Self-Fulfilling Debt Crises, Revisited: The Art of the Desperate Deal*

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Abstract

We revisit self-fulfilling crises by introducing an alternative equilibrium selection that involves bond auctions at depressed but strictly positive equilibrium prices. We refer to these auctions as "desperate deals," and discuss counterparts in the data. Quantitatively, auctions at fire-sale prices are crucial for generating realistic spread volatility. We also explore equilibria in which the sovereign repurchases debt when prices are low, and establish that welfare potentially improves. This suggests that Bulow and Rogoff's well-known critique of buy-backs does not extend to self-fulfilling debt crises.

1 Introduction

In this paper we explore a novel class of self-fulfilling sovereign debt crisis equilibria. We build on the familiar Cole and Kehoe (2000) framework in which a coordination failure can

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lead to a "failed auction" and subsequent default.[1] We extend this to incorporate self-fulfilling equilibria in which the sovereign auctions bonds at fire-sale – but strictly positive – prices. Such "desperate deals" are consistent with the experiences of emerging markets and recent European crisis countries, in which spreads are high and volatile but default remains relatively rare. The standard Cole-Kehoe equilibrium has difficulty explaining such episodes given the stark assumption that a crisis results in a price of zero for new issuances and default with probability one. We explore quantitatively the differences between our framework and the canonical model, and show that including fire-sale auctions as part of the equilibrium path is crucial for understanding the high volatility of spreads.

The framework we explore builds on the standard Eaton and Gersovitz (1981) model and the recent quantitative versions beginning with Aguiar and Gopinath (2006) and Arellano (2008). In particular, the government of a small open economy faces endowment risk and issues non-contingent (but defaultable) bonds to a pool of competitive foreign investors. The creditors involved in sovereign lending are risk-averse with finite wealth, and hence the sovereign pays a risk premium. As in Cole and Kehoe (2000), our timing convention allows the sovereign to default in the same period as a successful auction. Cole and Kehoe used this timing to support an equilibrium price of zero for any amount of bonds sold at auction, which in turn is supported by immediate default due to the inability to rollover maturing bonds. Cole and Kehoe considered an equilibrium selection in which bonds are auctioned at a positive prices in non-crisis periods, but conditional on the realization of a sunspot, creditors coordinate on the zero-price equilibria, triggering default.

The idea that some factor other than domestic fundamentals, such as creditor beliefs about the equilibrium behavior of other lenders, is compelling. Aguiar, Chatterjee, Cole, and Stangebye (in process) document a number of supporting facts regarding emerging market and European bonds. First, as is well known, emerging market spreads over benchmark risk-free bonds are volatile. Second, while large spikes in spreads are correlated with declines in output, the correlation is relatively weak. In fact, a sizable proportion of such spikes occur when growth is positive and in line with historical means. The same holds in the

[1] There are two main traditions in the self-fulfilling debt crisis literature, associated with Calvo (1988) and Cole and Kehoe (2000). Loosely speaking, the former tradition focuses on the link between prices today and budget sets (and incentives to default) tomorrow. See Lorenzoni and Werning (2013) and Ayres, Navarro, Nicolini, and Teles (2015) for recent papers in the Calvo tradition. The Cole and Kehoe (2000) model features multiple pairs of prices and contemporaneous default decisions that satisfy equilibrium conditions, with multiplicity reminiscent of a bank run. Recent papers in this tradition include Conesa and Kehoe (2011) and Aguiar, Amador, Farhi, and Gopinath (2015).

shorter sample of European crisis countries (Portugal, Ireland, Italy, Spain, and Greece). While the literature has shown some of the variation in spreads can be explained by shifts in measures of global risk premia, there remains a large and time-varying unexplained residual component. One possible interpretation of this residual source of risk is shifts in creditors beliefs about the behavior of other creditors. Recently, Bocola and Dovis (2016) performed an accounting exercise on the Italian debt crisis and found that shifts in the probability of a self-fulfilling crisis played a non-negligible role in explaining the spike in spreads.

As mentioned above, the failed auctions of the standard Cole-Kehoe model shed light on how creditor beliefs can play a role in generating defaults, and how this prospect affects government policy ex ante. However, in practice, sovereigns in crisis frequently escape default by issuing a minimal amount of bonds at low prices. As a motivating example, consider the case of Portugal.[2] Yields on Portugal's bonds increased in 2010. By the start of 2011, Portugal was in distress and having difficulty rolling over its maturing bonds. In January of 2011, it issued one billion euros in a "private placement" that was reportedly purchased by China.[3] This was not sufficient to stem the crisis, and in May of that year Portugal began to draw on emergency funding from the EU. In late 2012, the prospect of bonds maturing in 2013 loomed. In anticipation, the Portuguese debt agency re-purchased bonds maturing in September 2013 while issuing bonds maturing in 2015. This swap was accomplished not through default, negotiation, and restructuring, but rather was implemented via a dual auction.[4] The *OECD Sovereign Borrowing Outlook 2013* referred to this type of transaction as "market-friendly solutions to resume market access and to ease near-term redemption pressures." A benefit of the operation was to avoid the risk of a failed auction in 2013 when the original bonds matured.[5] As it turned out, Portugal did successfully auction bonds in 2013, but did so without the threat of a rollover crisis due to the maturity swap.

This narrative gives a sense in which a debt crisis involves a rich menu of possibilities, even in the absence of outright default and re-negotiation. We capture some of this richness

[2]We are grateful for conversations with Pedro Teles regarding Portugal's debt management during the crisis.

[3]See `http://ftalphaville.ft.com/2011/01/11/453471/p-p-p-ick-up-a-portuguese-private-placement/` and `http://uk.reuters.com/article/portugal-bonds-idUSLDE7061QG20110107`.

[4]See `http://www.igcp.pt/fotos/editor2/2013/Relatario_Anual/Financiamento_Estado_Port_uk.pdf` page 6.

[5]The Portuguese debt agency annual report for 2012 (`http://www.igcp.pt/gca/?id=108`) notes that "the management of the debt portfolio takes into account the refinancing profile of (IGCP) the debt, so as to avoid an excessive concentration of redemptions..." Its 2013 report states that its various operations "enabled the IGCP to accumulate levels of liquidity" which it used in part to reduce additional future commitments.

in a tractable manner by incorporating "desperate deals" as part of the equilibrium outcome during a "coordination failure." In particular, we follow Cole and Kehoe and introduce a sunspot that coordinates creditor beliefs between a relatively high equilibrium price schedule and a crisis price schedule. However, rather than the latter involving zero prices and immediate default, we consider an equilibrium price schedule which makes the government indifferent to default or repayment immediately after the auction. In our quantitative model, such prices typically imply spreads roughly 500 basis points higher than non-crisis periods, which is inline with many real-world episodes. This price schedule is rationalized by allowing the government to play a mixed-strategy over post-auction default, with the probability of default consistent with that period's equilibrium price schedule. As the government is indifferent, randomization is an acceptable best response to the equilibrium price schedule. In this sense, our approach corresponds to a worldview that debt crises push a sovereign to the brink of default, but whether default is actually realized is a random outcome that is independent of fundamentals, and, from the creditors' perspective, a matter of luck.

A few features of this approach are worthy of note. The equilibrium price schedule and the government's mixed-strategy response are part of a competitive equilibrium. While bargaining and re-negotiation are important aspects of sovereign default,[6] many emerging markets and all European crisis countries other than Greece managed their crises without resorting to outright default. The auctions we consider are arms-length transactions involving competitive prices. Moreover, as the prices are competitive, they are not bailouts. While bailouts are a feature of many crises, the economics of official assistance are relatively well understood. Our focus is on the less familiar market transactions that occur during crisis periods.

Although the desperate deals do not involve bargaining or transfers, they do benefit legacy bondholders (compared to default), and deliver the default-value to the government without the associated deadweight costs of default. In this sense, conditional on the occurrence of a crisis, the deals raise the efficiency of bond markets. Given the competitive nature of the bond market, the sovereign reaps this gain ex ante through better prices. We show that this has important implications for welfare as well as the willingness of the government to borrow despite the prospect of crises. An important analytical insight of the Cole and Kehoe (2000) model is that potential of a crisis, and the associated ex ante equilibrium price schedule, induces the government to delever in order to avoid being vulnerable to a self-fulfilling run.

[6] See Benjamin and Wright (2008), for example.

Replacing failed auctions with desperate deals mitigates this tendency.

We calibrate the model to Mexico and quantitatively contrast our benchmark model with desperate deals to the canonical Cole-Kehoe framework in which crises generate certain default. With desperate deals, we match key bond market regularities, including the average and standard deviation of bond spreads, average debt-to-income ratios, and a default frequency of twice every one hundred years, the latter being consistent with broad historical samples. In the Cole-Kehoe version of the model, the standard deviation of bond spreads is a factor of *twenty-five* times too small. While that model generates frequent enough defaults, the sovereign never borrows into high spreads. In our benchmark model, the government is more willing to accumulate debt, and, more importantly, willing to issue bonds at fire-sale prices when faced with the crisis price schedule.

Using our benchmark model, we also contrast defaults due to a coordination failure versus "fundamental" defaults in which the government defaults despite the creditors coordinating on the better equilibrium price schedule. The latter have a distinct boom-bust pattern, in which default is preceded by abnormally high growth followed by a large negative growth realization. The high growth generates high bond prices, inducing the government to leverage up. The relatively high level of debt leaves the sovereign unwilling to repay when an abnormally low growth outcome is realized. This pattern shares something in common with the data, but our empirical work suggests booms followed by large recessions represent only a fraction of debt crises in practice. Moreover, fundamental defaults do not generate an ex ante spike in spreads, as the low growth realization is largely unanticipated given the preceding Markov process for the endowment.

The model's defaults associated with coordination failures do not have an anticipatory boom and coincide with a relatively moderate contraction of endowment. Relatively high debt levels are also a necessary component of default, but in our benchmark these are frequently observed in the ergodic distribution due to the reasons discussed above. Given this vulnerability, a coordination failure generates a spike in spreads as the government issues bonds at "desperate deal" prices. The benchmark simulations rationalize why large recessions may not yield large jumps in spreads, while smaller recessions can be associated with extremely adverse outcomes, a pattern observed in many sovereign debt crises.

Finally, we consider crisis equilibria in which the government *repurchases* non-maturing bonds at fire-sale prices. As legacy bondholders prefer this outcome to default, ex ante prices are higher when such events are likely to occur. We show that this has ambiguous effects on ex ante government welfare. On the one hand, a better outcome in the event of a crisis increases the efficiency of bond markets, which the government captures via prices due to competitive markets for its bonds. On the other, more efficient markets encourages the sovereign to borrow more, raising the likelihood of a crisis going forward, which may lower prices. We explore two buy-back scenarios. When buy-backs occur only in response to self-fulfilling crisis, ex ante welfare increases relative to the Cole-Kehoe equilibrium. However, when buy-backs occur when fundamentals are bad enough to induce default even in the "good" equilibrium, ex ante welfare is reduced.

The rest of the paper is organized as follows. Section 2 lays out the model; Section 3 discusses equilibrium multiplicity and how we model rollover crises involving desperate deals; Section 4 discusses how we construct an equilibrium involving repeated rollover crises; Section 5 discusses calibration; Section 6 present the quantitative results; Section 7 contains a brief discussion of how including desperate deals compares with other approaches in the literature to mitigating the deadweight costs of default and the implications for welfare; and Section 8 concludes.

2 Environment

We consider a single-good, discrete-time environment, with time indexed by $t = 0, 1, \ldots$. The focus of the analysis is a small open economy that receives a stochastic endowment. The economy is small relative to the rest of the world in the sense that its endowment realizations and decisions do not affect the world risk-free interest rate. However, financial markets are segmented in the sense that the economy can borrow from a set of potential lenders with limited wealth. Consumption and saving decisions are made on behalf of the domestic economy by a sovereign government. In this section, we proceed by characterizing the domestic economy and the sovereign's problem, then turn to the lenders' problem, and conclude by defining an equilibrium in the sovereign debt market.

2.1 Endowment

The economy receives a stochastic endowment $Y_t > 0$ each period. The endowment process is characterized by:

$$Y_t = G_t e^{z_t}, \tag{1}$$

where

$$\ln G_t \equiv \sum_{s=1}^{t} g_s, \tag{2}$$

is the cumulation of period growth rates g_t, and z_t represents fluctuations around trend growth. We assume that g_t and z_t follow finite-state first-order Markov processes. The relevant state vector for the current endowment and its probability distribution going forward is (Y_t, g_t, z_t).

2.2 Financial Markets

The sovereign issues non-contingent bonds to a competitive pool of lenders (described below). Bonds pay a coupon every period up to and including the period of maturity, which, without loss of generality, we normalize to r^* per unit of face value, where r^* is the (constant) international risk-free rate. With this normalization, a risk-free bond will have an equilibrium price of one. For tractability, we consider a bond with random maturity, as in Leland (1994).[7] In particular, each bond matures next period with a constant hazard rate $\lambda \in [0, 1]$. We let the unit of a bond be infinitesimally small, and let maturity be *iid* across individual bonds, such that with probability one a fraction λ of any non-degenerate portfolio of bonds matures each period. The constant hazard of maturity implies that all bonds are symmetric before the realization of maturity at the start of the period, regardless of when they were purchased. Note as well the expected maturity of a bond is $1/\lambda$ periods, and so $\lambda = 0$ is a console and $\lambda = 1$ is one-period debt. While we vary λ across quantitative exercises, within any specific environment there is only one maturity traded. With these conventions, a portfolio of sovereign bonds of measure x receives a payment (absent default) of $(r^* + \lambda)x$, and has a continuation face value of $(1 - \lambda)x$.

Let B_t denote the outstanding stock of debt at the start of the period t. We do not restrict the sign of B_t, which allows the government to be either a net creditor ($B_t < 0$) or debtor ($B_t > 0$). Net issuances of new debt in the period is given by $B_{t+1} - (1-\lambda)B_t$, where $(1-\lambda)$ is the fraction of debt that does not mature in the current period. If $B_{t+1} < (1-\lambda)B_t$, then the government is repurchasing its outstanding debt rather than issuing new debt. To rule out Ponzi schemes, we place an upper bound on end-of-period debt as a ratio of current income: $\frac{B_{t+1}}{Y_t} \leq \bar{b}$, $\forall t$.

[7] See also Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012) and Arellano and Ramanarayanan (2012).

2.2.1 Timing

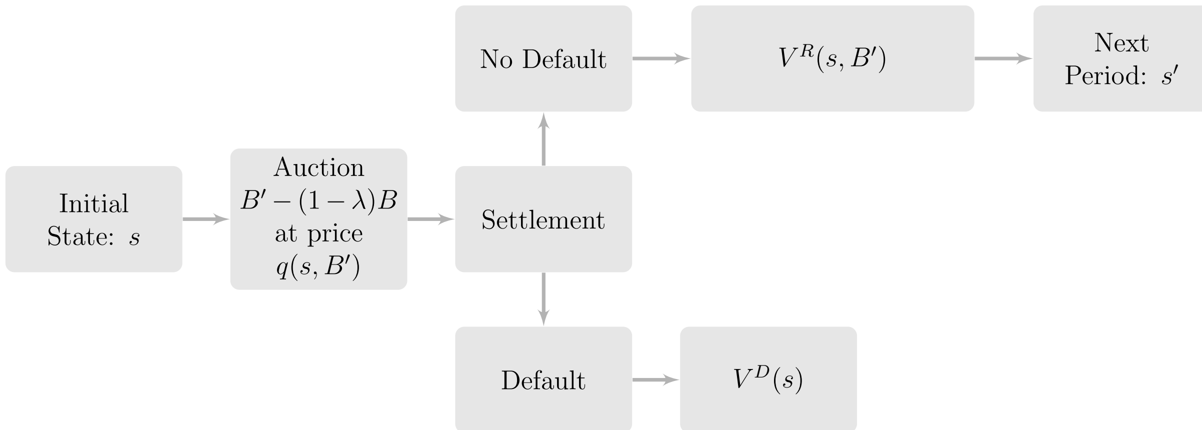


Figure 1: Timing within a Period

The timing of a period is depicted in Figure 1. Let s denote the aggregate state after the period's realization of exogenous state variables but before the period's consumption and debt-issuance decisions have been made. Specifically, $s = (Y, g, z, B, \rho)$, where (Y, g, z) characterize the current period endowment state; B is the inherited stock of debt; and ρ is a random variable that coordinates equilibrium beliefs and is discussed in Section 4. Other than B, the elements of s follow an exogenous Markov process. For notation, let S denote the possible state space for s, and let SB denote the state space augmented to include next period's debt:

$$SB \equiv \left\{ (s, B') | s \in S \text{ and } \frac{B'}{Y} \leq \bar{b} \right\}.$$

After observing the period's realized s, the government decides to auction $B' - (1 - \lambda)B$ units of debt, where B' represents the face value of debt at the start of the next period. There is one auction per period. While this assumption is standard, it does allow the government to commit to the amount auctioned within a period.[8]

We postpone the formal definition of equilibrium until Section 2.5, but to anticipate we shall consider equilibria in which endogenous variables are functions of the state $s \in S$. Let $q(s, B')$ denote the equilibrium price schedule. The government is large in its own debt market and internalizes the fact that it faces different prices depending on how much debt

[8]For an exploration of an environment in which the government cannot commit to a single auction, see Hatchondo and Martinez (undated); Lorenzoni and Werning (2013).

it auctions; hence in choosing, B' the government internalizes the entire price schedule as a function of B'. It is useful to define $x(s, B')$ as the equilibrium amount raised at auction per endowment, if positive, given an amount auctioned B' and a price schedule $q(s, B')$:

$$x(s, B') \equiv \max \left\{ q(s, B')(B' - (1 - \lambda)B), 0 \right\}. \tag{3}$$

The proceeds from auction are held in escrow until the government makes a repayment decision. In particular, the government can use these funds to pay its outstanding liabilities, but cannot draw on them for consumption unless all such payments are made. In particular, given outstanding debt-to-income B, the government is contractually obligated to pay λB in principal and r^*B in interest payments. These payments are financed through current endowment as well as the revenue raised by the auction of new debt. If the government makes its contracted payments, it consumes $C = Y - (r^* + \lambda)B + q(s, B')(B' - (1 - \lambda)B)$, and continues on to the next period with the new debt state implied by B'. We capture the value of the government in the repayment state by $V^R(s, B')$, which we define in the next subsection.

If the government defaults, the amount in settlement $x(s, B')$ is disbursed to all claimants on the basis of the face value of their claims. In particular, there are holders of current liabilities, totaling $(r^* + \lambda)B$, as well as holders of future liabilities, with a face value B'. In the period of default, each unit of such claims receives a payout R^D:

$$R^D(s, B') = \frac{x(s, B')}{B' + (r^* + \lambda)B}. \tag{4}$$

If $B' < (1 - \lambda)B$, then the government has repurchased bonds on net. In this case we assume that the original holders of the repurchased bonds receive their payment at the time of the auction and there are no funds left in escrow at the time of default. In this case, $R^D(s, B') = 0$. The assumption that bondholders receive payments (if any) in proportion to the face value of their claims reflects the *parri passu* and acceleration clauses typically in sovereign bond contracts.

In addition to losing any auction revenue, if the government fails to make all contracted payments in the period it enters the "default state" at the start of the next period. While in default, the government has no access to foreign financial markets. Moreover, in the default state the government loses part of its endowment, which proxies for economic disruptions that are a consequence of default in practice. Let ϕ denote the proportional loss of output, so that in the default state the government receives $(1-\phi)Y$ when the non-default state endowment is Y. While in default, the government has a constant hazard $\xi \in [0,1]$ of exiting the default state. Having exited default, the government no longer suffers an endowment penalty and regains access to foreign financial markets. The debt outstanding at the time of default is forgiven, implying that creditors are paid only the amount disbursed at settlement. Thus the default state runs from the period following failure to repay through the period before the economy regains access to financial markets. We denote the expected value conditional on default by $V^D(s)$.

The timing places the auction before the default decision. An important implication of this convention is that holders of newly issued bonds are not fully compensated if the government defaults immediately after the auction. In this regard, our timing deviates from that of Eaton and Gersovitz (1981), which has become standard in the quantitative sovereign debt literature. In the standard timing, the bond auction occurs after that period's default decision has been made. Thus newly auctioned bonds do not face within-period default risk. Our timing expands the set of equilibria relative to the Eaton-Gersovitz timing, and in particular allows a tractable way of introducing self-fulfilling debt crises.[9] We discuss such equilibria in detail below, and in particular discuss the separate roles of the initial auction and the "Settlement" process by which auction revenue is dispersed and existing liabilities are (or are not) paid.

2.3 The Government's Problem

The domestic economy is run by an infinitely lived sovereign government, which enjoys preferences over the sequence of aggregate consumption $\{C_t\}_{t=0}^{\infty}$ given by:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(C_t),$$

with $\beta \in (0,1)$ and

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

[9]The timing in Figure 1 is adapted from Aguiar and Amador (2014), which in turn is a modification of Cole and Kehoe (2000). The difference relative to Cole and Kehoe is that we do not allow the government to consume the proceeds of an auction if it defaults. See Auclert and Rognlie (2014) for a discussion of how the Eaton-Gersovitz timing in some standard environments has a unique Markov equilibrium, thus ruling out self-fulfilling crises.

with $\sigma \neq 1$. We assume that the sovereign has enough instruments to implement any feasible consumption sequence as a domestic competitive equilibrium, and therefore abstract from the problem of individual residents of the domestic economy. This does not mean that the government necessarily shares the preferences of its constituents, but rather that it is the relevant decision maker viz a viz international financial markets.

To ensure that the government's problem is well behaved, we require:

$$\max_{g} E_g \left\{ \beta e^{(1-\sigma)g'} \right\} < 1,$$

where the max is taken over elements of the Markov process for g_t.

Let $V(s)$ denote the start-of-period value for the government, conditional on the state s and the equilibrium price schedule q (which we suppress in the notation). Working backwards through a period, at the time of settlement the government has issued $B' - (1-\lambda)B$ units of new debt at price $q(s, B')$ and owes $(r^* + \lambda)B$.

If the government honors its obligations at settlement, its payoff is:

$$V^R(s, B') = u(C) + \beta \mathbb{E}\left[V(s')|s, B'\right], \tag{5}$$

with

$$C = Y - (r^* + \lambda)B + q(s, B')[B' - (1-\lambda)B]$$

Note that consumption is pinned down at settlement by the budget constraint; if the required consumption is negative, we define $V^R(s, B') = -\infty$, which is always dominated by default.

In the period of default, the sovereign's payoff is:

$$V^D(s) \equiv u(Y^D) + \beta \mathbb{E} V^E(s'), \tag{6}$$

where $Y^D = Ge^{\bar{z}}$ and V^E denotes the continuation value while in the default (exclusion) state:

$$V^E(s) = u((1-\phi)Y) + \beta(1-\xi)\mathbb{E}\left[V^D(s')|s\right] + \beta\xi\mathbb{E}\left[V(s')|s, B' = 0\right]. \tag{7}$$

Note that in the period of default, we evaluate Y^D at the mean *iid* endowment shock $z = \bar{z}$. This is done for computational reasons, and makes V^D (but not V^R) independent of the current z realization. Given the low variance of our calibrated z, this is not a crucial assumption. For expositional reasons, we do not impose the output punishment ϕ until the period after default. This simplifies the comparison of endowment growth in periods of default and repayment, and is innocuous given that ϕ can be scaled accordingly. Recall that ξ is the probability of regaining access to financial markets. Once this occurs, the punishment ϕ is no longer operative and the government resumes issuing bonds with no legacy debt.

The start-of-period value function is:

$$V(s) = \max \left\langle \max_{B' \leq \bar{b}Y} V^R(s, B'), V^D(s) \right\rangle, \ \forall s \in S. \tag{8}$$

Let $\mathcal{B}$ denote the policy function for B' generated by the government's problem, defined on S. We denote the policy function for default at settlement conditional on B' by $\mathcal{D}(s, B')$. As we discuss in detail below (Section 3), we allow the government to randomize over default when indifferent; that is, when $V^R(s, B') = V^D(s)$. Therefore, $\mathcal{D} : SB \to [0, 1]$ is the probability the government defaults at settlement. The policy function of consumption is implied by those for debt and default.

2.4 Lenders

We assume financial markets are segmented and only a subset of foreign agents participate in the sovereign debt market. This assumption allows us to introduce risk premia on sovereign bonds while treating the risk-free rate as parametric. For tractability, we assume that a set of lenders has access to the sovereign bond market for one period, and then exits, to be replaced by a new set of identical lenders. The short horizon of the specialist lenders is for tractability, avoiding the need to solve an infinite horizon portfolio problem and carry another endogenous state variable.

Specifically, each period a unit measure of identical lenders enter the sovereign debt market. Let W denote the aggregate wealth of the agents that can participate in the current period's bond market. The entering "young" lenders allocate their wealth across sovereign bonds and a risk free asset that yields $1 + r^*$. As noted above, the risk-free rate is pinned down by the larger world financial market, and specialists in the sovereign bond market can freely borrow and lend at this rate.

Recalling the timing of Figure 1, "old" lenders enter a period with B units of debt. A fraction λ of the representative portfolio matures, which is paid at settlement. We also assume that all coupon payments (on both maturing and non-maturing bonds) are paid at settlement. The remaining (ex-coupon) non-matured bonds, $(1-\lambda)B$ are sold to "young" lenders at the time of auction. In particular, new lenders purchase from the legacy lenders the stock of non-maturing bonds plus any new bonds the government auctions at the same time.[10] At the end of the auction, new lenders hold all non-maturing bonds.

With this timing, we can compute the return on bonds purchased in the current period by young lenders in state s when the government's end-of-auction stock of debt is B'. In particular, consider a young lender that purchases a unit-measure portfolio today, paying $q(s, B')$ at auction. If the government defaults in the current period, the young lender receives $R^D(s, B')$, where R^D is defined by (4). As the lender is still young, it can invest this amount in risk-free bonds. If the government does not default this period, the young lender holds the sovereign bonds into the next period.

Next period, the lender is now "old." It sells $1-\lambda$ at auction, and receives $q(s', B'')$, where B'' reflects next period's debt issuance decisions. In equilibrium, this will be $B'' = \mathcal{B}(s')$. The lender receives $q(s', B'')(1-\lambda)$ for these bonds regardless of the government's subsequent default decision. In addition, if the government does not default, the lender receives $r^* + \lambda$ at settlement. Otherwise, it receives $R^D(s', B'')(r^* + \lambda)$ at settlement.

Let δ and δ' denote indicator functions that take the value of one if the government defaults in the current or next period, respectively, and zero otherwise. The preceding implies that the realized return on a sovereign bond, denoted $\tilde{R}$, purchased at price $q(s, B')$ is given by:

$$\begin{aligned}\tilde{R} = \frac{1}{q(s, B')} \Big[& (1-\delta) q(s', B'')(1-\lambda) \\ & + \delta R^D(s, B')(1+r^*) \\ & + (1-\delta)\delta' R^D(s', B'')(r^* + \lambda) \\ & + (1-\delta)(1-\delta')(r^* + \lambda) \Big]. \end{aligned} \tag{9}$$

The first term on the right is the sale of non-maturing bonds at next period's auction, which only occurs if there is no default this period; the second term is the payment at settlement

[10] Our auction assumption is that payments due within the period, that is, coupons and matured bonds, are not sold to new lenders. This is done for tractability, as currently due payments will sell at a different price than new bonds.

in case of immediate default, which is then invested at the risk-free rate; the third term is the payment at settlement next period in case of default, scaled by the claims on coupons and matured principal; and the final term is the payment of coupon and principal absent default in either period. Note that while next period's price incorporates the government's default policy that period, the sale takes place before next period's default decision is made. Hence, it is not multiplied by $1 - \delta'$.

Lender's have preferences over wealth when old, W_o, given by:

$$v(W_o) = \frac{W_o^{1-\gamma}}{1-\gamma}.$$

The young lender's problem is to allocate a fraction μ of its wealth in sovereign bonds, and the remainder in risk-free bonds. Given the homogeneity of preferences, the optimal decision conditional on s and B' is defined by:

$$\mu^*(s, B') = \underset{\mu}{\operatorname{argmax}} \, \mathbb{E} \left[v \left((1-\mu)(1+r^*) + \mu \tilde{R} \right) \middle| s, B' \right], \tag{10}$$

where $\tilde{R}$ is given by (9). In forming expectations over $\tilde{R}$, the lender uses the equilibrium policy functions of the government:

$$\begin{aligned} \delta &= 1 \text{ with probability } \mathcal{D}(s, B'); \\ \delta' &= 1 \text{ with probability } \mathcal{D}(s', B'') \text{ in state } s'; \text{ and} \\ B'' &= \mathcal{B}(s'). \end{aligned}$$

The first-order condition for the lender's problem is the usual condition:

$$\mathbb{E} M(\tilde{R} - (1 + r^*)) = 0,$$

where $M = v'((1-\mu^*)(1+r^*) + \mu^* \tilde{R})$ is the stochastic discount factor. If lenders are risk-neutral, then we have that $\mathbb{E}\tilde{R} = 1 + r^*$, which is the usual case in the quantitative literature. When $\gamma > 0$, we will have a positive risk premium. In particular, $q(s, B')$ will be such that lenders receive the appropriate compensation for the probability of default plus any additional risk premium required to bear such risk. Note that the stochastic discount factor depends on μ^*. In equilibrium, the market for bonds must clear. In particular,

$$\mu^*(s, B')W = q(s, B')B'. \tag{11}$$

As B' increases, lenders devote more of their wealth to sovereign bonds, and therefore prices must fall to generate the appropriate risk premium to clear the market.

2.5 Definition of Equilibrium

Definition 1 (Equilibrium)**.** *An* equilibrium *consists of a price schedule q, government policy functions $\mathcal{B}$ and $\mathcal{D}$, and a lender portfolio policy function μ^* such that: (i) $\mathcal{B}$ and $\mathcal{D}$ solve the government's problem from Section 2.3, conditional on q and μ^*; (ii) μ^* solves the representative lender's problem* (10) *conditional on q and the government's policy functions; and (iii) market clearing: equation* (11) *holds for all $(s, B') \in SB$.*

Note that the market clearing condition requires that the price schedule clears the market at all potential B', even for debt position that occur off equilibrium. This is a perfection requirement which ensures that if the government were to deviate from $\mathcal{B}$ and issue a sub-optimal amount of debt, these bonds would be priced in a manner consistent with equilibrium behavior going forward.

3 The Logic of Rollover Crises

There are many equilibria in this environment. A primary contribution of the paper is to propose a new perspective on self-fulfilling debt crises and explore its quantitative implications. We focus on a "static" multiplicity, by which we mean that we can support multiple equilibrium outcomes in the current period holding constant equilibrium behavior going forward. In particular, for the same debt and output state variables and the same future continuation value and future price functions, the sovereign can be induced to default today if it is offered an adverse price schedule while it would strictly prefer to repay if it is offered a more generous price schedule. Moreover, the current and future default behavior of the sovereign in these two circumstances is consistent with the lenders optimally choosing to lend the prescribed amount at the prescribed price (i.e., each schedule is an equilibrium schedule). We refer to an auction with an adverse price schedule as a *rollover crisis*. As we shall see, not all such auctions result in default.

In this section, we explain the logic of rollover crises – beginning with the Cole-Kehoe class of crises and then move to our proposed "desperate deal" alternative. For expositional purposes, we first treat each type of crisis as a "one off" event holding fixed future equilibrium behavior, and then describe the fixed-point algorithm we use to compute an equilibrium in which crises happen with regularity.

3.1 Cole-Kehoe Multiplicity

To show the logic of a Cole-Kehoe crisis, we take as the premise an equilibrium price schedule, q, value functions $\{V, V^D, V^R\}$, and associated equilibrium policy functions, $\{\mathcal{B}, \mathcal{D}, \mu^*\}$. We will hold equilibrium behavior going forward fixed, and consider two different outcomes for the current period by altering creditors' "static" beliefs about intra-period behavior of other creditors and the government.

Consider a state $\tilde{s}$ that satisfies two characteristics. The first is that $\mathcal{B}(\tilde{s}) = \tilde{B}' > (1-\lambda)\tilde{B}$ such that $V^R(\tilde{s}, \tilde{B}') > V^D(\tilde{s})$. That is, the optimal policy of the government given $q(s, .)$ is to issue new debt and not default. The second criterion is that if the government does not issue new bonds, it weakly prefers to default at settlement, and strictly prefers to default if $B' = (1-\lambda)B$. That is, $V^R(\tilde{s}, B') < V^D(\tilde{s})$ for $B' = (1-\lambda)\tilde{B}$, and $V^R(\tilde{s}, B') \leq V^D(\tilde{s})$ for $B' < (1-\lambda)\tilde{B}$.

We now construct a self-fulfilling crisis of the Cole-Kehoe type. For notation, recall our state vector s includes an index of creditor beliefs, denoted ρ. Now consider two states, $\tilde{s}$ and $\hat{s}$ that differ *only* in creditor beliefs about this period's equilibrium behavior. Specifically, let $(\tilde{Y}, \tilde{g}, \tilde{z}, \tilde{B}) = (\hat{Y}, \hat{g}, \hat{z}, \hat{B})$, so all fundamentals are identical. Moreover, for any function of next period's state, $f(s')$, we have $\mathbb{E}[f(s')|\tilde{s}] = \mathbb{E}[f(s')|\hat{s}]$, so there is no difference about future expectations conditional on $\hat{s}$ versus $\tilde{s}$.

The only difference concerns creditor beliefs. Let $\tilde{\rho} \in \tilde{s}$ denote the creditor beliefs that support the government issuing positive debt $\tilde{B}'$ at positive prices and then repayment at settlement, as discussed above. However, let $\hat{\rho} = CK$ indicate that creditors coordinate on the Cole-Kehoe type rollover crisis. In particular, consider a candidate price schedule under the CK beliefs:

$$q(\hat{s}, B') = \begin{cases} 0 \text{ if } B' \geq (1-\lambda)\tilde{B} \\ q(\tilde{s}, B') \text{ if } B' < (1-\lambda)\tilde{B} \end{cases}$$

With this schedule, the government is unable to raise money from the bond market in the current period, but can repurchase debt at the same price schedule as $\tilde{s}$.

The candidate equilibrium repayment value of the government is:

$$V^R(\hat{s}, B') = \begin{cases} u\left(\tilde{Y} - (r+\lambda)\tilde{B}\right) + \beta E\left[V(s')|\tilde{s}, B'\right] \text{ if } B' \geq (1-\lambda)\tilde{B} \\ V^R(\tilde{s}, B') \text{ if } B' < (1-\lambda B). \end{cases}$$

Note that in the first line we use the fact that fundamentals and future beliefs are the same for $\hat{s}$ and $\tilde{s}$. The second line reflects that for $B' < (1-\lambda)\tilde{B}$, we have $q(\hat{s}, B') = q(\tilde{s}, B')$, and thus the repayment values are identical.

To fill in the remaining equilibrium objects, we have $V^D(\hat{s}) = V^D(\tilde{s})$, as creditor beliefs do not affect default payoffs, holding constant fundamentals and equilibrium behavior going forward. Finally, $V(\hat{s}) = \max\{\max_{B'} V^R(\hat{s}, B'), V^D(\hat{s})\}$.

To establish that our equilibrium conditions are satisfied at $\hat{s}$, recall that our choice of $\tilde{s}$ implied $V^R(\tilde{s}, (1-\lambda)\tilde{B}) < V^D(\tilde{s})$. Given that the continuation value $V(s')$ is weakly decreasing in B', we have for $B' \geq (1-\lambda)\tilde{B}$:

$$\begin{aligned} V^R(\hat{s}, B') &= u\left(\tilde{Y} - (r+\lambda)\tilde{B}\right) + \beta E\left[V(s')|\tilde{s}, B'\right] \\ &\leq u\left(\tilde{Y} - (r+\lambda)\tilde{B}\right) + \beta E\left[V(s')|\tilde{s}, (1-\lambda)\tilde{B}\right] \\ &= V^R(\tilde{s}, (1-\lambda)\tilde{B}) \\ &< V^D(s'). \end{aligned}$$

Hence, if the government were to issue $B' \geq (1-\lambda)\tilde{B}$ when facing $q(\hat{s}, B')$, it will default at settlement. This rationalizes $q(\hat{s}, B') = 0$ on this domain. Note that this relies on the fact that lenders are atomistic and individually cannot purchase enough debt to sway the government's default decision. For $B' < (1-\lambda)B$, prices, repayment value, and default value, are all the same in the two scenarios. Hence, $\{q(\hat{s}, B'), V^R(\hat{s}, B')$ are an equilibrium for $B' < (1-\lambda)B$ as well. In this manner, we have constructed a new equilibrium price schedule that is zero for any issuances and equals the “non-crisis” price schedule for repurchases.

This establishes that given a set of fundamentals and fixed beliefs about future equilibrium play, we can support two different outcomes in the current period. In one, creditors are willing to lend and the government uses the auction revenue to rollover debt at settlement. In the other, maturing debt must be paid out of current endowment. If debt is high enough and output low enough, the government prefers to default, supporting the zero price for new bond issuances.

The defining feature of the Cole-Kehoe crisis is a zero price for new debt and immediate default. We now propose an alternative class of crises in which bonds are issued in equilibrium at depressed, but positive, prices.

3.2 Desperate Deals

To illustrate the logic of desperate deals, we follow the previous example and begin with an equilibrium price schedule q and associated value functions. Let $\tilde{s}$ be the same as the previous example as well, indicating a set of fundamentals and beliefs that support issuing $\tilde{B}' > (1-\lambda)B$ at a price $q(\tilde{s}, \tilde{B}')$ and repayment at settlement. Given the price schedule $q(\tilde{s}, B')$, there may be other levels of debt issuance, albeit sub-optimal, that also imply repayment at settlement (but possibly default in the future). We denote this “no-immediate-default” set for issuances by $\mathbb{B}(\tilde{s})$:

$$\mathbb{B}(\tilde{s}) \equiv \{B' | V^R(\tilde{s}, B') > V^D(\tilde{s})\}. \tag{12}$$

The first premise of $\tilde{s}$ discussed above ensures that $\mathcal{B}(\tilde{s}) \in \mathbb{B}(\tilde{s})$. The second premise implies if $B' \leq (1-\lambda)\tilde{B}$, then $B' \notin \mathbb{B}(\tilde{s})$.

As in the previous example, we now consider alternative beliefs for today's auction, holding constant fundamentals and future equilibrium behavior. Specifically, let $\bar{s}$ agree with $\tilde{s}$ on fundamentals (Y, g, z, B) and for future expectations. The only difference is that $\tilde{\rho} \neq \bar{\rho}$. In particular, let $\bar{\rho} = DD$ indicate our “desperate deals” beliefs, which we describe below. As before, common fundamentals implies $V^D(\bar{s}) = V^D(\tilde{s})$.

The key to the desperate deals construction is that for any $B' \in \mathbb{B}(\tilde{s})$, there is a price that makes the government indifferent to default at settlement. In particular, for the fundamentals implied by $\bar{s}$ and a given choice $B' \in \mathbb{B}(\tilde{s})$, define $q_{DD}(\bar{s}, B')$ by:

$$u\left(\bar{Y} - (r+\lambda)\bar{B} + q_{DD}(\bar{s}, B')(B' - (1-\lambda)\tilde{B})\right) + \beta\mathbb{E}[V(s')|\bar{s}, B'] = V^D(\bar{s}). \qquad (13)$$

To see that a q_{DD} exists for each $B' \in \mathbb{B}(\tilde{s})$, note that the left-hand side of (13) is continuous and strictly increasing in the value of q_{DD} for any $B' > (1-\lambda)\tilde{B}$. Moreover, by the definition of $\mathbb{B}(\tilde{s})$, the left hand side is strictly greater than $V^D(\bar{s})$ for $q_{DD}(\bar{s}, B') = q(\tilde{s}, B')$ and strictly less for $q_{DD}(\bar{s}, B') = 0$. Hence there exists a $q_{DD}(\bar{s}, B') \in (0, q(\tilde{s}, B'))$ at which the equality holds.

Our candidate desperate deals price schedule is:

$$q(\bar{s}, B') = \begin{cases} q_{DD}(\bar{s}, B') \text{ if } B' \in \mathbb{B}(\tilde{s}) \\ q(\tilde{s}, B') \text{ if } B' \notin \mathbb{B}(\tilde{s}). \end{cases}$$

Note that the equilibrium conditions hold at $\bar{s}$ for $B' \notin \mathbb{B}(\tilde{s})$, as prices and values are identical to $\tilde{s}$ on this domain. The question is how to support $q(\bar{s}, B')$ for $B' \in \mathbb{B}(\tilde{s})$. By the definition of q_{DD} in equation (13), we have $V^R(\bar{s}, B') = V^D(\bar{s})$ for all $B' \in \mathbb{B}(\tilde{s})$. That is, the government is indifferent over repayment and default. As $q_{DD}(\bar{s}, B') > 0$ for all such B', for this to be an equilibrium the government cannot default with probability one. However, this does not pose a hurdle because a positive price can be supported in equilibrium by having the government randomize over repayment and default at settlement.[11]

To see how this works, assume for simplicity that lenders are risk neutral, and let $\mathcal{D}(\bar{s}, B') \in (0, 1)$ denote the equilibrium probability of default at settlement given issuances $B' \in \mathbb{B}(\tilde{s})$. Market clearing in the bond market requires that the lenders break even; that is for each B', $\mathcal{D}$ satisfies:[12]

$$q_{DD}(\bar{s}, B') = \mathcal{D}(\bar{s}, B')R^D(\bar{s}, B') + (1 - \mathcal{D}(\bar{s}, B'))\, q(\tilde{s}, B'), \qquad (14)$$

[11] Our mixed strategy equilibria builds on Aguiar and Amador (2014), which allowed for off-equilibrium debt buybacks during a rollover crisis. We extend this idea to allow the government to issue debt during a rollover crisis on the equilibrium path. Aguiar and Amador (2014) were concerned with the government repurchasing its debt if secondary prices were zero during a rollover crisis. They ruled these out by allowing the government to randomize over default when indifferent, supporting positive prices that made the government indifferent between repurchasing and defaulting. To make this an off equilibrium phenomenon, Aguiar and Amador explored the equilibrium in which the government always chose zero buybacks and default with probability one over buying back debt and randomizing.

[12] For risk averse lenders, we replace equation (14) with the appropriate portfolio problem and market clearing condition. For $B' \in \mathbb{B}(\tilde{s})$, there is always an interior $\mathcal{D}(s^*, B')$ that clears the market.

where $R^D(\bar{s}, B') = q_{DD}(\bar{s}, B') \left(\frac{B' - (1-\lambda)\bar{B}}{B' + (r+\lambda)\bar{B}} \right)$ is the recovery value in the event of default. Since $0 < B' - (1-\lambda)\bar{B} < B' + (r+\lambda)\bar{B}$, the $\mathcal{D}(s, B')$ that solves (14) is in $(0, 1)$. The first term on the right of equation (14) is the probability of default at settlement times the recovery value. The second term is the probability of repayment at settlement; in this event, the bond is worth the price in state $\tilde{s}$. The desperate deal bond is therefore a lottery ticket that defaults at settlement with probability $\mathcal{D}(\bar{s}, B')$, but pays out at settlement a "non-crisis" bond with probability $1 - \mathcal{D}(\bar{s}, B')$. We now describe some additional features of the desperate deals environment.

3.3 Notable Features of Desperate Deals

The defining feature of the Cole-Kehoe crisis is the inability to raise any money at auction. In contrast, the desperate deals price schedule is positive for debt issuances, but less than the price under the non-crisis beliefs for the same fundamentals. From the government's perspective, the price schedule leaves them indifferent to default. From the lenders' perspective, they are purchasing a lottery ticket that immediately loses value with some positive probability, but otherwise pays off the value that the bond would trade at under non-crisis beliefs. In this way, the equilibrium reflects the situation in which bond prices are positive but depressed, pushing the government up against its indifference condition. It then becomes a random outcome whether the bonds retain their value into the next period.

We view the mixed strategy as capturing other facets of the government's repayment decision that generate default risk. A primary example is the political payoff to the current incumbents regarding the popularity of repayment versus default, which may vary stochastically and generate uncertainty between an auction and the repayment of maturing bonds. In the spirit of (Harsanyi, 1973), this can serve as a rationale for mixed strategies.

Another feature of desperate deals is that they occur at equilibrium prices. This is not a bargaining outcome in which creditors or the government threaten to "walk away." The lenders have no incentive to hold out from the auction, as at the margin they are indifferent to participating or not. Thus, while this results in a positive price for legacy lenders who are selling their bonds at the same time, this is not a partial default or haircut in the usual sense.

While the government is indifferent to the amount issued in a crisis at the "indifference" price schedule, the legacy lenders are not. In particular, they would like the government to choose an amount that maximizes the price of non-maturing bonds. However, given these are arms length transactions, there is no market mechanism to induce the government to select the surplus maximizing policy. This is the natural counterpart of the competitive assumption that there is no way for the government to "select" the equilibrium price schedule that does not involve a rollover crisis. Finally, and related to the last point, the ability to issue bonds in a crisis reduces the deadweight loss associated with rollover crises. In particular, depending on the realization of the mixed strategy randomization, some rollover crises are not followed with immediate default. We shall see that given a certain probability of a rollover crisis next period, the ability to engage in desperate deals will therefore raise bond prices ex ante. This encourages the government to borrow more in equilibrium and makes the bond market a more efficient provider of inter-temporal smoothing and insurance.

4 Equilibrium Construction

We now describe how we construct our benchmark equilibrium in which beliefs fluctuate between "non-crisis" and "desperate deals" price schedules. As usual, we do so by solving a fixed point problem. That is, we construct a candidate equilibrium price schedule, value functions, and policies, and then check whether the optimality and market clearing conditions are satisfied at each state s given future equilibrium behavior. In particular, let $\{q, V, V^D, V^R\}$ and associated policies $\{\mathcal{B}, \mathcal{D}, \mu^*\}$ be our candidate equilibrium. Moreover, fix a first-order Markov process for beliefs ρ and the endowment process (Y, g, z).

To proceed, recall that a key element of our timing assumption is that the auction happens before the current period's default decision. This is contrary to the "Eaton-Gersovitz" (EG) timing in which the government makes its default decision before the auction occurs. The EG timing involves an additional level of intra-period commitment. It is useful to consider how lenders would price a bond *if* the government were able to commit not to default in the *current* period, but lack such commitment in future periods.

In particular, given our candidate equilibrium going forward, suppose that the government could commit to repay this period for any B'. What would be the price of such a bond in such a situation? In the lender's problem (10), this corresponds to setting $\delta = 0$ for the current period, and solving for a price schedule that correctly anticipates the government's equilibrium policy functions next period, given today's s and bond issuance B'. Let $\{\mu_{EG}, q_{EG}\}$ be a pair that clears the bond market under such temporary commitment. That is, faced with $q_{EG}(s, B')$ today, equilibrium behavior in the future, and no default today ($\delta = 0$), lenders' chose a portfolio share μ_{EG} such that $\mu_{EG}(s, B')W = q_{EG}(s, B')B'$. In the risk-neutral lender case, q_{EG} will be the discounted expected equilibrium value of the bonds at next period's auction.

Now consider the government's decision temporarily facing $q_{EG}(s, B')$ in today's auction. In particular, define:

$$V_{EG}^R(s, B') \equiv u\left(Y - (r + \lambda)B + q_{EG}(s, B')[B' - (1 - \lambda)B]\right) + \beta\mathbb{E}[V(s')|s, B'], \tag{15}$$

where continuation values are given by our candidate equilibrium. This is the value of repayment if the government faced q_{EG} at auction.

Our "non-crisis" beliefs builds on this assumption of intra-period commitment. However, if $V_{EG}^R(s, B') < V^D(s)$, then the assumption that $\delta = 0$ is not credible, as the government will default at settlement given the opportunity. On the other hand, if $V_{EG}^R(s, B') \geq V^D(s)$, repayment is time consistent absent commitment. Therefore, the "non-crisis" equilibrium beliefs will coordinate on q_{EG} whenever this is consistent with repayment at settlement. In particular, let $\rho = EG$ denote creditor beliefs that the equilibrium price schedule will be q_{EG} for (s, B') such that repayment $V_{EG}^R(s, B') \geq V^D(s)$:

$$q(s, B') = q_{EG}(s, B') \text{ if } \rho = EG \text{ and } V_{EG}^R(s, B') \geq V^D(s).$$

Correspondingly, the default policy $\mathcal{D}(s, B') = 0$ in these states.

Note that if fundamentals are bad enough, there may not be a B' such that $V_{EG}^R(s, B') \geq V^D(s)$. In this case, the government will default with probability one, even under non-crisis beliefs. It is useful to denote this "default" set by $\mathbb{D}$:

$$\mathbb{D} \equiv \left\{ s \in S \,\middle|\, \max_{B'} V_{EG}^R(s, B') < V^D(s) \right\}. \tag{16}$$

If $s \in \mathbb{D}$, the government defaults regardless of beliefs. We consider such defaults as "fundamental" defaults, as their is no set of current beliefs that support repayment. However, keep in mind that fundamental defaults involve expectations of future equilibrium payoffs, and hence factor in the possibility of rollover crises in the future.

If $s \notin \mathbb{D}$, the government can do better than default with the appropriate choice of B'. To complete the characterization of prices for $\rho = EG$, we need to specify prices if the government were to choose a sub-optimal level of issuance that left them with an incentive to default at settlement. For such issuances, we set $q(s, B')$ as low as possible consistent with default at settlement. For $B' \geq (1-\lambda)B$, this implies a price of zero. For buy-backs, we need to ensure that the price is high enough that the government cannot repurchase its debt and avoid default. This is done as in (Aguiar and Amador, 2014) by allowing the government to repurchase debt and randomize at settlement. However, in our benchmark model we select that such buy-backs never occur on the equilibrium path under non-crisis beliefs ($\rho = EG$).

Taking stock, for non-crisis beliefs $\rho = EG$, the government faces the intra-period commitment prices when credible. That is, if $s \notin \mathbb{D}$, the government issues bonds and repays at settlement with probability one. If $s \in \mathbb{D}$, the government defaults regardless of beliefs due to poor fundamentals.

We now turn to crisis beliefs. As noted in the previous section, there may be a set of fundamentals in which a desperate-deals price schedule can be supported in equilibrium. Recall in that discussion that we started with a state in which repayment was optimal if the government could issue new debt, but default was optimal otherwise. With q_{EG} and V_{EG}^R in hand, let $\mathbb{C}$ define the set of such states:

$$\mathbb{C} \equiv \left\{ s \in S \,\middle|\, \max_{B' > (1-\lambda)B} V_{EG}^R(s, B') > V^D(s) \right.$$
$$\& \max_{B' < (1-\lambda)B} V_{EG}^R(s, B') \leq V^D(s)$$
$$\left. \& \; V_{EG}^R(s, (1-\lambda)B) < V^D(s) \right\}.$$

The first condition says the government will repay if it can issue debt at q_{EG}; the second condition says default is weakly optimal if it repurchases debt at q_{EG}; and the final condition states that if it neither issues nor repurchases, the government strictly prefers to default. We refer to $\mathbb{C}$ as the "crisis" set as we can support a desperate deal price schedule for such states.

Note that the first condition implies that $\mathbb{C} \cap \mathbb{D} = \emptyset$. That is, fundamentals in the crisis set are not so terrible that default is unavoidable. On the other hand, the next two conditions in the definition of $\mathbb{C}$ imply that fundamentals are bad enough that the government needs to issue debt in order to guarantee repayment at settlement. Let $\mathbb{NC}$ denote the non-crisis set, defined as the complement of $\mathbb{C} \cup \mathbb{D}$ on S. For $s \in \mathbb{NC}$, fundamentals are good enough that the government is willing to repay even if they cannot issue bonds in the current period.

If $s \notin \mathbb{C}$, we cannot support a desperate deal price schedule. In particular, either $s \in \mathbb{D}$

and there will be a fundamental default even at the favorable price schedule q_{EG}, or $s \in \mathbb{NC}$, and the government is willing to repay even if it does not issue bonds in the current period. Thus the crisis set contains intermediate fundamentals and inherited debt levels, in which creditors' static beliefs can sway the government's repayment decision.

Our focus is therefore to construct a crisis scenario for $s \in \mathbb{C}$. As in the previous section, let $\rho = DD$ denote our desperate deal beliefs. Paralleling (12), for $s \in \mathbb{C}$, let $\mathbb{B}_{EG}(s)$ be defined as:

$$\mathbb{B}_{EG}(s) \equiv \{B' | V_{EG}^R(s, B') > V^D(s)\}. \tag{17}$$

For these levels of debt issuances, the government will repay if prices are q_{EG}, but default if prices of new issuances are zero (as $s \in \mathbb{C}$). For $B' \in \mathbb{B}_{EG}(s)$, let $q_{DD}(s, B')$ be defined by the indifference condition given by equation (13) evaluated at fundamentals $(Y, g, z, B) \in s$:

$$u\left(Y - (r + \lambda)B + q_{DD}(s, B')[B' - (1 - \lambda)B]\right) + \beta \mathbb{E}[V(s')|s, B'] = V^D(s). \tag{18}$$

We then set:

$$q(s, B') = q_{DD}(s, B') \text{ if } \rho = DD \text{ and } s \in \mathbb{C} \text{ and } B' \in \mathbb{B}_{EG}(s). \tag{19}$$

As discussed in the previous section, to support the desperate deal prices, we need to choose a default probability that clears the bond market. For $B' \in \mathbb{B}_{EG}(s)$, there is always a $\mathcal{D}(s, B') \in (0, 1)$ that clear the market at prices $q_{DD}(s, B') \in (0, q_{EG}(s, B'))$.

To complete the description of the equilibrium, for $s \in \mathbb{NC} \cup \mathbb{D}$, current beliefs are irrelevant to the default decision. In this case, prices are the same as the non-crisis prices associated with the same fundamentals. That is, the prices are the same as $\rho = EG$, holding constant (Y, g, z, B). Similarly, if $s \in \mathbb{C}$ but $B' \notin \mathbb{B}_{EG}(s)$, then the B' is such that the government will not repay at settlement even at non-crisis prices. In this case, we set the price equal to the non-crisis price for the corresponding fundamentals, which will be zero for issuances and high enough to rule out buy-backs in equilibrium.

The defining feature of q_{DD} is that the government is indifferent to default for any $B' \in \mathbb{B}_{EG}(s)$. This implies it is indifferent to the choice of B'. As part of the equilibrium selection, we posit a point in the policy correspondence; that is, we select $\mathcal{B}(s) \in \mathbb{B}_{EG}(s)$. For our benchmark, we assume the government auctions off $\frac{\lambda}{2}B$, issuing enough to pay off half the stock of maturing bonds. That is, $\mathcal{B}(s) = \left(1 - \frac{\lambda}{2}\right) B$ for $s \in \mathbb{C}$ and $\rho = DD$.[13]

In this fashion, starting from the conjectured equilibrium tuple $\{q, V, V^R, V^D\}$, and a Markov process for exogenous states (including beliefs), we have constructed a new price schedule and associated value functions for $\rho \in \{EG, DD\}$. Given future equilibrium behavior, the new price schedule and value functions satisfy the equilibrium conditions. We iterate this process until convergence; that is, until the constructed equilibrium outcomes at each state s are consistent with the conjectured beliefs for future equilibrium behavior.

5 Calibration

We now calibrate the model and explore its quantitative predictions. We use Mexico as our reference economy.

Technology

For the endowment process, we assume the growth rate process is governed by

$$g_{t+1} = (1 - \rho_g)\bar{g} + \rho_g g_t + \varepsilon_{t+1}$$

where $\varepsilon \sim N(0, \sigma_g^2)$. The transitory component of output z_t is assumed to be *iid*, orthogonal to ε_t and to have mean zero and variance σ_z^2. The implied growth rate of log output is

$$y_{t+1} - y_t = g_{t+1} + z_{t+1} - z_t + \varepsilon_{t+1}.$$

We estimate this model using quarterly Mexican constant-price GDP for the period 1980Q1 through 2015Q1. The estimated parameter vector is reported in Table 1. The estimates suggest that the stochastic trend is the primary driver of GDP fluctuations for Mexico, consistent with Aguiar and Gopinath (2007).

With these parameters in hand, we discretize the process for g using Tauchen's method using 50 grid points spanning $\pm 3\sigma_g/\sqrt{1-\rho_g^2}$. The *iid* z shock is drawn from a continuous Normal distribution truncated at $\pm 3\sigma_z$. When taking expectations, we numerically integrate

[13]For $\rho = EG$, we have a well defined optimum for $s \notin \mathbb{D}$, namely $\mathcal{B}(s) = \text{argmax}_{B'} V^R(s, B')$. For $s \in \mathbb{D}$, the government will default with probability one at settlement and prices are zero, and we assume the government does not auction any bonds. Recall that with our settlement assumption, the government is indifferent to auctioning bonds when it plans to default, as it does not keep the proceeds. This marks a difference with the original Cole-Kehoe timing, in which the government could take the auction proceeds and then default.

over z's continuous distribution by evaluating at 11 grid points.

Preferences

The coefficient of relative risk aversion for the sovereign and the creditors is set to 2. The government's discount factor is set through the moment matching procedure described below.

Financial Markets

We set the risk-free interest rate at 1 percent quarterly (hence, 4 percent annually). The average maturity length is set to 8 quarters, that is $\lambda = 1/8$, which implies a Macaulay duration of 6.4 quarters. This is shorter than the average maturity (or duration) observed in many emerging markets. However, maturity length is not constant over time and tends to shorten when the probability of a crisis is high (Broner, Lorenzoni, and Schmukler, 2013; Arellano and Ramanarayanan, 2012). Moreover, much of a country's short-term debt is issued domestically (whether in dollars or local currency), and thus is not reflected in the average maturity of external debt. For simplicity, our model only has external debt of constant maturity, raising the question of how to accurately capture a world in which maturity varies over time and the amount due (and to whom) in any given quarter is not uniform. Given our focus on crises, we set the average maturity length to a value that is relatively short.

While excluded from financial markets, a fraction ϕ of output is lost each period. Note that the cost of default is linear in output, as in Aguiar and Gopinath (2006). In contrast, Arellano (2008) introduced a non-linear cost of default which made default disproportionately more costly in good endowment states and "forgiven" – at least in terms of output costs –in low endowment states. The Arellano specification amplifies the impact of endowment fluctuations in the decision to default while also making default a better insurance option in low-endowment states. This helps the model generate additional volatility of spreads and frequency of default, but does so by making endowment risk more important rather than less. The empirical facts outlined above and in complementary work like Tomz and Wright (2007) suggests that this pulls the model in the wrong direction relative to the data.[14] The parameter ϕ is set below by matching moments.

[14]Moreover, Aguiar, Chatterjee, Cole, and Stangebye (in process) show the Arellano non-linear cost requires a very volatile endowment process to generate volatile spreads. In particular, the endowment process calibrated to Mexico used here is not sufficiently volatile. This indicates that the typical calibration to Argentina's more volatile output process is not representative of other emerging markets.

In addition to lost output, default also brings exclusion from financial markets. We set the re-entry probability after default to 0.125 quarterly; that is, the average duration of default is two years. This is in the range documented by Gelos, Sahay, and Sandleris (2011) for the 1990s, but lower than Tomz and Wright (2013)'s median of 6.5 years using a much longer sample. Again, this is not crucial given that we scale the value of default by choosing the parameter ϕ.

Simulated Matched Moments

We calibrate the remaining moments by simulating the model and matching targeted empirical moments. In particular, the remaining parameters are the probability of transiting from $\rho = EG$ to $\rho = DD$, and vice versa; the government's discount factor; the wealth of the creditors, which we assume is a constant proportion of endowment $w = \frac{W}{Y}$; and the proportion of output lost during default. We assume that beliefs follow an *iid* process over time; that is, $\Pr(\rho' = DD|\rho = EG) = \Pr(\rho' = DD|\rho = DD)$. This leaves one transition probability and three parameters. We set these to match the average debt-to-GDP ratio, the average spread defined below by (20), and the standard deviation of spreads, using Mexico as our empirical counterpart, as well as an average default rate of 2.0 percent per annum, which is in line with the estimates of Tomz and Wright (2013) using a broad sample of countries over a relatively long time period.

Specifically, we match the average external Mexican debt to annual GDP (both in US dollars) for the period 2002Q1 through 2014Q3. The average over this period is 16.4 percent, which translates into a quarterly debt-to-income ratio of 65.6 percent. This measure of debt includes external debt by the government as well as banks. A longer time series exists for a narrower stock of debt issued by the federal government. This series suggests that debt levels were higher in the 1990s and have been falling in the 2000s and 2010s. Hence our measure of 65 percent may be an understatement.

We also match the mean and standard deviation of the spread over US bonds for Mexican debt. The average EMBI spread for Mexico over the entire period is 3.4 percent, with a standard deviation of 2.5 percent. For the simulated model, we compute the spread implied by the equilibrium price. Specifically, denote by $r(s, B')$ the implicit yield of a risk-free bond paying r^* each period and maturing with probability λ that is purchased at a price $q(s, B')$:[15]

$$r(s, B') = \frac{r^* + \lambda}{q(s, B')} - \lambda. \tag{20}$$

The implied quarterly "spread" is then $r(s, B') - r^*$, which we annualize before comparing to the EMBI data.[16]

While moment matching involves simultaneously matching four moments by varying four parameters, we can provide a heuristic guide regarding which moments are particularly important for determining which parameter based on how the model behaves when we have varied parameters. In particular, the debt-to-GDP ratio is sensitive to the choice of the punishment ϕ. Given a level of debt-to-GDP, the propensity to default is sensitive to the discount factor β. As we will discuss in detail below, the risk of a rollover crisis is important for generating the empirical volatility of spreads, which pins down the probability $\rho = DD$. Finally, given the risk of default and spread volatility, the average spread reflects an average risk premium that is sensitive to lenders' wealth.

These four targets, the model counterparts (under the column "Benchmark Model"), and the associated parameters are reported in Table 2. The model is simulated 1.5 million times, as default is a rare event. The model is able to hit all four targets precisely. To do so, the probability of a rollover crisis is 10 percent. As we shall see, this does not mean that a crisis occurs every other quarter on average. In particular, a crisis requires that the debt is high enough and other fundamentals bad enough that a rollover crisis can be supported in equilibrium; that is, $s \in \mathbb{C}$. As debt is an endogenous state variable, the government has the ability to avoid a rollover crisis. This is will be a key element of the discussion to follow. Given the vulnerability to default in general and a rollover crisis in particular, we need a fairly low discount factor (0.82 quarterly) to ensure the government accumulates the target debt levels. Upon default, the government loses 7 percent of its endowment. The final parameter, creditor wealth to GDP, is a factor of 3.

The fact that u and v are homogenous functions and the budget set for the government's

[15]That is, $q(s, B') = \sum_{k=1}^{\infty} (1 + r(s, B'))^{-k} (1 - \lambda)^{k-1} [r^* + \lambda]$.

[16]In the simulated model, the mean debt-to-income ratio and the spread is conditional on not being in the default state. More specifically, we compute the mean conditional on being out of the default state for at least 25 quarters. The reason we condition on being in good credit standing for a significant period of time is that the government exits default status with zero debt. Zero debt after default is not a realistic feature of the model and hence we focus on the ergodic distribution conditional on having sufficient time to rebuild debt.

Table 1: Parameters I: Set Prior to Simulation

Parameter	Value	Source
Endowments:		
$(1-\rho_g)\bar{g}$	0.0034	Mexico GDP Data 1980Q1-2015Q1
ρ_g	0.445	
σ_g	0.012	
σ_z	0.003	
Preferences:		
Sovereign CRRA (σ)	2	Standard
Creditor CRRA (γ)	2	Standard
Financial Markets:		
Quarterly Risk-free rate (r^*)	0.01	Standard
Reciprocal of Avg. Maturity (λ)	0.125	N/A
Default Re-entry Prob (ξ)	0.125	Gelos et all (2011)

Note: Pre-set parameters for calibrated model.

problem is homogenous of degree one in Y implies that the level of endowment Y is not a relevant state variable for the equilibrium price schedule and associated policies, where recall that the policy functions for debt issuance and bond demand have been defined as ratios to current endowment Y.[17] Therefore we may solve for an equilibrium price schedule and the associated government's and lender's problems on a truncated ("detrended") state space that omits Y. With these policies in hand, we can simulate the economy by drawing a sequence $\{g_t, z_t\}_{t=0}^{\infty}$ and then iterating on the detrended policy functions to obtain equilibrium paths for debt-to-income as well as prices and default outcomes. Finally, the path of endowment Y_t can be constructed from $\{g_t, z_t\}_{t=0}^{\infty}$, and the level of any endogenous variable can be obtained by scaling up its ratio with income.

[17]See Alvarez and Stokey (1998) for a formal treatment of dynamic programming with homogenous functions.

Table 2: Parameters II: Simulated Method of Moments

Target Moment	Data	Benchmark Model	No Deals (Cole-Kehoe)
Debt-to-Income (Quarterly)	65.6%	65.6%	63.9%
Mean Annualized Spread $r(s, B') - r^*$	3.4%	3.4%	3.5%
Quarterly Std Dev of Annualized Spread	2.5%	2.5%	0.1%
Default Frequency (Annually)	2.0%	2.0%	2.3%

Parameter	Value
Crisis Probability $\Pr(\rho = DD)$	10%
Discount factor (β)	0.84
Default Cost (ϕ)	6.8%
Creditor Wealth Relative to Y ($w = \frac{W}{Y}$)	3.75

Note: The top panel reports the empirical moments and the model counterparts for our benchmark model and the alternative without "Desperate Deals." The bottom panel are the values of the four parameters calibrated from matching the benchmark moments in the top panel to their empirical counterparts.

6 Results

In discussing the quantitative results of the model, it is useful to contrast it with an environment in which the sovereign cannot issue bonds in a rollover crisis, as in much of the literature since Cole and Kehoe (2000). In Table 2 we add the moments from the alternative model in the column labelled "No Deals." This model has the same parameters as the benchmark, but with a bond issuance policy of zero during crises (that is, $B' = (1-\lambda)B$). Recall that the government is indifferent to the amount issued (or defaulting) when faced with the rollover crisis price schedule and thus zero issuance and default is also an equilibrium outcome of the model, as discussed in Section 3. In the "No Deals" equilibrium selection, conditional on a crisis the equilibrium price is zero and the sovereign default's probability is one.

The key equilibrium objects are the government's debt issuance and default policy functions and the corresponding price schedule. In Figure 2 Panel (a) we plot $q^{EG}(s, B')$ as a function of B'/Y. As discussed above, the model is homogenous in Y and thus debt normalized by income is the relevant state variable for equilibrium prices, values, and policies. Recall that q^{EG} is the non-crisis equilibrium price schedule assuming the government does not default in the current period. We evaluate s at the mean of g. As q^{EG} is forward looking, it is independent of the inherited B and the realization of the *iid* endowment shock z.

As is usual in these models, the price schedule is highly non-linear. The relevant region is in the neighborhood of the mean debt-to-income level of 65.6%. Figure 3 depicts the ergodic distribution of debt-to-income in our simulated model, conditional on at least 25 quarters having passed since the most recent default. The figure indicates a fairly tight distribution around the calibrated mean, a point we discuss below. In Panel (b) of Figure 2, we plot the price schedule over the tighter range relevant for the equilibrium debt distribution.

Figure 2 also depicts the "No Deals" alternative equilibrium price schedule. This schedule is shifted down relative to the benchmark, reflecting that forward looking lenders are pricing in the possibility of a rollover crisis and default; as the "No Deals" model generates a default during a crisis with probability one, ex ante prices are depressed. Correspondingly, in Figure 3, the distribution of debt is shifted to the left as well. Absent the desperate deals, the bond market is less efficient, and hence the government is unable to indulge its impatience to the same extent as in the benchmark model.

Figure 4 depicts the benchmark price schedule absent a crisis for three values of g; namely, the mean endowment realization and plus or minus three standard deviations of the unconditional distribution of g. The figure zooms into the relevant debt levels surrounding the mean debt-to-income. A high realization of g bodes well for future endowments, and thus future bond prices. Thus, the price schedule shifts up and out for high realizations of g. This reflects that in an incomplete markets environment, default is relatively attractive for low output realizations.

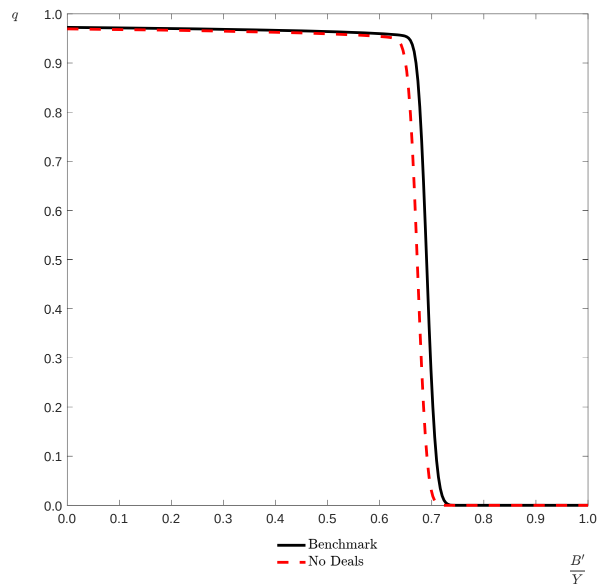
The nonlinearity of the price schedules in Figures 2 and 4 play an important role in equilibrium dynamics. Specifically, consider the revenue by auctioning $B' - (1 - \lambda)B > 0$ units of debt: $x(s, B') = q(s, B')(B' - (1 - \lambda)B)$. For the purposes of intuition, assume that the auction revenue function x is differentiable in B', and consider the the revenue raised from an additional unit of debt:

$$\frac{\partial x(s, B')}{\partial B'} = q(s, B') + \frac{\partial q(s, B')}{\partial B'}(B' - (1 - \lambda)B).$$

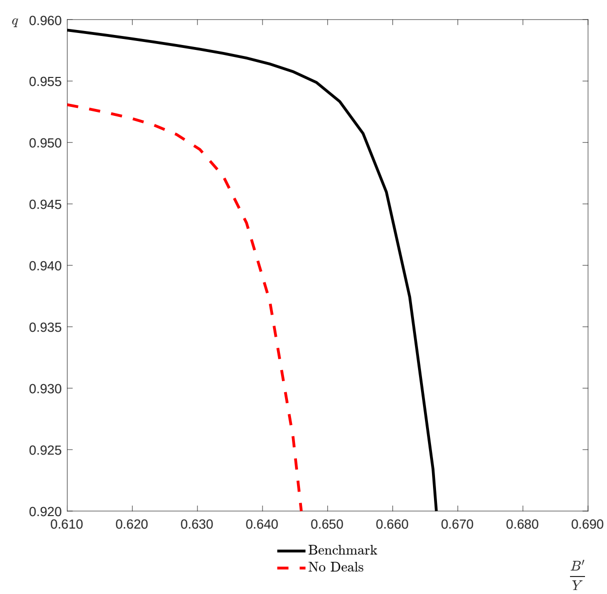
The first term is the average price of bonds issued, while the second term captures that an additional unit of debt lowers the price of all bonds sold at auction. This latter effect reflects that all bonds are of equal seniority, and so the marginal bond has the same risk profile as the infra-marginal bonds. From Figure 2, we see that the second term is large and negative

Figure 2: Equilibrium Price Schedule: No Crisis

(a) Entire Debt Domain



(b) Relevant Debt Domain

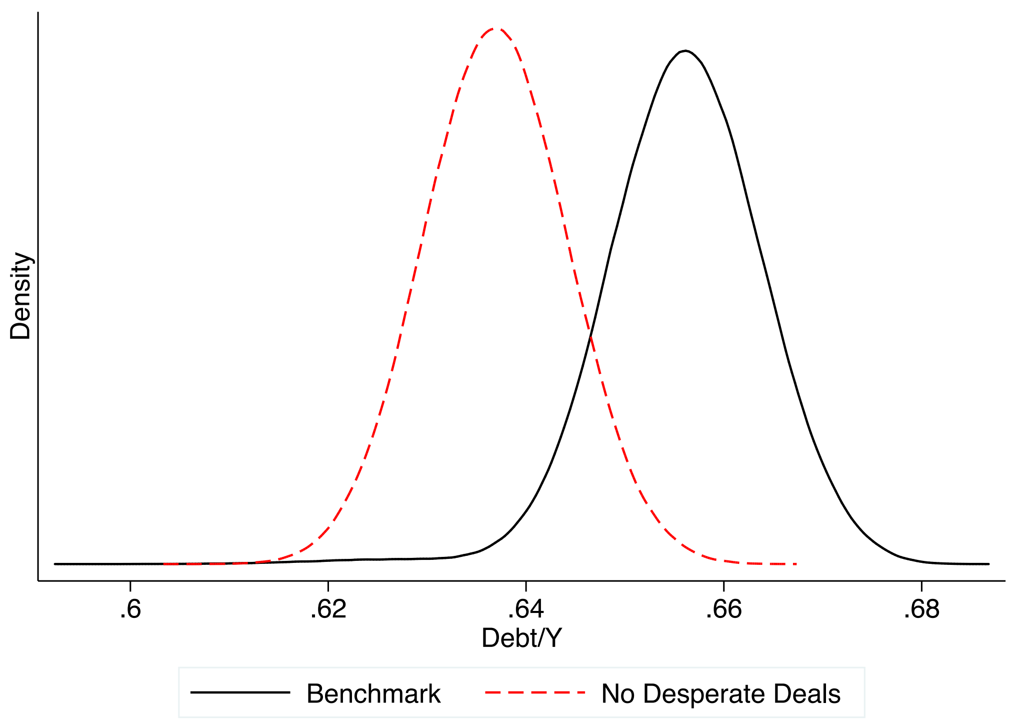


Note: This figure depicts $q^{EG}(s, B')$ as a function of B'/Y, with $g = \bar{g}$, $z = 0$, $\rho = EG$ and $B/Y = 0.656$. As discussed in the text, the "EG" price schedule assumes no default in the current period and $\rho = EG$. The solid line is the benchmark schedule and the dashed line is the "No Deals" alternative. Panel (a) depicts the entire debt domain, while panel (b) zooms into the domain that is relevant in the ergodic distribution.

for debt issuances in the neighborhood of the mean debt-to-income level. As a consequence, despite the fact that $\beta << q(s, B')$ near the ergodic mean, the government does not wish to issue additional bonds in the nonlinear region of $q(s, B')$.

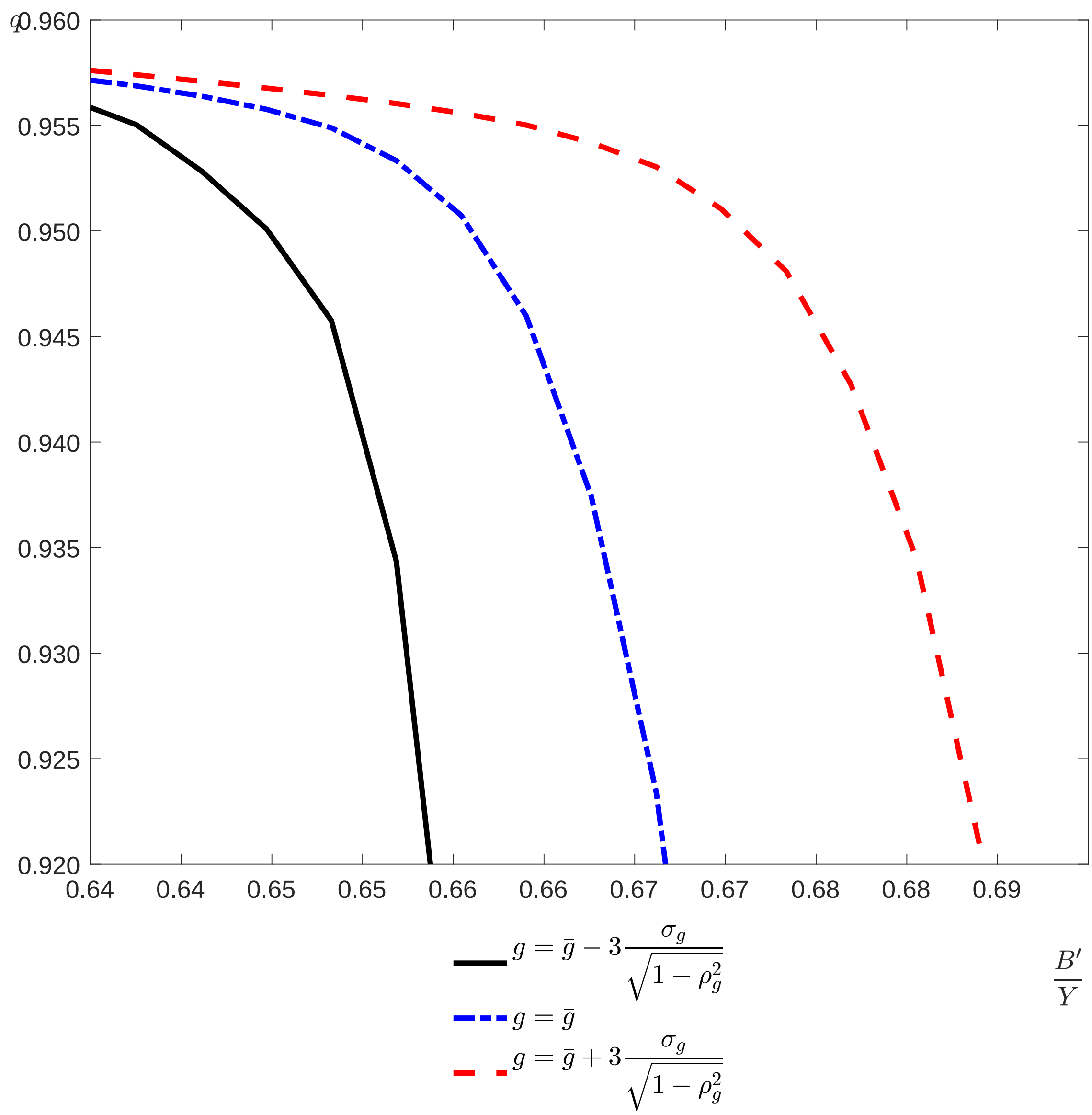
There are a number of important consequences of the preceding discussion. One is that the government does not venture far beyond the mean debt-to-income ratio in equilibrium, as shown in Figure 3. Second, debt issuances are pro-cyclical. From Figure 4, we see that the nonlinear portion shifts to the right in response to a high g realizations, as default is less likely going forward. This encourages additional borrowing, given the government's impatience. Figure 5 depicts the policy function for the same three values of g depicted in Figure 4. The high-g policy lies above the mean-g policy, which in turn lies above the low-g policy. Moreover, near the 45 degree line, the policy functions are very flat. This indicates that the government levers up and down very quickly in this region in response to shocks to g. Finally, the nonlinear price schedule results in the government not borrowing enough to raise spreads substantially, as high spreads (low q) are associated with regions in which the price is highly elastic. This discourages borrowing and lowers the volatility of spreads. This generates a fairly low volatility of spreads absent the desperate deals, a point we discuss in detail below.

Figure 3: Ergodic Distribution of Debt



Note: This figure depicts the kernel density of debt-to-income for the benchmark model simulation (solid line) and the "No Deals" alternative. The distributions are conditional on no default within the last 25 quarters.

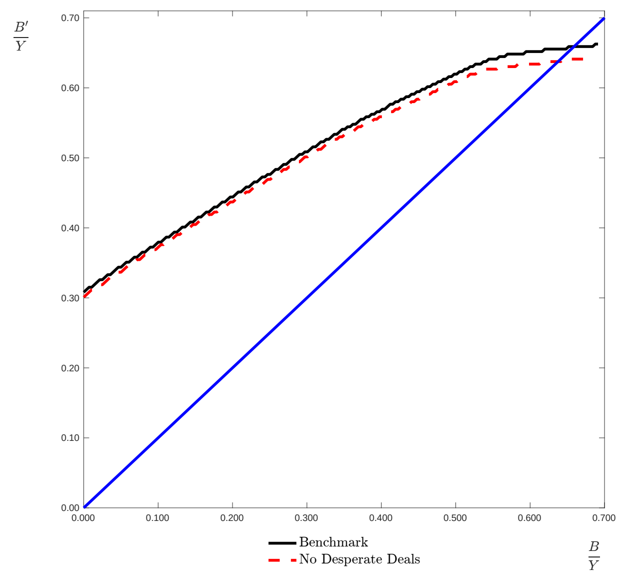
Figure 4: Benchmark Equilibrium Price Schedules: Response to g



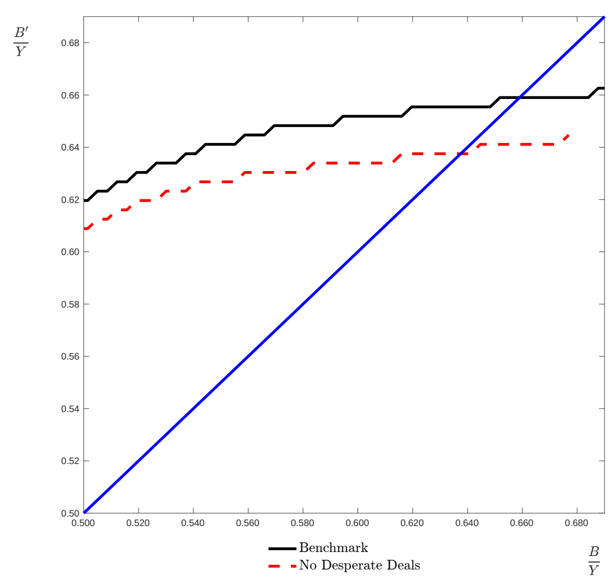
Note: This figure depicts the benchmark $q^{EG}(s, B')$ as a function of B' for different values of g. The top, dashed schedule is the highest g in our discretization; specifically three standard deviations of the unconditional g distribution above the mean. The lowest, solid line is three standard deviations below the mean, and the middle schedule corresponds to the mean g. The schedule is evaluated at $z = 0$, $\rho = EG$, and $B/Y = 0.656$, the ergodic mean.

Figure 5: Debt Issuance Policy Functions

(a) Full Domain

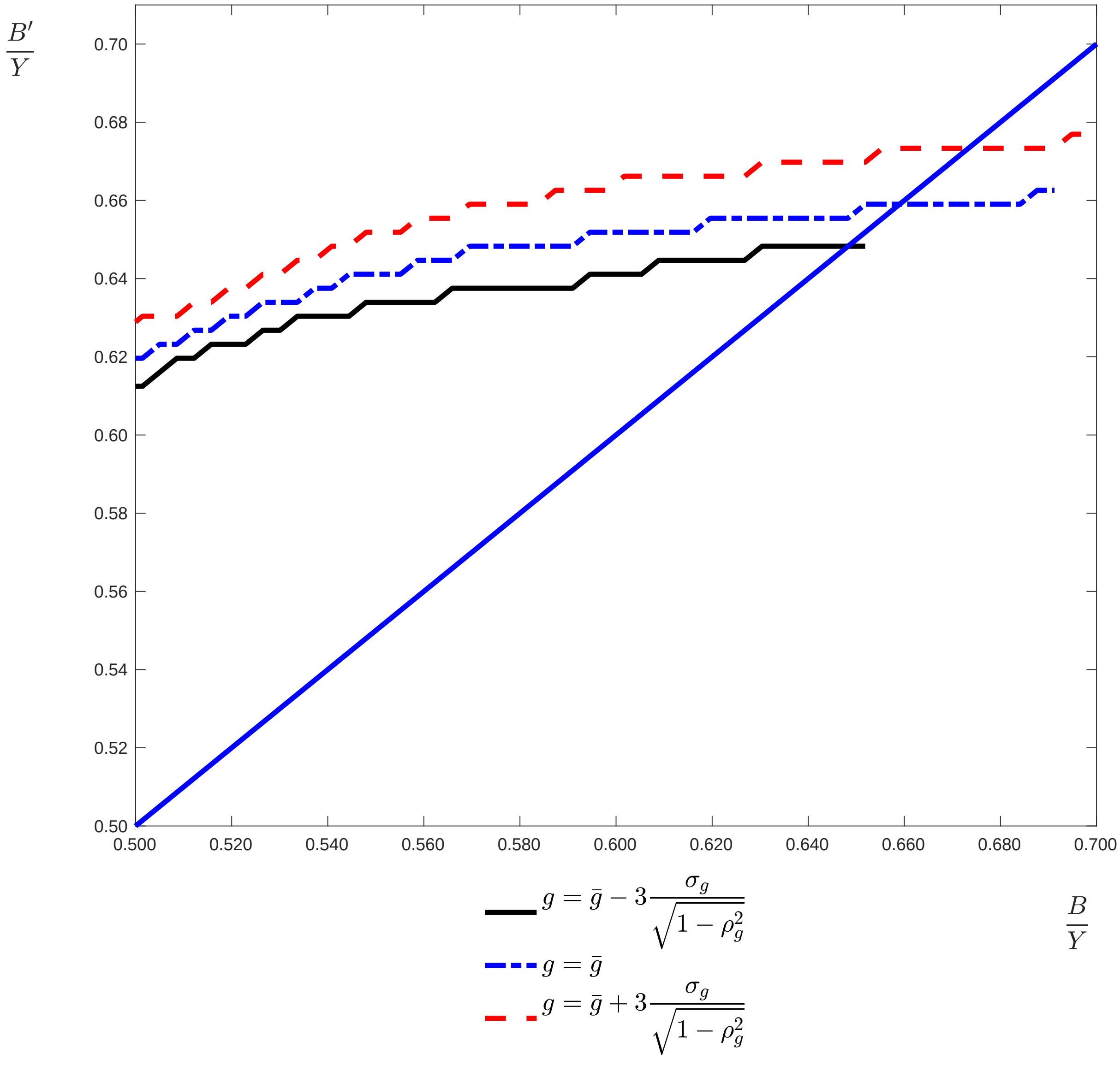


(b) Relevant Domain



Note: This figure depicts the bond issuance policy function $\mathcal{B}$, normalized by Y, as a function of B/Y. The solid line is the benchmark policy, and the dashed line is the "No Deals" alternative. The schedules are evaluated at the mean values of g and z and for $\rho = EG$. Panel (a) depicts the policy function over the entire debt domain, while Panel (b) focuses on the part of the domain relevant for the ergodic distribution.

Figure 6: Benchmark Debt Issuance: Shocks to g



Note: This figure depicts the benchmark model's bond issuance policy function $\mathcal{B}$, normalized by Y, as a function of B/Y for various realizations of g, evaluated at $z = 0$ and $\rho = EG$.

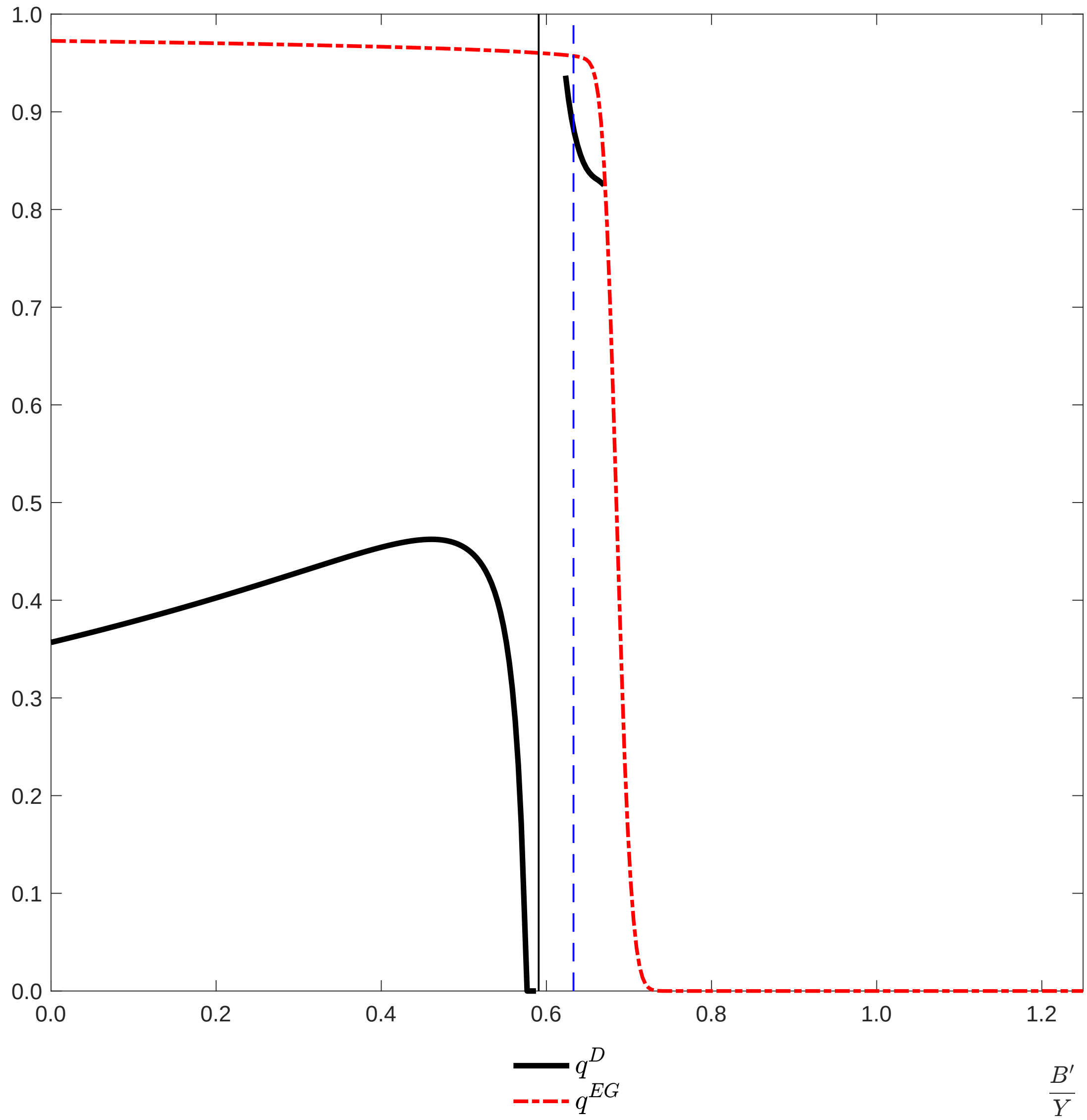
The price conditional on a crisis is depicted in Figure 7. Specifically, the figure depicts q^D, which makes the sovereign indifferent to default, as a function of issuances B'/Y. This is evaluated at a point in the crisis zone; that is, $s \in \mathbb{C}$. The figure also includes q^{EG} as reference. The solid vertical line is the $(1-\lambda)B$, which indicates the B' associated with zero issuances today. At this point, there is no price that makes the sovereign indifferent to default as the price at zero issuances is irrelevant. To the left of the this line, the sovereign is repurchasing bonds. To the right, the sovereign is issuing bonds. The dashed vertical line is $B' = (1-\lambda/2)B$, which is our equilibrium issuance policy.

Recall that for $s \in \mathbb{C}$, the sovereign strictly prefers to default if it does not auction bonds. Now consider the buy-back region, to the left of the solid vertical line. The cost of this is reduced consumption and the benefit is reduced debt going forward. In the buy-back region, for small repurchases, the sovereign needs a very low price to make it indifferent. For arbitrarily small buy backs, the price would need to be negative, which is therefore not sustainable in equilibrium (that is, for small buybacks, we have $B' \notin \mathcal{B}(s)$). The non-monotonicity reflects how the sovereign trades off the need to reduce consumption today to repurchase bonds with the benefit of exiting the period with less debt.

To the right of the no-issuance reference line, the sovereign is raising money at auction. More issuances raises consumption today, but also reduces the continuation value due to higher debt going forward. For small issuances, indifference requires a high price. As the price cannot exceed q^{EG}, there is no equilibrium for small issuances (again, $B' \notin \mathcal{B}(s)$ for small issuances). As issuances increase, there is an equilibrium price that makes the sovereign indifferent. Again, the non-monotonicity reflects the tradeoff of raising more at auction today against exiting the period with higher debt. At some level of issuances, the indifference price exceeds q^{EG}, which again cannot be supported as an equilibrium.

The government's policy function, the equilibrium price schedule, and the stochastic processes for endowment combine to generate a spread distribution. The ergodic distribution is depicted in Panel (a) of Figure 8. Most of the distribution is concentrated around the mean spread of 3.4%, with a long right tail during rollover crises. Panel (b) zooms in on the non-crisis part of the distribution by plotting the distribution conditional on $q(s, B') = q^{EG}(s, B')$; that is, no crisis. Absent a crisis, there is a fairly tight distribution of spreads, a feature we discuss below in detail. In Panel (b) we also plot the spread distribution of the "No Deals" alternative. Absent deals, the figure depicts the full distribution of spreads absent default, as crisis periods always generate defaults and a price of zero. The "No Deals" distribution is

Figure 7: Equilibrium Price Schedule: Crisis

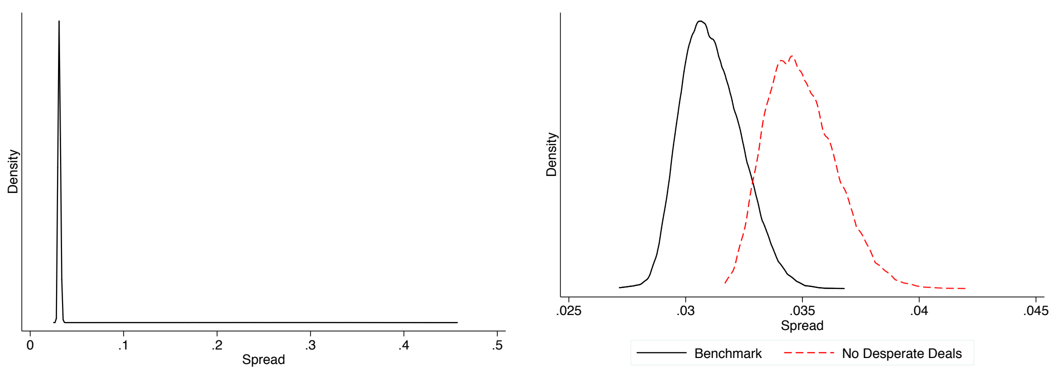


Note: Crisis bond price schedule, q^D (solid) and non-crisis schedule, q^{EG} (dashed), as a function of B'/Y evaluated at $g = -0.0212$, $z = -0.0008$ and $B/Y = 0.652$. The solid vertical line is $(1-\lambda)B/Y$ and the dashed vertical line is $(1-\lambda/2)\,B/Y$

Figure 8: Ergodic Distribution of Annualized Spreads

(a) Unconditional

(b) Conditional on No Crisis



Note: This figure depicts the simulated distribution of spreads. Panel (a) depicts the distribution of spreads in the benchmark model including rollover crises. Panel (b) depicts the distribution of spreads conditional on no rollover crisis for the benchmark model (solid line) and the "No Deals" alternative (dashed line).

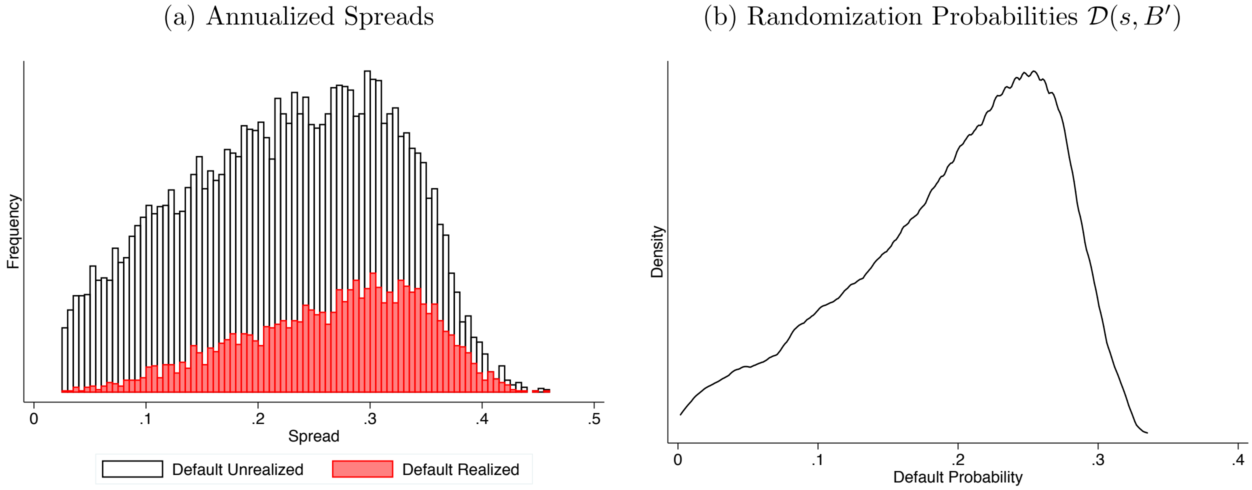
similar to the conditional distribution of the benchmark, indicating the importance of crisis deals in generating the volatility of the spread in the benchmark model.

Figure 9 considers spreads during crisis events in the benchmark model. Panel (a) plots the distribution conditional on a rollover crisis. We separate the events that result in repayment from those in which the randomization comes up default. As required by equilibrium, the crises with a higher spread are more likely to generate a subsequent default. Moreover, the distribution covers a range of spreads that encompass magnitudes observed during events like Mexico 1994 and Greece 2012. Panel (b) depicts the distribution of the randomization probabilities during crises. The mean of this distribution is 0.20; that is, conditional on a rollover crisis, the sovereign defaults 20% of the time.

In the benchmark, the sovereign is in a crisis quarter, that is $\rho = DD$ and $s \in \mathbb{C}$, only 1.3% of the time. This is despite the fact that the exogenous probability of $\rho = DD$ is calibrated to be 10 percent. The rarity of rollover crises therefore reflects the fact that the government avoids the Crisis Zone. In particular, the sovereign is in the Crisis Zone only 13.0% of the non-excluded quarters. Exposing itself to a rollover crisis is costly ex ante due to the equilibrium price schedule. A rollover crisis therefore requires both a high level of debt, but also a relatively negative growth shock.

To see this, Figure 10 depicts the distribution of growth conditional on a rollover crisis; that is, conditional on $s \in \mathbb{C}$ and $\rho = DD$. The crisis distribution is shifted to the left,

Figure 9: Crisis Spreads



Note: Panel (a) depicts the histogram of spreads in the benchmark model conditional on a rollover crisis. The unfilled bars denote episodes that did not result in a default, while the shaded bars depict the distribution conditional on a subsequent within-period default at settlement. Panel (b) depicts the simulated distribution of the government's mixed-strategy probability of default, $\mathcal{D}$, during rollover crises.

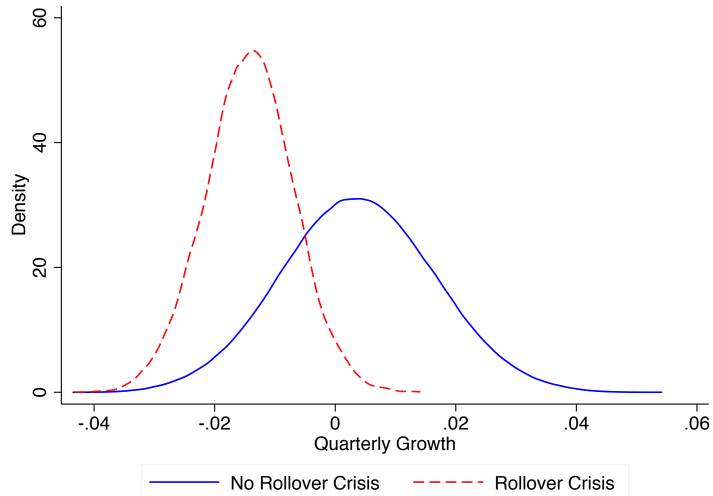
indicating that self-fulfilling crises in our model involve a combination of a shift in beliefs and a negative output realization. The mean growth conditional on a crisis is -1.4%, compared to 0.4% for non-crisis quarters. Moreover, 97% of rollover crises are associated with negative growth.

6.1 Default Post-Mortems

We now turn to default episodes in the benchmark model. Table 3 reports simulated moments conditional on default in the current quarter. For the benchmark, we can see that spreads spike during a default episode and growth is relatively low. Moreover, we see that two-thirds of defaults coincide with a rollover crisis. In the "No Deals" alternative this fraction is nearly 94%.

To obtain a better sense of the nature of rollover-crisis defaults, in Figure 11 we perform an event study analysis in the benchmark model's simulation. In particular, we normalize $t = 0$ as the quarter of default and then explore mean behavior in the preceding five quarters. The solid line depicts default events that occur with a rollover crisis ($\rho = DD$ and $b \in \mathbb{C}$), which we label "self-fulfilling" defaults. The dashed line depicts defaults that occurs outside

Figure 10: Growth and Rollover Crises



a rollover crisis ($\rho = EG$ or $b \notin \mathbb{C}$). We label the latter defaults as "fundamental" defaults as the default would occur that period regardless of the realization of ρ; of course, the fact that future crises could occur play a role in the default decision today as these events are embedded in the value of repayment.

Panel (a) of Figure 11 plots the mean growth leading up to a default event. For "fundamental" defaults we see a boom-bust pattern. Two quarters prior to default tends to generate high growth, which is then followed by a mediocre growth realization the period before default. The default itself coincides with a large negative growth realization. This pattern is the classic fundamental driven default; the high growth induces the government to borrow, and then if a large negative growth shock occurs while the economy is so highly leveraged the sovereign defaults. For self-fulfilling defaults ex ante growth is not particularly elevated, and default itself coincides with a mildly negative growth realization. The self-fulfilling defaults are thus associated with relatively minor recessions.

Panel (b) depicts the trajectory of debt before default. We see the increase in debt typical before a fundamental default, which reflects the boom period just discussed. A relatively high debt level is also necessary to sustain a self-fulfilling crisis in equilibrium, although the level is less than that associated with a fundamental default.

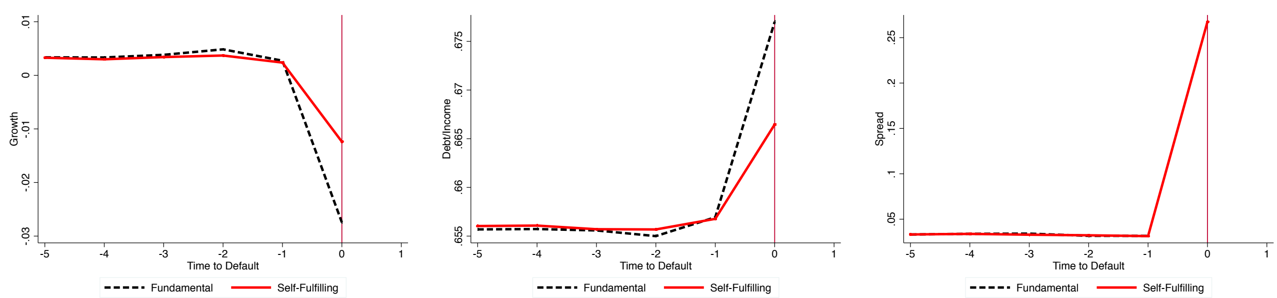
Finally, Panel (c) depicts spreads. For fundamental defaults, there is hardly any increase

Table 3: Defaults

	Benchmark	No Deals
Conditional Mean $r - r$	26.8	NA
Conditional Mean $\Delta(r - r^{*})$	23.6	NA
Conditional Mean Δy	-1.8	-1.8
Conditional Fraction $\Delta y < 0$	95.7%	98.8%
Conditional Mean $\frac{B}{Y}$	67.0%	65.0%
Conditional Fraction $\rho = DD$	62.8%	93.9%

Figure 11: Default Event Studies

(a) Growth (b) Debt/Income (c) Spreads



in spreads prior to the default, and spreads are undefined in the period of default. The fundamental defaults combine the shift up in the price schedule during the boom period and the sovereign's best response of adding debt in response, keeping spreads largely unchanged. The default then occurs because an unusually large negative growth rate is realized after a relatively large positive growth rate; as low growth is relatively unlikely to follow high growth, spreads do not anticipate the fundamental default (other than the unconditional risk in all quarters). This contrasts with self-fulfilling crises, in which spreads spike in the quarter of the default as the government issues debt at very low prices. Creditors understand the risk of imminent default and charge accordingly.

7 Discussion: The Value of Desperate Deals

We now discuss why including desperate deals has a significant impact on how the bond market operates. Viewed in isolation, the ability to issue debt at the q^D price schedule is immaterial to the sovereign; in both the benchmark and the canonical "No Deals" equilibrium, the government's payoff is the default value. Similarly, as the deals occur at competitive prices, the new lenders are indifferent at the margin to buying bonds in periods of distress. However, the legacy lenders strictly prefer a crisis equilibrium with a positive price for non-maturing bonds. In this sense, adding desperate deals generates a more efficient outcome conditional on a rollover crisis (and, of course, an even better outcome is generated by no crisis altogether).

Looking forward to the possibility of a positive secondary market price for their bonds, the legacy lenders are more willing to purchase bonds ex ante. This encourages the government to indulge its impatience and borrow, raising its non-crisis welfare. In particular, the sovereign is more likely to borrow into the crisis region, generating the extreme fluctuations in spreads observed in the data.

There are other ways to improve the efficiency of the bond market in models of this type. Looking at equilibria absent rollover crises is one approach, but this removes the high volatility at the same time. For example, Aguiar and Gopinath (2006) explore default in a model with a similar endowment process but no self-fulfilling crises and find a very stable spread.

Contrasting with this is the approach taken by Arellano (2008), who introduces nonlinear default costs. Specifically, defaults occurring during low endowment realizations are not associated with an output loss, reducing the deadweight cost of default. Moreover, this makes non-contingent bonds better insurance, as default is partially "forgiven" when output is low (which is the typical default scenario). The sovereign is thus willing to borrow at fairly high spreads, generating empirical volatility of spreads. However, this requires a volatile output process (Arellano calibrates to Argentina). Aguiar, Chatterjee, Cole, and Stangebye (in process) calibrate an Arellano-type nonlinear default cost using Mexican data and find much of the volatility in spreads disappears. Moreover, as default costs are sensitive to output, this increases the importance of income in spread fluctuations, contrasting with the modest role of fundamentals in explaining spreads (see Aguiar, Chatterjee, Cole, and Stangebye (in process)).

Efficiency could also be enhanced by renegotiating under the threat of default. If bargaining is efficient, this eliminates the deadweight costs of default. Yue (2010) explores such a model using an endowment calibration similar to Aguiar and Gopinath (2006) and generates volatile spreads. Yue's renegotiations happen immediately, and thus the sovereign is never punished for default on the equilibrium path. However, Benjamin and Wright (2008) document that in practice it takes many years for defaults to be resolved. Thus introducing bargaining during default may not spare the sovereign the costs of default. However, in practice defaults are resolved through partial repayment, which compensates creditors. Our "Desperate Deals" scenario shares this aspect, although the partial payments occur through competitive secondary market trades rather than default resolution. This feature is compelling as many crisis episodes are not associated with default.

This discussion of making bond markets more efficient highlights a key aspect of the models. While adding a sunspot and self-fulfilling crises conceivable can generate a volatile bond market, the sovereign always has the option of avoiding the drama by not borrowing. This is what happens in our "No Deals" alternative. While the government defaults in response to a combination of high debt, low output, and a run on their bonds, the sovereign never ventures into the region of the state space in which spreads are particularly volatile. Impatience is not enough, as the nonlinear prices essentially ration government debt. Viewed in this way, the volatility associated with real life markets must be relatively benign to support creditors willing to lend and governments willing to borrow. In this study, the mitigating factor is the ability to issue bonds at fire-sale prices in crisis episodes.

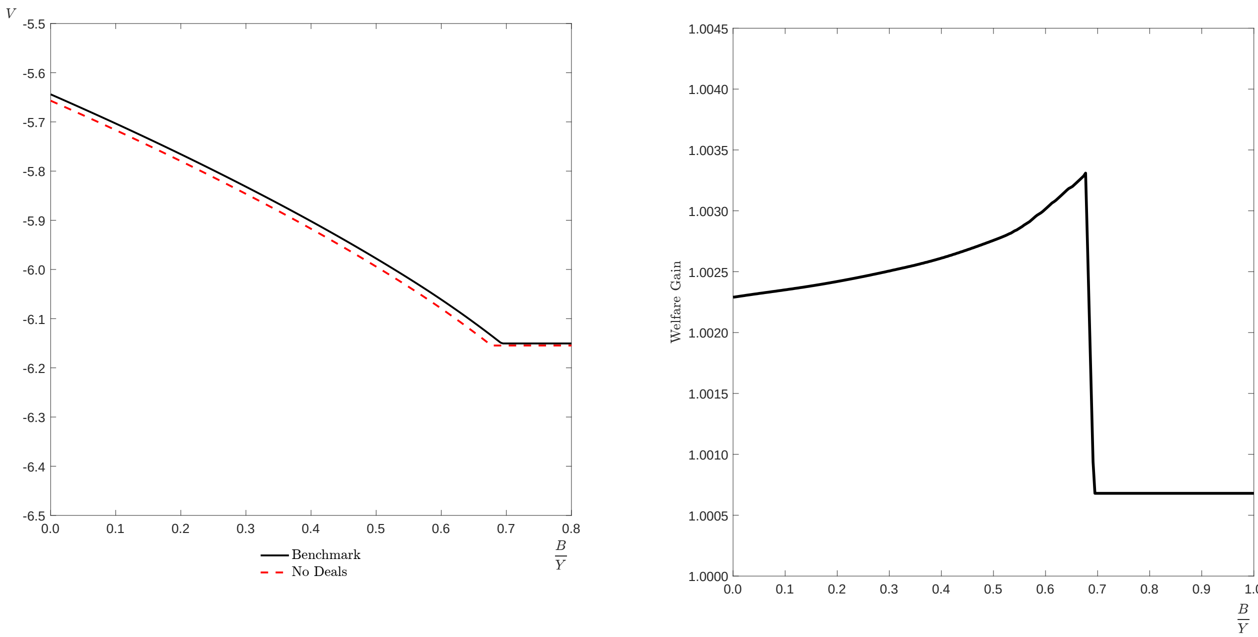
In Figure 12 Panel (a) we plot the non-crisis value functions in our benchmark and our "No Deals" alternative as a function of debt-to-income, holding constant g and z at their mean value. Both values are strictly declining until the debt level at which the sovereign prefers to default rather than repay. The benchmark value is slightly higher than the alternative. The gap in the default region is still slightly positive due to the possibility of re-entry.

To translate the gap in values into economically meaningful units, Panel (b) computes the sovereign's welfare gain from including desperate deals in terms of consumption. Specifically, the figure plots the proportional increase in consumption every period in the "No Deals" equilibrium that would make the sovereign indifferent to switching to the benchmark equilibrium, conditional on the initial B.[18] The figure indicates the welfare gains are small.

[18]This ratio is computed as $\left(\frac{V(s)}{V^{ND}(s)}\right)^{\frac{1}{1-\sigma}}$, where V and V^{ND} denote the benchmark and "No Deals" value

Figure 12: Value of Desperate Deals

(a) Value Functions (b) Consumption Equivalent Welfare Gain



Note: Panel (a) plots V for the benchmark model (solid) and the alternative model with the "No Deals" (dashed), as a function of debt-to-income B/Y when $\rho = EG$. The remaining values of s are $g = \bar{g}$ and $z = 0$. Panel (b) plots the consumption equivalent welfare gain from desperate deals as a function of B, evaluated at the same g and z as Panel (b). Specifically, the figure depicts the proportional increase in consumption every period that would make the sovereign indifferent between the model without desperate deals and the benchmark. This is given by the ratio of Panel (a)'s benchmark value function to the alternative value function evaluated at each B/Y, raised to the power $1/(1-\sigma)$.

As a rule, the gain is less than one-half of one percent. In part, this reflects the fact that conditional on a crisis the sovereign obtains the default value regardless of the equilibrium. Thus the welfare gains accrue solely from the higher ex ante prices preceding a crisis. While these prices are slightly higher (see figure 2) conditional on debt, the average spread (absent a crisis) is similar across the two equilibria. This is due to the fact that the sovereign simply borrows a little more in the benchmark equilibrium. Figure 12 tells us that this additional borrowing has a small impact on the sovereign's welfare. Such small welfare gains are not unusual in models of uninsurable business cycle risk. What is perhaps more striking is that the dramatic increase in spread volatility represents a net positive gain in welfare.

functions, respectively, depicted in Figure 12 Panel (a).

7.1 Debt Buy-Backs, Revisited

To gain a better understanding of how desperate deals improve welfare, we consider two variants on our benchmark model. Both alternatives has the sovereign *buying back* debt when default is imminent. Specifically, in the first alternative we assume the government repurchases non-maturing debt during a rollover crisis. Like the benchmark model's issuances, these repurchases take place at the q^D price schedule that leaves the government indifferent. We posit that the government repurchases ten percent of its non-maturing debt.[19]

The second alternative concerns behavior during a "fundamental default." Recall that a fundamental default is defined as a default that occurs even if the government faces the Eaton-Gersovitz price schedule, q^{EG}. These occur when debt is relatively high and a low endowment is realized and is the area of focus of the quantitative sovereign debt literature. For a fundamental default, there is no level of issuances that is sustainable in equilibrium at positive prices, as the government prefers to default rather than issue at the best possible price schedule q^{EG}. However, buy backs at low enough prices may be consistent with equilibrium, as the government may be indifferent to defaulting versus repurchasing debt at the price schedule q^D. Like in the previous paragraph's model, we assume the sovereign repurchases 10% of the non-maturing bonds during fundamental default episodes.

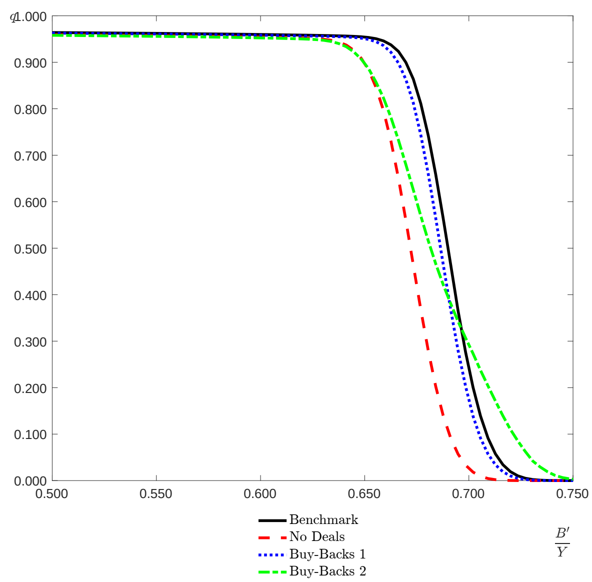
Figure 13 shows the government's price schedules under various scenarios. As we can see from the figure, debt issuance or buybacks through desperate deals during a rollover crises shift up the equilibrium price schedule fairly uniformly relative to the no-deals scenario (over the range in which there is some risk of default). However, desperate buybacks during rollover crises does so by less than debt issuance. In contrast, desperate buybacks during *fundamental* crises has very little impact on the price schedule for low risk portions of the schedule (where q is only moderately below the risk-free level) however, it shifts it out quite a bit for high-risk portions. As a result it actually twists price schedule relative to the benchmark desperate deals scenario.

The reason that desperate buybacks during rollover crises do not impact on the price schedule as must as desperate issuance can be understood from Figure 7. Buybacks do not raise the price as much as issuances and hence result in smaller gains to the legacy lenders.

The reason for twisting of the schedule with buybacks during fundamental crises is two-fold: (i) these crises only occur in fairly extreme portions of the debt-output space, and (ii)

[19]Specifically, if a rollover crisis occurs in period t, the government sets $B_{t+1} = 0.9(1-\lambda)B_t$.

Figure 13: Equilibrium Price Schedule: Alternative Models



Note: This figure depicts $q^{EG}(s, B')$ as a function of B'/Y, with $g = \bar{g}$, $z = 0$, $\rho = EG$ and $B/Y = 0.656$. As discussed in the text, the "EG" price schedule assumes no default in the current period and $\rho = EG$. The solid line is the benchmark schedule, the dashed line is the "No Deals" model, the dotted line indicates the alternative with repurchases during rollover crises, and the dash-dotted line indicates repurchases during fundamental defaults.

the fact that they reduce the incentive to delever in these events means that they encourage the country to borrow slightly more during adverse regions of the state space. The first factor is salient in extreme portions of the state space where the debt level is high and this pushes out the price schedule. However, in more moderate portions of the state space, the second factor and the threat of future dilution is key, which explains why the buybacks-during-fundamental-crises schedule is actually slightly below the no-deals schedule right where the two schedules bend.

The welfare gains from desperate buybacks and issuances during rollover crises relative to no-deals are unsurprising. They are small because these events are rare and only have a modest effect on the price schedule. Desperate issuance has the best price schedule of these three and has the largest ex ante welfare level, desperate buyback during rollover crises has the next best schedule and its ex ante welfare level lies between no-deals and issuance.

The welfare gains from desperate buybacks during fundamental crises do not follow in any obvious way from the price schedule. However, because the extreme portion of the price schedule where prices are higher under this scenario is reached very rarely, it ends up worse than either of the other desperate deals scenarios. The surprise here is that desperate buybacks during fundamental crises ends up with a very slightly lower ex ante welfare level than no-deals.

Our results on implications and efficacy of debt buy-backs have interesting implications for a long-standing debate on their benefits. Buybacks of debt emerged as potential policy tool during the Latin American crisis of the late 1980s, with Brazil, Chile and Mexico undertaking large billion dollar plus repurchases. In a classic article, Bulow and Rogoff (1988) pointed out that the very low prices on sovereign debt before a buyback was proposed did not reflect the price at which the buyback would actually occur. They argued that the debt would be bought back at the equilibrium price after the buyback, and thus the lenders would receive most or all of the gains from the buyback.

Statically speaking, the buybacks in our model suffer from exactly the feature that Bulow and Rogoff noted, and as a result, the country is indifferent between defaulting and doing either a "desperate deal" net issuance or buyback under either a rollover or a fundamental crisis. However, dynamically speaking, the prospect of future transfers to creditors under these scenarios does generate equilibrium effects (fundamentally, the transfers occur because the government avoids the deadweight losses from default with some probability). In particular, they have the potential to be welfare improving, where this improvement comes via a more favorable price schedule for government debt. However, a better price schedule also exacerbates the time-consistency problem of the government: with a better price schedule, investors rationally expect the government to borrow more in the future, which works to push down the price schedule via the dilution effect. What we see in our quantitive results is exactly the sort of mixed results that this calculus suggests, with buybacks during rollover crises being ex ante welfare improving and those during fundamental crises welfare reducing.

8 Conclusion

In this paper we extended the nature of self-fulfilling crises to include bond issuances at fire-sale prices during a rollover crisis. This was motivated by the fact that crises in practice are often associated with positive issuances at abnormally high spreads. The addition of these "desperate deals" change the nature of equilibrium spreads in the quantitative model, particularly increasing the volatility of spreads. Absent such deals, the volatility of spreads is an order of magnitude too small, despite the presence of self-fulfilling crises and defaults as frequent as in the benchmark. In the no-deals model, the sovereign either deleverages or defaults in response to adverse credit conditions. With deals, the government is willing to endure the high and volatile spreads due to crises as they are indifferent to repayment and default in such situations. However, creditors strictly prefer the positive prices of such deals, conditional on a crisis, and thus are willing to purchase bonds ex ante at more favorable prices for the issuer. This latter effect induces more borrowing on the part of the government as well as higher ex ante welfare, despite the volatility of spreads. The nature of desperate deals in the model, and the associated equilibrium behavior, provides a lens to interpret the interest rate crises used to motivate the analysis.

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Appendix Tables

Table A1: Sovereign Spreads: Summary Statistics

Country	Mean $r - r^\star$	Std Dev $r - r^\star$	Std Dev $\Delta(r - r^\star)$	95th pct $\Delta(r - r^\star)$	Frequency Crisis
Argentina	15.25	17.59	6.10	7.17	0.18
Brazil	5.60	3.93	1.74	2.04	0.09
Bulgaria	5.24	4.86	1.29	1.55	0.03
Chile	1.46	0.57	0.34	0.34	0.00
Columbia	3.48	2.06	0.88	2.45	0.05
Hungary	1.82	1.54	0.57	0.88	0.02
India	2.25	0.54	0.47	0.85	0.00
Indonesia	2.85	1.37	0.98	0.73	0.02
Latvia	1.57	0.34	0.16	0.17	0.00
Lithuania	2.46	0.92	0.48	0.98	0.00
Malaysia	1.75	1.22	0.75	0.81	0.03
Mexico	3.45	2.53	1.34	1.27	0.05
Peru	3.43	1.96	0.84	1.82	0.06
Philippines	3.43	1.53	0.75	1.36	0.04
Poland	1.91	1.38	0.54	0.67	0.01
Romania	2.71	1.02	0.49	0.68	0.00
Russia	7.10	10.96	4.78	1.75	0.06
South Africa	2.26	1.16	0.68	0.99	0.03
Turkey	3.95	2.17	0.95	2.05	0.05
Ukraine	7.60	6.07	3.50	5.77	0.11
Pooled	4.31	6.76	2.29	1.58	0.05

Table A2: Sovereign Spreads: Summary Statistics

Country	Mean $\frac{B}{4*Y}$	Corr $(\Delta(r - r^\star), \Delta y)$	Corr $(r - r^\star, \%\Delta B)$	Corr $(\Delta(r - r^\star), \%\Delta B)$
Argentina	0.38	-0.35	-0.22	0.08
Brazil	0.25	-0.11	-0.18	-0.01
Bulgaria	0.77	0.09	-0.20	0.06
Chile	0.41	-0.16	-0.18	-0.11
Columbia	0.27	-0.29	-0.40	-0.07
Hungary	0.77	-0.24	-0.56	-0.05
India	0.82	-0.32	0.04	-0.65
Indonesia	0.18	-0.43	-0.03	0.07
Latvia	0.49	-0.18	-0.12	-0.16
Lithuania	1.06	-0.25	-0.17	-0.31
Malaysia	0.54	-0.56	-0.33	0.24
Mexico	0.16	-0.4	0.23	-0.13
Peru	0.48	-0.01	-0.39	-0.05
Philippines	0.47	-0.16	0.06	0.09
Poland	0.57	-0.09	-0.35	-0.38
Romania	0.61	0.5	0.42	-0.33
Russia	NA	-0.45	-0.30	0.02
South	0.26	-0.14	-0.38	-0.24
Turkey	0.38	-0.34	-0.20	0.08
Ukraine	0.64	-0.49	-0.60	-0.07
Pooled	0.46	-0.27	-0.19	0.01