness, modeled as a latent statistical factor. From the Job Openings and Labor Turnover Survey, we measure coefficients relating jobfilling rates by industry to the same latent factor. We exploit the statistical principle that, in a body of indicators that each respond to a single latent common factor plus an idiosyncratic random factor, the common latent factor is the first principal component of the indicators.

The paper then turns to the role of fluctuations in tightness in the determination of unemployment, total labor input, and output. Recessions in general and the last one in particular involve sharp increases in unemployment followed by protracted declines. The DMP model treats unemployment as a state variable, responding to tightness, which is a jump variable, through a law of motion that involves lags. Nonetheless, unemployment tracks tightness quite accurately in annual data. Total labor input in the economy is also highly correlated with tightness, even though fluctuations in labor-force participation and variations in hours per worker are not intrinsically connected to tightness but do influence labor input.
And the relation of tightness to output fluctuations is strong, despite the importance of fluctuations in the growth of total factor productivity.

1  Tightness as a Sufficient Statistic for Jobfinding and Jobfilling Rates

1.1  The standard DMP setup

We start from the constant-returns matching function relating the flow of hires $H$ to the stocks of jobseekers $N$ and vacancies $V$:

$$H = m(N, V).$$  \hfill (1)

Tightness, called $\theta$, is defined as the ratio of vacancies to jobseekers :

$$\theta = \frac{V}{N}. \hfill (2)$$

The jobfinding rate $f$ is:

$$f(\theta) = \frac{H}{N} = m(1, \theta). \hfill (3)$$

The jobfilling rate $q$ is:

$$q(\theta) = \frac{H}{V} = \frac{f(\theta)}{\theta}. \hfill (4)$$

Theorem 1. In the standard DMP setup, with fixed matching function, tightness $\theta$ is a sufficient statistic for the jobfinding rate $f(\theta)$ and the jobfilling rate $q(\theta)$.

In other words, two observed variables, the job-finding rate and vacancy-filling rate, are functions of only a single variable, the tightness measure, taken as the vacancy/unemployment rate, $\theta$.

1.2  Using $T$ as the measure of tightness with one type of jobseeker

In this setup, tightness is defined as vacancy duration:

$$T = \frac{V}{H}. \hfill (5)$$

Constant returns of the matching function implies

$$1 = m \left( \frac{N}{H}, \frac{V}{H} \right) = m \left( \frac{1}{f(T)}, T \right). \hfill (6)$$

This defines the jobfinding rate function $f(T)$ implicitly. The job-filling rate is

$$q(T) = \frac{H}{V} = \frac{1}{T}. \hfill (7)$$
Theorem 2. In the DMP setup with vacancy duration $T$ serving as the measure of tightness, with fixed matching function and one type of jobseeker, tightness $T$ is a sufficient statistic for the jobfinding rate $f(T)$ and the jobfilling rate $q(T)$.

Again, $f$ and $q$ are functions of a single measure of tightness, $T$.

1.3 Multiple jobseekers and multiple types of employers

? introduced a matching function that accommodates multiple types of jobseekers. Now we consider a generalization of that matching function that also accommodates multiple industries. Let $i$ index heterogeneous categories of jobseekers with jobfinding rates $f_i(T)$ and let $j$ index heterogeneous employers with jobfilling rates $q_j(T)$. Both depend on a measure of tightness, $T$, that is not observed directly. The function $f_i(T)$ is continuous and increasing, with $f_i(0) = 0$ and $\lim_{T \to \infty} f_i(T) = 1$ and the function $q_j(T)$ is continuous and decreasing, with $q_j(0) = 1$ and $\lim_{T \to \infty} q_j(T) = 0$. The vector $\mathbf{N}$ gives the number of jobseekers of each type and the vector $\mathbf{V}$ gives the number of vacancies for each type of employer.

Definition: A constant-returns matching function with heterogeneous jobseekers and employers is a function $m(\mathbf{N}, \mathbf{V})$ giving the volume of matches as a function of the vectors of jobseekers and vacancies that is first-degree homogeneous and semi-strictly increasing in the vector $\mathbf{X} = [\mathbf{N}, \mathbf{V}]$.

Theorem 3. Let

$$G(\mathbf{X}, T) = \sum_i N_i f_i(T) - \sum_j V_j q_j(T).$$

For any $\mathbf{X}$, there exists a unique value of tightness, $T(\mathbf{X})$, such that $G(\mathbf{X}, T(\mathbf{X})) = 0$. $m(\mathbf{X}) = \sum_i N_i f_i(T(\mathbf{X}))$ is a constant-returns matching function.

Proof: $G(\mathbf{X}, 0) = -\sum_j V_i < 0$. $\lim_{T \to \infty} G(\mathbf{X}, T) = \sum_i N_i > 0$, so there is a $T^*(\mathbf{X})$ with $G(\mathbf{X}, T^*(\mathbf{X})) > 0$. $G(\mathbf{X}, T)$ is continuous in $T$, so there is a $T(\mathbf{X})$ such that $G(\mathbf{X}, T(\mathbf{X})) = 0$ by the intermediate value theorem. $m$ gives the volume of matches because it equals the number of jobs found, which by construction equals the number of jobs filled. $T(\mathbf{X})$ is unique because $G(\mathbf{X}, T)$ is strictly increasing in $T$. $m$ is first-degree homogeneous because a scaling of $\mathbf{X}$ leaves $T(\mathbf{X})$ unchanged and raises $\sum_i N_i f_i(T(\mathbf{X}))$ in proportion. □
**Theorem 4.** In the setup with multiple types of jobseekers and multiple types of employers, and with fixed parameters of the jobfinding rate functions $f_i(T)$ and the jobfilling rate functions $q_j(T)$, tightness $T = T(X)$ is a sufficient statistic for $f_i(T)$ and $q_j(T)$.

We take the local linear approximation,

$$f_i(T) = \alpha_i^f + \beta_i^f T$$

and

$$q_j(T) = \alpha_j^q - \beta_j^q T.$$  \hspace{1cm} (10)

In many cases, we stack the parameter vectors as $\alpha$ and $\beta$.

**Theorem 5.** In the setup with multiple types of jobseekers and multiple types of employers, and with fixed parameters $\alpha$ and $\beta$, $T$ is a sufficient statistic for the jobfinding rates, $\alpha_i^f + \beta_i^f T$, and the jobfilling rates, $\alpha_j^q - \beta_j^q T$.

### 1.4 Implications

Equilibrium in the DMP class of models occurs at the point of zero profit

$$\kappa \cdot T = J$$

Here $J$ is the capital gain an employer receives for a successful hire. It is the difference between the present value of the new worker’s contribution to revenue, $P$, and the present value of the worker’s wage, $W$:

$$J = P - W.$$  \hspace{1cm} (11)

Under the assumption of parameter stability, the sufficient-statistic property of $T$ has the strong implication that only $T$ belongs on the right-hand side of an equation for the jobfinding and vacancy-filling rates—all other variables are excluded. Joint fluctuations in jobfinding and vacancy duration result from changes in the job value $J$ and not from any shift of the matching function. For example, there cannot be seasonal shifts in the jobfinding or jobfilling rates arising from the presence of seasonal shifts of the matching function—any seasonality of jobfinding and jobfilling must result from contemporaneous seasonal movements of $J$. These implications rely on the zero-profit condition and thus need not hold for employers to whom a zero-profit condition does not apply. For this reason, we will apply our model only to data on private-sector vacancies and exclude government vacancies.
2 Statistical Framework

We consider the following model of the relation between an indicator (jobfinding or jobfilling rate) and the underlying unobserved tightness factor, $T_t$:

$$x_{i,t} = \alpha_i + \beta_i T_t + \epsilon_{i,t}.$$  \hspace{1cm} (12)

Here $x_{i,t}$ is indicator $i$ in month $t$, $\alpha_i$ is a measure-specific constant, $\beta_i$ is the response of measure $i$ to tightness, $T_t$ is the latent tightness index discussed earlier, and $\epsilon_{i,t}$ is an idiosyncratic component, including measurement error, assumed to be uncorrelated over time and across tightness measures. We let $i$ run over both the jobfinding series and the jobfilling series. This statistical model imposes strong restrictions on the data. In particular, to the extent that there are secular trends or predictable seasonality in the tightness data, those trends must enter the data through underlying tightness $T_t$; the data series should not be separately detrended or seasonally adjusted before we estimate the common factor $T_t$. This strong restriction is consistent with the our theory of the matching function with multiple categories of jobseekers and employers, under which trends or seasonality in productivity, labor demand, reservation wages, or any other variable affecting jobseekers’ or firms’ behavior affect job-finding rates or vacancy duration only by inducing trends or seasonality in aggregate tightness $T_t$. We test this restriction below.

If the variance of $\epsilon_{i,t}$ is the same across measures $i$ and over time, and if $\epsilon_{i,t}$ is uncorrelated across $i$ and $t$, as we have assumed, $T_t$ is the first principal component of the collection of time series $\{x_{i,t}\}$. The first principal component is defined as the eigenvector corresponding to the largest eigenvalue of the variance-covariance matrix of $\{x_{i,t}\}$. Under the assumption that $\epsilon_{i,t}$ is homoskedastic and uncorrelated across $i$ and $t$, there are only two eigenvalues, the larger of which is $\sigma^2 + \mathbb{E}(T_t^2)$ and corresponds to the eigenvector $\{T_t\}$. As long as the heteroskedasticity and serial correlation are bounded, ? show that the first principal component will still be a consistent estimator of $T_t$.

There are good reasons to believe that the measurement error has different variance for different tightness measures—perhaps so different that the principal-components estimator will not work. The job-finding rate comes from an entirely different data source from the vacancy duration, so the variances of the measurement errors associated with the two different data sources may differ substantially. And within each of these two categories of tightness measures, different measures are estimated from samples of different sizes. For
example, in an average month, 39 observations contribute to the estimated job-finding rate for unemployed people who have entered the labor force within the past three weeks, while 46,500 observations contribute to the estimated rate of job-to-job transitions. Based on the effect of sample size on the variance of an estimate, we would expect the variance of the measurement error to be more than 1,000 times larger for new entrants to the labor force than for job-to-job transitions. Other differences may also cause the measurement error to vary across tightness measures—for example, probabilities close to 0 or 1 can be estimated more precisely than probabilities close to 0.5.

In our baseline specification, we standardize each series $x_{i,t}$ to have mean 0 and variance 1:

$$
\tilde{x}_{i,t} = \frac{x_{i,t} - \mu_i}{\sigma_i},
$$

where $\mu_i$ and $\sigma_i$ are the time-series mean and standard deviation of $x_{i,t}$. We calculate the first principal component of the standardized indicators.

In most cases, standardization should reduce heteroskedasticity. If the response coefficient $\beta_i$ were the same for all $i$, heteroskedasticity in $\epsilon_{i,t}$ would be the only source of different standard deviations $\sigma_i$ for the different series $i$, and standardization would remove all of the heteroskedasticity in $\epsilon_{i,t}$. In practice, because $\beta_i$ varies across series, standardization only approximately removes the heteroskedasticity. In the limit as the differences arising from heterogeneity in the $\beta_i$ dominate the differences arising from the heterogeneity of the variances of $\epsilon_{i,t}$, standardizing makes the heteroskedasticity worse—see ?. We check how our results differ if without standardizing the data. In the appendix, we also report a robustness check where we normalize each series by an estimate of the variance of the measurement error in that series, an approach that makes more statistical assumptions but is potentially more robust to differences in $\beta_i$.

Principal-components analysis provides not only a method for recovering the common latent tightness factor $T_t$, but also a way to test for whether the data actually satisfy the single-factor model in equation (12), or the standardized version of the equation. ? provide a set of criterion functions for estimating the number of factors in a collection of time series. Each criterion function is a penalized measure of the residual sum of squares from the factor model; the estimated number of factors is the number that minimizes the criterion function. We use these criterion functions to estimate the number of factors that drive our time series. We do this for the raw data, for the standardized data, and for data that we have detrended.
and seasonally adjusted by obtaining residuals from regressions of each series on a linear trend and month-of-year dummies. Our theoretical prediction is that one factor (or a very small number of factors) should drive all of the series, even before we remove trends and seasonality, but after we remove heteroskedasticity by standardizing the data. Like Chamberlain and Rothschild’s, Bai and Ng’s method assumes that heteroskedasticity and cross-sectional and time series dependence of the errors $\epsilon_{i,t}$ are bounded but does not require that the errors actually be white noise. Thus, the method is appropriate even if standardization does not entirely remove heteroskedasticity, and even if there is some cross-sectional or time series dependence in the data. This dependence could arise, for example, because the CPS is a rotating panel and because the samples used to calculate the different job-finding rates are not independent.

In our statistical model, equation (12), the sign and overall level of the index of tightness, $T_t$, are not identified—any candidate could be multiplied by any positive or negative factor constant across $i$ and the corresponding $\beta_i$ coefficients divided by the same factor. We normalize $T_t$ so that it has the same standard deviation within our sample as does the aggregate ratio of the number of job openings to the number of monthly hires in JOLTS, and so that it is positively correlated with this ratio. These choices are a pure normalization to make the results easy to relate to published data and have no substantive content.

3 Data

For the job-finding rate, we use data from ?—see that paper and its online replication files for the details. We create 16-month activity histories from the Current Population Survey for all respondents of working age. Then we calculate the frequencies of employment following an observation month, up to 15 months later, conditional on the activity in the observation month. For example, for people observed in third month of their presence in the survey, who were classified as unemployed on account of loss of a permanent job, we calculate the fraction who were employed eight months later, in the 11th month of their presence in the survey. We refer the length of the time from the conditioning month to the month of potential employment as the span—eight months, in this example. We calculated the conditional frequencies of employment for all respondents for all possible spans. These are 1, 2, 3, 12, 13, 14, and 15 months—shorter spans treat short-lived and long-lived jobs equally while longer spans focus on the transition rate into longer-lasting jobs. (In what
follows, we report results only for 1- and 12-month spans; results with 2- and 3-month spans are similar to those with 1-month spans, and results with 13- to 15-month spans are similar to those with 12-month spans.) We considered 16 job-seeking categories: 13 categories of unemployed workers distinguished by the reason for unemployment and reported duration of unemployment; 2 categories of workers out of the labor force, distinguished by whether they say they want to work; and employed workers, who may make a job-to-job transition. In our earlier paper, we adjusted the job-finding rates for changes in demographics, but the adjustments had essentially no effect, so here we use the original tabulations without adjustment. Figure 1 shows annual averages of the raw data for the 16 measures for 1-month spans, without attempting to label them. Figure 2 shows the same measures for 12-month spans. The rates vary a great deal in average level, but common movement is apparent, especially the plunge in the recession that began at the end of 2007.

Figure 3 shows the data from JOLTS on (the negative of) jobfilling rates, as annual averages. Note that most of the rates exceed one—meaning that the typical volume of hires during a month exceeds the number of jobs open at the end of the month. The data on jobfilling rates should be considered a daily or continuous rate stated at monthly levels. There is a good deal of heterogeneity across industries in levels and in relative volatility, but no doubt that they tend to move together.
Figure 2: Jobfinding Rates for 16 Categories of Jobseekers over 12-Month Span

Figure 3: (Negative of) Jobfilling Rate by Industry from the 16 Industries in JOLTS
Both the jobfinding rates and the jobfilling rates vary with tightness, in opposite directions. They have entirely different data sources—measurement errors do not impart any common movements to the two. In both cases, the common source of movement is apparent, though the measures differ in their average levels and in the magnitude of the response to the underlying impetus.

Figure 4 shows all 32 monthly time series we analyze in standardized form—16 series on job-finding rates by category of job seeker, for one-month job-finding spans, and 16 series on jobfilling rates by industry. Figure 5 shows the same series aggregated to annual frequency. The series generally move closely together and most of the differences between them appear to be white noise, a hypothesis we will test formally below.

4 Results

Table 1 shows the estimated number of factors driving the collection of 32 time series, for different versions of the data. Bai and Ng provide three criterion functions for estimating the number of common factors. We almost always find the same number of common factors by all three criteria. The different criterion functions apply only to the estimate of the number of factors; the factors themselves are estimated identically, by principal components. The
Figure 5: Standardized Annual Data on Jobfinding and Jobfilling Rates, for One-Month Jobfinding Spans

table also shows the percentage of the variance in the data explained by each of the first two factors.

We analyze six versions of the data. The first is the raw data. In the next version, we detrend each series by computing residuals from a linear regression on a time trend. In the third version, we detrend and seasonally adjust each series by computing residuals from a linear regression on a time trend and month-of-year dummies. The remaining three versions are identical to the first three except that we standardize each series to have mean 0 and variance 1, after detrending and seasonally adjusting, if applicable. We consider seven variants of each of these six versions—each variant uses a different number of months as the span to measure job-finding rates. Table 1 shows only the results for standardized data and for 1- and 12-month spans. Results for all 42 estimations are available in the online backup for the paper.

If we do not standardize the data, we almost always estimate that 30 or 31 factors underly the data. In other words, heteroskedasticity is so high that each series is portrayed as having its own driving force. Even so, the series clearly have much in common, with the first principal component explaining about half of the total variance of the data, and the second principal component explaining around 20 percent of the total variance. These findings hold
Table 1: Number of Factors According to the Bai-Ng Criteria, and Percentage Explained by the First Two Factors

<table>
<thead>
<tr>
<th>One-month span</th>
<th>Bai-Ng criterion</th>
<th>Variance explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjustment</td>
<td>Number of factors</td>
<td>First factor</td>
</tr>
<tr>
<td>None</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Trend</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Trend and seasonals</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12-month span</th>
<th>Bai-Ng criterion</th>
<th>Variance explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adjustment</td>
<td>Number of factors</td>
<td>First factor</td>
</tr>
<tr>
<td>None</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Trend</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Trend and seasonals</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

whether we remove trends and monthly seasonals before estimating the factors or not, and regardless of the length of the job-finding span.

Reducing heteroskedasticity by standardizing the data reveals that the data have low dimension. Without detrending or seasonally adjusting the data, we almost always find that two factors drive the 32 time series, though for 1- and 2-month job-finding spans we sometimes find three factors. In the standardized data, separately detrending the series generally does not reduce the estimated number of factors, indicating that, to the extent a trend is relevant in explaining the behavior of the data, it is a common trend, as our model predicts. Further adjusting the data, by removing monthly seasonals, tends to reduce the estimated number of factors to 1 instead of 2. This finding suggests there may be some role for seasonality in explaining the data beyond what the model allows, but it is a limited role, much less than the number of factors we would find if each series had its own seasonal pattern. In the standardized data as well, the decomposition of variance shows that the series have much in common, with the first principal component—our estimate of $T$—explaining 20 to 32 percent of the total variance. Note that the theory does not call for $T$ to explain all or even a majority of the variance. Each series is potentially measured with error, and these measurement errors contribute to the variance, perhaps substantially. What the theory demands is that, apart from the single common factor $T$, the series appear to be white noise—which is what we find.
Figure 6 shows our estimates of $T$ for one- and 12-month spans, extracted from the standardized data, with detrended and seasonally adjusted data, smoothed by taking annual averages of the underlying monthly estimates. The values of $T$ are quite similar. They track the business cycle, with a contraction in 2001 and a deeper contraction in 2008 and 2009. These values are normalized to have the same standard deviation as the overall average duration of vacancies reported in JOLTS and are deviations from means.

Table 2 shows the estimates of the coefficients $\beta_i$ reflecting the relation of the jobfinding rate to the unobserved common tightness factor, $T_t$, based on the detrended and seasonally adjusted data. The loadings are all positive, as theory leads us to expect, but there are significant differences in the loadings of different series. For unemployed people, the estimates are generally between 0.15 and 0.4—an increase in tightness of 0.1 months of vacancy duration corresponds to an increase in the jobfinding rate of 1.5 to 4 percentage points—but there is considerable heterogeneity within the job-finding rates. In particular, the job-finding rates of people who have quit or lost permanent jobs have the strongest relation to $T$, while the job-finding rates of people on temporary layoff move less with $T$.

Table 3 shows the estimates of the $\beta$s for the 16 private industries distinguished in JOLTS. Because the unobserved factor $T_i$ is normalized to match the standard deviation of the av-
<table>
<thead>
<tr>
<th>Industry</th>
<th>Span, months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Out of labor force</td>
<td>0.021</td>
</tr>
<tr>
<td>Want job</td>
<td>0.087</td>
</tr>
<tr>
<td>Working</td>
<td>0.013</td>
</tr>
<tr>
<td>Recently laid off</td>
<td>0.149</td>
</tr>
<tr>
<td>Recently lost permanent job</td>
<td>0.324</td>
</tr>
<tr>
<td>Temp job recently ended</td>
<td>0.163</td>
</tr>
<tr>
<td>Recently quit</td>
<td>0.295</td>
</tr>
<tr>
<td>Recently entered</td>
<td>0.179</td>
</tr>
<tr>
<td>Recently re-entered</td>
<td>0.266</td>
</tr>
<tr>
<td>On layoff for months</td>
<td>0.248</td>
</tr>
<tr>
<td>Lost permanent job months ago</td>
<td>0.254</td>
</tr>
<tr>
<td>Temp job ended months ago</td>
<td>0.207</td>
</tr>
<tr>
<td>Quit months ago</td>
<td>0.250</td>
</tr>
<tr>
<td>Entered months ago</td>
<td>0.196</td>
</tr>
<tr>
<td>Re-entered months ago</td>
<td>0.215</td>
</tr>
<tr>
<td>Long-term unemployed</td>
<td>0.159</td>
</tr>
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</table>

Table 2: Estimated Slopes of the Jobfinding Rate with Respect to Tightness

Average ratio of vacancies to hires as published in JOLTS, the loading factors for the negative of industry hires-vacancies ratios are distributed around minus one. Industries in the goods economy—manufacturing, wholesale trade, and transportation—have coefficients above one in absolute value, along with mining, construction, and real estate. Health and financial services have low coefficients in absolute value. The jobfilling rate in the construction industry is about 20 times more responsive to $T$ than the jobfilling rate in finance. Jobfilling rates are generally more responsive to $T$ than are jobfinding rates.

5 The Role of Tightness in Aggregate Fluctuations

In this section, we make the case that a substantial part of the overall movements in aggregate output and an even larger fraction of the movements of aggregate labor input are associated with fluctuations in labor-market tightness. We examine the relationship of the measure of tightness derived earlier in the paper with aggregate output and employment, using the version based on a one-month span. Our measure of output is the equally weighted average of real gross domestic product and gross domestic income—see Table 1.17.5 of the U.S. National
<table>
<thead>
<tr>
<th>Industry</th>
<th>Span, months</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Mining</td>
<td>-2.923</td>
</tr>
<tr>
<td>Construction</td>
<td>-11.121</td>
</tr>
<tr>
<td>Durables</td>
<td>-2.726</td>
</tr>
<tr>
<td>Nondurables</td>
<td>-2.283</td>
</tr>
<tr>
<td>Wholesale</td>
<td>-2.369</td>
</tr>
<tr>
<td>Retail</td>
<td>-1.611</td>
</tr>
<tr>
<td>Transportation</td>
<td>-2.922</td>
</tr>
<tr>
<td>Information</td>
<td>-1.444</td>
</tr>
<tr>
<td>Financial</td>
<td>-0.401</td>
</tr>
<tr>
<td>Real estate</td>
<td>-1.611</td>
</tr>
<tr>
<td>Professional services</td>
<td>-0.915</td>
</tr>
<tr>
<td>Education</td>
<td>-1.240</td>
</tr>
<tr>
<td>Health</td>
<td>-0.501</td>
</tr>
<tr>
<td>Entertainment</td>
<td>-7.044</td>
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<tr>
<td>Accommodation</td>
<td>-2.459</td>
</tr>
<tr>
<td>Other services</td>
<td>-1.075</td>
</tr>
</tbody>
</table>

Table 3: Estimated Slopes of the Industry-Level Jobfilling Rate with Respect to Tightness
Table 4: Simple Regressions of Aggregate Annual Log-Change Variables on the Log Change of Labor-Market Tightness

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient and standard error</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate output</td>
<td>0.224 (0.034)</td>
<td>0.81</td>
</tr>
<tr>
<td>Aggregate hours of work</td>
<td>0.255 (0.034)</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Income and Product Accounts. Our measure of labor input is total hours of work from the Bureau of Labor Statistics. Both variables are reported in John Fernald’s productivity spreadsheet distributed by the San Francisco Federal Reserve Bank.

Table 4 describes the joint distributions of output and tightness, and hours of work and tightness, in the form of simple regressions of the change in the logs of the two variables on the change in the log of tightness. Figure 7 shows the actual and fitted value of the output regression and Figure 8 the same for the aggregate hours regression. The fitted values based on just the tightness track the two recessions of the period since 2001 quite accurately. Despite the fact that output fluctuations include many influences that do not operate through the tightness channel, such as total factor productivity, capital accumulation, and labor-force participation, tightness accounts for more than half of the variance of output growth. And despite the roles of participation, population growth, job-loss, and other non-tightness-related forces, tightness accounts for almost three-quarters of the variance of the annual log-change of total hours of work.

Of course, these regressions have no causal interpretation. Fundamental forces cause tightness to move together with annual output and total hours. Rather, the regressions make a good case that labor-market tightness is a central channel of annual-frequency fluctuations in key aggregate variables. An account of macro fluctuations in a model without a tightness channel is likely to be misleading.

Figure 9 shows the actual and fitted values of the change in the log of the number of unemployed people on tightness, based on a simple regression on the change in tightness.
Figure 7: Actual Values of the Annual Log-Change in Aggregate Output, with Fitted Values from a Regression on Log-Changes in Tightness

Figure 8: Actual Values of the Annual Log-Change in Aggregate Hours of Work, with Fitted Values from a Regression on Log-Changes in Tightness
It is common among macroeconomists involved in policymaking to measure tightness and slackness in terms of output, as the gap between actual and potential real GDP. The Congressional Budget Office maintains such a calculation and updates it for consistency with the current version of the historical data from the National Income and Product Accounts. Figure 10 shows the actual and fitted values of the change in the agency’s gap measure, stated as a decimal, derived from a simple regression on the change in the tightness measured developed in this paper.

Both unemployment and the CBO output gap are highly correlated with tightness. Unemployment has close connections to tightness—transition rates from unemployment to employment are important inputs to our calculation of tightness. But unemployment is not solely a function of tightness, either in theory or in fact. Flows into unemployment vary over time and are not necessarily related to tightness. Outflow rates from unemployment to out-of-the-labor-force are substantial and are also not necessarily related to tightness. Another reason unemployment is not a function of contemporaneous tightness alone is that unemployment is a state variable. Its law of motion cumulates the difference between inflows and outflows. It takes a few months for unemployment to rise fully to the changes in flows that occur at the beginning of a recession. Initially, outflows fall and inflows rise, so
unemployment begins to rise. Inflows return to normal fairly soon, but outflows begin to rise because they are the product of the outflow rate and the level of unemployment, and the latter is rising. Soon, unemployment reaches its new higher stochastic equilibrium level. The process unwinds when the outflow rate rises back to normal and unemployment glides back to normal. Transition rates in the labor market are high enough so that this process is barely visible in annual data.

An extensive literature on the “ins and outs of unemployment” considers flows into and out of unemployment separately as determinants of the level of unemployment. Not surprisingly, as Figure 11 shows, outflows measured in the CPS are highly correlated with our measure of tightness, because somewhat more than half of the outflows from unemployment are to employment, and our measure of tightness includes that flow. More surprising is that the inflow rate to unemployment (ratio of inflows to the number of unemployed people) is equally highly correlated with tightness, as shown in Figure 12. One reason could be that the declines in the value of a worker to a firm, $J$, that result in a slacker labor market also lead to increased separations. However, separations are positively, not negatively, correlated with tightness—quits rise more in tight markets than layoffs fall. The reason for the close relation between the inflow rate to unemployment and tightness is that, as first observed,
transitions between jobs and from out-of-the-labor-market to jobs involve less intervening
unemployment in tighter markets.

The correlation of output and tightness can be broken into two parts: (1) the direct effect
derived from the counterfactual that the unemployed would have been employed but for the
labor-market friction that gives rise to unemployment, and (2) a residual effect from the
correlations of tightness with productivity growth, capital intensification, labor quality, and
hours of work per employed person. We study this issue in terms of Solow’s log-linearization
of the aggregate production function:

$$\Delta \log Y = \Delta \log A + \alpha \Delta \log K + (1 - \alpha)[\Delta \log Q + \Delta \log(1 - u) + \Delta \log H^*]]. \quad (14)$$

Here $Y$ is aggregate output, $A$ is total factor productivity, $\alpha$ is the elasticity of the production
function with respect to capital, $K$ is capital, $Q$ is labor quality, $u$ the unemployment rate,
measuring the direct effect, and $H^* = H/(1 - u)$ is hours in the market (working and looking
for work), where $H$ is hours at work.

Table 5 shows the standard deviations of the terms in the decomposition. The standard
deviation of the log-change in output itself is 2.49 percent per year. The single largest
among the terms of the decomposition is total factor productivity, with a standard deviation

Figure 11: Log-Change in Outflow Rate from Unemployment with Fitted Value from Re-
gression on Log-Change in Tightness
of 1.43 percent per year. Hours in the market is second, at 1.06 percent. The direct effect of unemployment variation is 0.73 percent. Capital and labor quality are substantially smaller contributors are than the other terms. The standard deviations are not additive—the variances would be additive if the terms were not correlated, but they are correlated.

The right-most column of the table show the covariances of the terms with our tightness measure, normalized by the covariance of output and tightness, so the covariances of the terms sum to 1. The covariances reveal the reason that labor-market tightness appears less important in this reckoning than in the simple regressions discussed at the beginning of this section. The covariance of labor tightness with productivity is substantial. The simple regression of output change on tightness change in Table 4 assigns credit to tightness for that covariance whereas this decomposition assigns it to productivity.

6 Labor-Market Model

The purpose of the model developed in this section is to investigate the role of tightness in a model with a conventional DMP matching function, but with two features that extend the DMP framework in directions that help in that investigation. One is credible bargaining, as in ?, and the other is the addition of an economic decision about participation in the labor
Table 5: Standard Deviations and Covariance Fractions Based on the Decomposition of Output Growth

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation, percent</th>
<th>Covariance fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>2.49</td>
<td>1</td>
</tr>
<tr>
<td>Labor share × labor quality</td>
<td>0.19</td>
<td>-0.069</td>
</tr>
<tr>
<td>Labor share × employment rate</td>
<td>0.73</td>
<td>0.296</td>
</tr>
<tr>
<td>Labor share × hours to market</td>
<td>1.06</td>
<td>0.439</td>
</tr>
<tr>
<td>Capital share × capital</td>
<td>0.32</td>
<td>0.003</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>1.43</td>
<td>0.330</td>
</tr>
</tbody>
</table>

market. The specification ranges from the traditional assumption of the DMP literature that participation is completely inelastic—all individuals participate as searching or working—to the polar opposite introduced by ?, where non-participants are indifferent to participation and participation is infinitely elastic at a threshold.

6.1 Financial environment

The states of the economy, denoted $s$, follow a Markov process with transition matrix $\pi_{s,s'}$. In state $s$, the Arrow state price of consumption in a succeeding state $s'$ is $\pi_{s,s'} \beta m_{s'}/m_s$. Here $\beta$ is an overall discount factor and $m_s$ is a state-contingent valuation, which would be marginal utility in a representative-consumer economy. I normalize $m_1 = 1$.

The productivity of the representative worker, $x$, grows stochastically, so it is not state-contingent. Its growth rate is state-contingent:

$$g_{s,s'} = \frac{x'}{x}. \quad (15)$$

Values are stated relative to the current value of productivity. For example, a flow payoff is written $y_s x$, and $y_s$ is the amount of the payoff in productivity units. Its capital value, $Y_s x$, satisfies the present-value condition

$$Y_s x = \sum_{s'} \pi_{s,s'} \beta \frac{m_{s'}}{m_s} y_{s'} x'. \quad (16)$$

Dividing both sides by $x$ yields

$$Y_s = \sum_{s'} \omega_{s,s'} y_{s'}. \quad (17)$$
Here

\[ \omega_{s,s'} = \pi_{s,s'} \beta \frac{m_{s'}}{m_s} g_{s,s'} \]  

(18)
is the Arrow state price adjusted for productivity growth.

### 6.2 Turnover and labor-market frictions, with participation decision

The mechanics of the labor market follow the standard principles of Diamond-Mortensen-Pissarides—see ? and ?. A fraction of the members of the population are out of the labor force each period, a fraction are searching for work, and remainder are working. Employers recruit workers from among the searchers by posting vacancies. The variable \( \theta_s \), the ratio of vacancies to unemployment, indexes the tightness of the labor market. The jobfinding rate depends on \( \theta_s \) according to the weakly increasing function \( f = \min(1, \mu \theta^n) \). The jobfilling rate, the probability that a vacancy will match with a job-seeker, is the decreasing function, \( q = f / \theta \). The jobfinding rate enters the model as a probability, so it is constrained not to exceed 1. The jobfilling rate enters as a rate—all non-negative values are interpretable. The matching function corresponding to the jobfinding and jobfilling rates is

\[ M(U, V) = \min(U, U^{1-\eta} V^n), \]  

(19)

which has constant returns and is increasing in the jobseeking count, \( U \), weakly increasing in the vacancy count, \( V \), and weakly concave.

The separation rate—the per-period probability that a job will end—is a constant, \( \psi \).

When a job-seeker and vacancy match, the pair make a wage bargain, resulting in a wage contract with a present value of \( W_s \). Three values characterize the job-seeker’s bargaining position. If out of the labor force, a person achieves a value \( N_s \). If unemployed, the jobseeker achieves a value \( U_s \). Upon finding a job, she receives a wage contract worth \( W_s \) and also anticipates a value \( C_s \) for the rest of her career, starting with the period of job search that follows the job. While searching, a job-seeker receives a flow value \( z \) per period. All of these values are stated in units of productivity.

The flow value accruing to an individual who is not participating in the labor market (neither searching nor working) is \( y_s \). The Bellman value for non-participation is

\[ N_s = y_s + \sum_{s'} \omega_{s,s'} \max(N_{s'}, U_{s'}), \]  

(20)
reflecting the non-participant’s option to begin searching in the next period. The unemployment Bellman value $U_s$ satisfies

$$U_s = z + \sum_{s'} \omega_{s,s'} \left[ f_s (W_{s'} + C_{s'}) + (1 - f_s) \max(N_{s'}, U_{s'}) \right],$$  \hspace{1cm} (21)$$

reflecting the searcher’s option to exit the labor market. The subsequent career value, $C_s$, satisfies

$$C_s = \sum_{s'} \omega_{s,s'} \left[ \psi \max(N_{s'}, U_{s'}) + (1 - \psi) C_{s'} \right].$$  \hspace{1cm} (22)$$

Note that we assume that work is always sufficiently valuable to preclude an endogenous separation ($W_s + C_s \geq \max(N_s, U_s)$). The job-seeker’s reservation wage value is $W_s = U_s - C_s$, the value sacrificed by taking a job.

Workers produce output with a flow value of 1 in productivity units. The present value, $X_s$, of the output produced over the course of a job, in productivity units, satisfies:

$$X_s = 1 + (1 - \psi) \sum_{s'} \omega_{s,s'} X_{s'}.$$  \hspace{1cm} (23)$$

$X_s$ is the potential employer’s reservation wage value, the highest value an employer would agree to.

The DMP model assumes free entry for employers, so the expected profit from initiating the recruitment of a new worker by opening a vacancy is zero. Thus employer pre-match cost equals the employer’s expected share of the match surplus. The incentive to deploy the resources is the employer’s net value from a match, $X_s - W_s$. Recruiting to fill a vacancy costs $\kappa$ per period. The zero-profit condition is:

$$q_s (X_s - W_s) = \kappa.$$  \hspace{1cm} (24)$$

Notice that the zero-profit condition holds for each value of the state $s$, so recruiting effort varies with $s$.

6.3 Credible bargaining

We assume that a jobseeker who matches with an employer engages in alternating-offer bargaining over the wage, as in ?. For reasons explained in that paper, the unique Nash equilibrium of the alternating offer bargaining game occurs when both parties are indifferent between accepting a pending offer and making a counter-offer one bargaining period
later. The indifference condition for the worker, when contemplating an offer $W_s^E$ from the employer, against making a counter-offer of $W_s^K$, is

$$W_s^E + C_s = \delta U_s + (1 - \delta) \left[ z + \sum_{s'} \omega_{s,s'} (W_{s'}^K + C_{s'}) \right]. \quad (25)$$

The similar condition for the employer is

$$X_s - W_s^K = (1 - \delta) \left[ -\gamma + \sum_{s'} \omega_{s,s'} (X_{s'} - W_{s'}^E) \right]. \quad (26)$$

We assume that the wage is the average of the two values:

$$W_s = \frac{1}{2} (W_s^E + W_s^K). \quad (27)$$

In equilibrium, the receiving party always accepts the first offer. The alternating-offer structure matters only through the off-equilibrium incentives it provides to the parties at the time of the first offer.

### 6.4 Equilibrium with credible bargaining

The state variable of the model, $s$, encodes the driving forces, which are the stochastic discounter $M_{s,s'} = \beta m_{s'}/m_s$ and productivity growth $g_{s,s'}$. An equilibrium is a set of vectors, solving equations (20), (21), (22), (23), (24) (25), (26), and (27).

\[
\{\theta_s, N_s, U_s, C_s, P_s, W_s^E, W_s^K, W_s\},
\]

If the support of $y$ contains only sufficiently low values, the equilibrium will have all individuals searching or working. Conversely, if the support contains only sufficiently high values, nobody will work. In the interesting case, there will be a threshold value $y^*_s$ in state $s$ so that individuals will participate if $y^*_s \leq y$ and not participate otherwise. Under our assumption of no endogenous separations, jobholders automatically continue to participate, so the threshold only applies to individuals who are not employed.

### 6.5 Heterogeneity

The labor market is fully segmented into markets for each flow value of non-work, $y$. The cdf of the flow value within the entire population is $F(y)$. The dispersion of $F$ is inversely related to the sensitivity of participation to its determinants—if there is no dispersion, participation is infinitely responsive at its threshold value.
6.6 Personal transition matrix and stochastic-equilibrium participation rate

The standard DMP model creates a two-state personal transition matrix, with a job-finding rate of $f$ and a job-losing rate of $\psi$. The fraction of the work force unemployed is a state variable obeying these transition rates. Many authors have noted that the actual unemployment rate is close to the stochastic-equilibrium rate for realistic value of the job-finding rate. In the three-state model developed here, the non-zero off-diagonal transition rates are (using obvious notation for the three states):

If $U'' \geq N'$:
- from $N$ to $U$: 1
- from $U$ to $E$: $f$
- from $E$ to $U$: $\psi$

If $U'' < N'$:
- from $U$ to $N$: 1
- from $E$ to $N$: $\psi$

We conclude that, applying the same approach as has been frequently used in the two-state model, the analog in the three-state is to assign probabilities of zero, $\psi/(f + \psi)$, and $1 - \psi/(f + \psi)$ to $N$, $U$, and $E$ when $U'' \geq N'$ and 1 to $N$ when $U'' < N'$. Thus the labor-force participation rate is the fraction of the population with $U'' \geq N'$. This approximation may be somewhat less accurate than the one for the two-state case, because the separation rate is not nearly as high as the jobfinding rate. People who are continuing in a job but will leave the labor force when separated are counted as non-participants in this scheme.

7 Specification and Parameters

7.1 State space

The model implies that each observable state-contingent variable should have the same value for all observations assigned to the same state. In practice, that goal is beyond reach. A finite record limits the number of states for which it is possible to estimate the transition
The hope is that for a well-chosen small number of states, the model still reasonably approximates the behavior of the unattainable model that matches the observed data.

For the labor market, the state needs to record the pronounced cyclical variable, tightness, \( \theta_s \). For the stock market, the price/dividend ratio is an influential state variable based on its forecasting power for subsequent returns. As the basic theme of this paper predicts, the correlation of \( \theta \) and \( P/d \) is fairly high. Accordingly, I constructed the state space by calculating the average of \( \theta \) and \( P/d \) weighted by the inverses of their standard deviations, and then formed 5 equally populated bins based on that weighted average. I also removed an upward linear time trend in \( P/d \). I used the data for the S&P 500 stock portfolio from Robert Shiller’s website.

Table 6 shows the monthly transition matrix, and the average values by state, of tightness \( \theta_s \) and the price/dividend ratio, for the period from January 1948 through May 2015. Figure 13 shows the time series for labor-market tightness, \( \theta \), and the values assigned by the state space bins, based on the averages of the of \( \theta \) and \( P/d \) in each bin. The discretization based on the average index is reasonably successful.

The matrix of state-contingent values of productivity growth \( g_{s,s'} \) appears in the supporting spreadsheet for this paper. Consistent with the lack of a relationship between productivity growth and labor-market tightness revealed in Figure ??, the state space captures the general growth of productivity but only part of its randomness. For reasons to be discussed shortly, the omission of part of the random movements in productivity has almost no effect on the conclusions about the role of discount movements in the determination of unemployment.
7.2 Parameters and variable values common to the DMP model and this paper

Values and sources are:

κ = 0.213, the vacancy holding cost as a ratio to productivity, from \( ? \)

\( \eta = 0.5 \), the elasticity of the Cobb-Douglas matching function, from \( ? \)

\( u = 0.055 \), unemployment at the calibration point

\( \psi = 0.0345 \), the monthly job separation rate, from Shimer (0.10 per quarter)

\( \theta^* = 0.59 \), the ratio of vacancies to unemployment at calibration point, taken as the value in state 3

\( f = (1 - u)\psi/u \), the monthly job-finding rate, from the stationary condition for unemployment

\( \mu = (\theta^*)^{-\eta}f \), the matching efficiency parameter, from the matching function

\( q^* = f/\theta^* \), the vacancy-filling rate

\( z = 0.4 \), the flow value from unemployment as a ratio to productivity, from Shimer
7.3 Parameters of credible bargaining

The parameter $\delta$ controls the probability of interruption of bargaining. If $\delta$ is above 0.2 per month, the model behaves much like the one in ? and has the property pointed out there, that driving forces have little effect on unemployment thanks to the strong equilibrating effect of the wage. The parameter $\gamma$ is the flow cost to employers from delay in bargaining, as a ratio to productivity. Conditional on the earlier labor-market parameters including the flow benefit of unemployment of $z = 0.4$, $\gamma$ controls the overall tightness of the labor market.

To infer the values of these two parameters, I solve the model and the parameters jointly in a system comprising all the equations of the model plus two additional restrictions: (1) that tightness in state 3, $\theta_3$, is its observed value of 0.59, and (2) that the difference between tightness in state 5 and in state 1 is its observed value of $1.01 - 0.38 = 0.63$. Roughly speaking, the first of these pins down $\gamma$ and the second $\delta$. The resulting values are $\gamma = 0.57$ delay costs in productivity units and $\delta = 0.013$ interruptions per month. The low value of $\delta$ substantially isolates the wage from conditions in the labor market and thus makes labor-market tightness $\theta$ quite sensitive to the discount rate. The value is somewhat below half the hazard of employment ending. The resulting state-contingent values of tightness are quite close to their actual values.

7.4 Participation function

As noted earlier, we take the participation function to be the fraction of the population with unemployment value $U_s$ at least as high as the out-of-labor-market value $N_s$. The participation rate rises with the state number, $s$—there are some people who only participate in stronger markets. The first column of Table 7 shows the U.S. participation rate by state $s$, $p_s$, since 1949. Participation is only slightly positively associated with tightness as measured by the state.

To find the participation function implied by the model and match it to the actual behavior of participation, we proceed in two steps. On any grid of values of $y$, participation will be complete in all five states below a value $y_L$ and zero above another threshold $y_H$. We locate these values using a fairly coarse grid. We use $y_L = 0.93$ and $y_H = 0.96$. Then, on a fine grid with 1000 values we locate the thresholds for participation in all 5, then 4, down to zero states, with rising $y$. The model’s participation function $\tilde{p}_s$ for the subpopulation with $y \in [y_L, y_H]$ is shown in the second column of the table. Participation is highly sensitive
<table>
<thead>
<tr>
<th>State</th>
<th>Actual participation rate</th>
<th>Participation rate in subpopulation</th>
<th>Model’s participation function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6283</td>
<td>0.0679</td>
<td>0.6283</td>
</tr>
<tr>
<td>2</td>
<td>0.6296</td>
<td>0.3117</td>
<td>0.6291</td>
</tr>
<tr>
<td>3</td>
<td>0.6302</td>
<td>0.5255</td>
<td>0.6298</td>
</tr>
<tr>
<td>4</td>
<td>0.6306</td>
<td>0.7123</td>
<td>0.6306</td>
</tr>
<tr>
<td>5</td>
<td>0.6314</td>
<td>0.8492</td>
<td>0.6314</td>
</tr>
</tbody>
</table>

Table 7: Actual and Model-Based Participation Function by State

To tightness in this narrow subpopulation. The model cannot identify the distribution of $y$ outside the subpopulation because participation is complete for $y < y_L$ and zero for $y > y_H$, irrespective of the state. But it can identify the mass below $y_L$, which we call $\rho_L$ and the mass above $y_H$, which we call $\rho_H$.

On the assumption that $y$ is distributed uniformly in the narrow subpopulation, the following two equations hold:

$$p_1 = \tilde{p}_1(1 - \rho_L - \rho_H) + \rho_L$$  \hspace{1cm} (29)

and

$$p_5 = \tilde{p}_5(1 - \rho_L - \rho_H) + \rho_L.$$  \hspace{1cm} (30)

This is a linear system of two equations in the two unknowns $\rho_L$ and $\rho_H$. The solution is $\rho_L = 0.6257$ and $\rho_H = 0.3663$. The residual probability of the interval $[y_L, y_H]$ is 0.0080. Less than one percent of the population is in the region where participation is locally responsive to its payoff. The model’s participation function, shown in the right-hand column of Table 7, is the linear interpolation within the values in $[y_L, y_H]$. 
References


