

# The Black–Scholes–Merton Model as an Idealization of Discrete-Time Economies

Seminar given at U. Chicago on November 5

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On November 5, at the University of Chicago, I will be giving a research seminar on the question: For which discrete-time financial market models, where trading takes place “rapidly,” does the classic Black–Scholes–Merton (BSM) model provide a good (economic) idealization?

The larger motivation for this work is to explore when and whether discrete-time models of economic phenomena “converge” in economic terms to their mathematical limits. The immediate project, about which I will talk, concerns the relatively simple case of models of financial markets, which are relatively simple because all information is held in common.

Following seminal work on this topic by William Sharpe and John Cox, Steve Ross, and Mark Rubinstein, it is generally “appreciated” that, if the discrete-time market models are driven by binomial random walks, then BSM provides a good idealization, but if the discrete-time market models are driven instead by random walks with one-step supports of more than two possible stock prices, then BSM is not a good idealization.

In some respects (not of great economic significance), the first part of this “common understanding” is wrong. And the second part is very definitely wrong.

In the seminar, I will present an overview of these ideas. The overview is based on a monograph and two papers:

- The monograph has the same title as this précis of the seminar. It is published by Cambridge University Press, in the Econometric Society Monograph Series, and is available from Amazon.com. (Search for Kreps Black–Scholes–Merton.)
- “Asymptotic Synthesis of Contingent Claims in a Sequence of Discrete-Time Markets,” by Walter Schachermayer and myself, is available at SSRN: <https://ssrn.com/abstract=3402645>. (Note that the paper available on SSRN is a revision, with a September 2019 date.)
- “Convergence of Optimal Expected Utility for a Sequence of Discrete-Time Markets,” also with Walter Schachermayer, is available at SSRN: <https://ssrn.com/abstract=3417898>

Both papers are deep mathematical dives into the two criteria by which we judge whether a sequence of discrete-time market models is asymptotically idealized by the BSM model: Can an investor asymptotically synthesize any (reasonable) contingent claim that

she desires? If she is an expected-utility maximizer, is what she can get along the sequence in terms of her expected utility what she can get at the limit, in the BSM economy? I will sketch the answers, and in the case of the first paper, I will sketch the proof of the main result, but otherwise I will not go into details of the proofs in the talk. (At Northwestern, I am giving a minicourse in which the details will be covered.)

If you are familiar with Arrow's classic paper "The role of securities markets..." and the basic ideas of the Black–Scholes–Merton model of a two-securities market, no prior reading will be necessary. But if you do wish to do in-advance reading, the best program would be to get the monograph, read Chapter 1, review Chapters 2 and 3 (in the case of Chapter 3, it may be more than a review), then concentrate on Chapters 5, 7, and 9, and finally proceed to the two papers.