Firm-to-Firm Trade:

Imports, Exports, and the Labor Market[1]

Jonathan Eaton,[2] Samuel Kortum,[3] and Francis Kramarz,[4]

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[2]Pennsylvania State University, jxe22@psu.edu

[3]Yale University, samuel.kortum@yale.edu

[4]CREST(ENSAE), kramarz@ensae.fr

Abstract

Firm-level customs and production data reveal both the heterogeneity and the granularity of individual buyers and sellers. We seek to capture these firm-level features in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for buyer-seller connections and the share of labor in production broadly consistent with observations on French manufacturers and their customers in other countries of the European Union. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

1 Introduction

International economists have begun to exploit data generated by customs records, which describe the finest unit of trade transactions. These records expose the activity of individual buyers and sellers that underlie aggregate trade flows, which had been the object of earlier quantitative analysis in international trade.

We seek to capture both the heterogeneity and the granularity in individual buyer-seller relationships in a general equilibrium framework that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

In contrast to standard production theory, we model a firm's technology as combining a set of tasks. Each task consists of a set of subtasks that can be performed by labor, which can be of different types appropriate for different tasks. But labor competes with intermediate goods produced by other firms which can also perform these subtasks. Firms may thus look very different from one another in terms of their production structure, depending on the sellers of intermediate goods that they happen to encounter. A firm's cost in a market thus depends not only on its underlying efficiency, but also on its luck in finding low-cost suppliers. An implication is that an aggregate change, such as a reduction in trade barriers, can reduce the share of labor in production by exposing producers to more and cheaper sources of supply.

Our model is complementary to recent work of Oberfield (2017) in which a producer's cost depends not only on its own efficiency but the efficiencies of its upstream suppliers. It is also complementary to recent work of Chaney (2014) and Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), with trade the consequence of individual links formed between buyers and sellers over time. In order to embed the framework into general equilibrium, however, our analysis here remains static, more in line with the two-stage model of production in Bernard, Moxnes, and Ulltveit-Moe (2017).[1] Our model also relates to Garetto (2013), in that firms and workers compete directly to provide inputs for firms.[2]

We proceed as follows. Section 2 discusses motivating facts culled from data on the customers of French manufacturers in 24 EU destinations. Section 3 develops our model of firm-to-firm trade. Section 4 analyzes its implications for aggregate outcomes such as bilateral trade and wages. Section 5 turns to firm-level implications for our motivating facts,

[1] Bernard, Moxnes, and Saito (2015) apply this model to micro data from Japan to evaluate the effects of a new high-speed train line on firms' supplier networks.

[2] In addition to the work already cited, our paper relates closely to a number of active areas. One is recent work on exports and the labor market, including Caliendo and Rossi-Hansberg (2012), Egger and Kreickemeier (2009), Felbermayr, Prat, and Schmerer (2008), Helpman, Itskhoki, and Redding (2010), and Hummels, Jørgenson, Munch, and Xiang (2014). Another is quantitative work focussing on firm-level imports, including Biscourp and Kramarz (2007), Blaum, Lelarge, and Peters (2014), Bricongne, Lionel, Gaulier, Taglioni, and Vicard (2012), Caliendo, Monte, and Rossi-Hansberg (2015), Frías, Kaplan, and Verhoogen (2009), Helpman, Itskhoki, Muendler, and Redding (2017), Irarrazabal, Moxnes, and Ulltveit-Moe (2013), Klein, Moser, and Urban (2010), Kramarz (2009), and Kramarz, Martin, and Mejean (2015). A third is other theories of networks or input-output interactions, including Acemoglu and Autor (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Lucas (2009), and Luttmer (2015). A fourth is recent work on networks in trade. Notable papers are Tintelnot et al. (2017), Kikkawa et al. (2017), and Miyauchi (2018).

and others. Section 6 discusses our strategy to estimate the model's parameters. Section 7 concludes.

2 Basic Facts about Firm-to-Firm Trade

For the year 2005 we observe French exporters of manufactured goods and the firms that buy from them in each of 24 EU countries, which we will call destinations, observing the amount sold to each buyer in each country. Appendix A describes these data. We organize these data in terms of some definitions and identities.

We denote by N_{nF} the number of French exporters to destination n and by $\bar{B}_{nF}$ the average number of buyers per French exporter in that destination. Multiplying these two we get what we call the number of *relationships* R_{nF} between French exporters and their buyers in n:

$$R_{nF} = \bar{B}_{nF} N_{nF}. \tag{1}$$

At the aggregate level we observe the value of total absorption (purchases for final demand or for use as intermediates) X_n of manufactured goods by destination n, which we call market size. We also observe the fraction π_{nF} that is spent on manufactures from France, which we call French market share. We can decompose the value of manufactures X_{nF} shipped from France to destination n as the product of market share and market size:

$$X_{nF} = \pi_{nF} X_n.$$

The three regressions in Table 1 report the results of regressing N_{nF}, R_{nF}, and $\bar{B}_{nF}$ against π_{nF} and X_n (all in logs).[3] The first regression shows how the number of French exporters N_{nF}

[3] Note that each of the left-hand side variables emerge from two other decompositions of total French exports

in a market varies with market size and market share. Both elasticities are positive and less than one. Since the effect of market share on entry exceeds that of market size, average French sales per firm are larger in larger markets given overall French exports.[4] The second regression shows how the number of relationships R_{nF} in a market varies with market size and market share. Both coefficients are larger than for N_{nF}, with the coefficient on market share around unity. An implication is that sales per relationship are larger when market size accounts for larger French exports but not when when larger French exports are accounted for by larger French market share. The third regression shows that the number of relationships per buyer increase with market size and market share each with an elasticity around one third.[5]

We now turn to an exploration of how these various magnitudes and others vary across destinations.

X_{nF}:

$$
\begin{aligned}
X_{nF} &= \hat{x}_{nF} N_{nF} \\
X_{nF} &= \bar{x}_{nF} R_{nF}
\end{aligned}
$$

where $\hat{x}_{nF}$ is average total sales per French exporter in market n and $\bar{x}_{nF}$ is average sales per relationship in market n. The two are connected by the identity:

$$\hat{x}_{nF} = \bar{B}_{nF} \bar{x}_{nF}$$

[4]Eaton, Kortum, and Kramarz (2011) explored the same relationship using data from 1986 with 112 foreign destinations. Coefficients on both market size and market share were somewhat larger.

[5]Note that, as dictated by the accounting identity (1), the coefficients in the first and third regressions sum to the corresponding coefficients in the second.

2.1 French Exporters

In parallel with the first regression in Table 1, the x in Figure 1 plots the number of French exporters in market n, N_{nF} (in thousands) against market size X_n (total manufacturing absorption in U.S. dollars), showing simply how larger markets attract more French exporters. (Figures 1, 2, and 3 are all on log-log scales.) The number of exporters vary by a factor of about 50. Malta, the smallest market in our data, attracts the second smallest number of French exporters. The most popular destination is Belgium, reflecting France's large market share there. The largest destination, Germany, is the second most popular market.

2.2 French Relationships

In parallel with the second regression in Table 1, the x in Figure 2 plots French relationships relative to French market share, R_{nF}/π_{nF}, against market size X_n. The relationship is tight with a slope in line with the regression coefficient of 0.83 on $\ln X_n$ in the second regression.

2.3 Buyers per French Exporter

In parallel with the third regression in Table 1, the x in Figure 3a plots the mean number of buyers per French exporter in market n, $\bar{B}_{nF}$ against market size X_n. Note for Malta the mean is barely above 1 (the theoretical minimum) while for Germany the number is nearly 10.[6]

The mean number of buyers masks vast heterogeneity across French exporters in terms of

[6] Our findings in Table 1 and Figure 3a on buyers per firm are in line with evidence from Norwegian exporters reported in Bernard, Moxnes, and Ultveit-Moe (2017), their Figures 1 and 2 in particular.

their number of clients. The x in Figure 3b plots (somewhat boringly) the median rather than the mean. The median is simply 1 in the smaller destinations and 2 in the larger ones. More interesting is Figure 3c, where the x plots the number of buyers per French exporter for the French exporter at the 99th percentile in terms of number of buyers. In the smaller markets the number of buyers is less that 10 but exceeds 100 in two of the largest ones.

To consider how the number of buyers a French exporter has in a given destination correlates with the firm's export activity elsewhere, the x in Figure 3d plots the average number of buyers in Germany (on the y-axis) of French firms that also export to the market indicated (by the 3-letter abbreviation) against the number of firms exporting both to Germany and to that other market (on the x-axis).

Where the destination is DEU (Germany itself) the figure simply reports the average number of buyers per seller from Figure 3a (around 10) against the total number of French exporters to Germany (over 20,000). But for the around 1,600 that also export to Estonia (the least popular alternative destination), the average number of buyers in Germany is nearly 40. Very generally, as the number selling to the third market declines, the average number of buyers per exporter in Germany rises. Firms that succeed in penetrating a less popular market also succeed in finding more buyers in Germany.

2.4 French Sellers per Buyer

Looking at the relationship from the buyers' side, The x's in Figure 4a report the mean number of French sellers per buyer in each market against French market share π_{nF}. The number varies between just below 2 to over 3.5, rising somewhat with French market share.

The dark bars in Figure 4b show the distribution of the number of French sellers per buyer in Germany. Almost 70 percent of German buyers have only one French seller, but there are a small share of buyers for whom the number exceeds 33.

2.5 French Labor Share

Our model pertains not only to the connections between firms and their customers in different destinations, but also to how firms procure their inputs. Standard general equilibrium models treat the production function as common across categories of firms, with the prediction that firms in the same category facing common factor prices in Walrasian input markets would employ inputs in the same proportion.

To assess the appropriateness of this approach, we look at payments to production labor by French manufacturing firms as a fraction of their total variable costs, defined as the sum of intermediate purchases and payments to production labor. The x in Figure 5 plots the distribution of the production labor share across these French manufacturing firms. Note that the share varies very continuously between 0 and 0.6.

2.6 Sales per Buyer

So far we have not considered statistics related to the volume of sales to individual buyers. We now turn to the average value sold to a buyer in n by a French exporter $\bar{x}_{nF}$ (introduced in footnote 3). Parallel to Figure 3d (and with the same values on the x-axis), the x's in Figure 8 plots $\bar{x}_{nF}$ (on the y-axis) fixing country n (Germany buyers) while conditioning on French exporters that also sell in third countries (indicated by the 3-letter label). While more noisy than Figure 3d, the slope of the relationship is also negative. Firms that succeed in penetrating a less popular market also succeed in selling more to each German buyer. A similar relationship is obtained if we consider buyers from some other country, such as Belgium.

3 A Model of Production through Random Encounters

Our model seeks to address the granularity and heterogeneity of firms' relationships with buyers in different destinations. It also seeks to understand the heterogeneity of firms' production decisions.

Our basic framework is Ricardian. We consider a world with a set of $i = 1, 2, ..., \mathcal{N}$ countries. Each country has an endowment of L_i^l workers of type $l = 0, 1, 2, ..., \mathcal{L}$.

3.1 Technology

A producer j in any country i can make a quantity of output $Q(j)$ by combining a discrete set of $K + 1$ tasks, indexed by $k = 0, ..., K$. Task k in turn combines m_k subtasks labelled ω. All producers must perform the same $K + 1$ tasks but may perform each task with different subtasks. We denote the set of subtasks used by firm j for task k as $\Omega_k(j)$ and define the number of such subtasks as:

$$m_k(j) = |\Omega_k(j)| \,.$$

The production function for firm j is specifically:

$$Q(j) = z(j) \prod_{k=0}^{K} \left(\frac{1}{\beta_k} \left(\sum_{\omega \in \Omega_k(j)} x_k(j, \omega)^{(\sigma_k - 1)/\sigma_k} \right)^{\sigma_k/(\sigma_k - 1)} \right)^{\beta_k}, \tag{2}$$

where $z(j)$ is the overall efficiency of producer j, $x_k(j, \omega)$ is the input of subtask ω of task k (discussed further below), $\sigma_k > 1$ is the elasticity of substitution between subtasks for task k, and β_k is the Cobb-Douglas share of task k, satisfying $\beta_k > 0$ and

$$\sum_{k=0}^{K} \beta_k = 1.$$

We treat the number of tasks K along with the σ_k's and β_k's, as common across all firms. We denote the $(K+1) \times 1$ vector of number of subtasks per task for firm j as:

$$\mathbf{m}(j) = [m_0(j), m_1(j), ..., m_K(j)]'.$$

In any country i we denote the set of possible values of $\mathbf{m}$ as $\Omega_{\mathbf{m}}$.

In the special case in which $m_k = 1$ for all k, (2) reduces to a Cobb-Douglas production function. The point of introducing more than a single subtask is to match more flexibly our data on firm-to-firm trade. Allowing heterogeneity across firms along this dimension captures the observation that some firms have a very large number of suppliers and others very few. An elasticity of substitution greater than one among subtasks explains why a given buyer tends to purchase more when buying from a seller with more buyers.

We assume that any subtask ω of k can be performed either by the unique type of labor appropriate for that task, denoted $l(k)$, or with an input produced by another firm. We allow $K \geq \mathcal{L}$, so that one type of labor might be able to perform subtasks of several different tasks. We denote the set of tasks that labor of type l can perform as Ω_l. Worker productivity performing subtask ω of k for a firm j is $q_k(j, \omega)$. A subtask can also be performed by an appropriate intermediate input produced by another firm. From firm j's perspective, labor and the available inputs are perfect substitutes for performing any subtask. Hence it chooses whatever performs the subtask at lowest cost.

To summarize, we can describe firm j's technology in terms of its overall efficiency $z(j)$, the number of subtasks it requires for each task $\mathbf{m}(j)$, and its worker efficiency at performing each subtask, $q_k(j,\omega)$. We now turn to deriving an expression for the firm's unit cost.

We assume that, if a firm hires labor to perform a subtask of k, it does so in a standard Walrasian labor market in country i, in which labor of type l has a wage w_i^l. Hence we define the wage for task k as $w_{k,i} = w_i^{l(k)}$.

In finding intermediates, however, buyers match with only an integer number of potential suppliers, either because of search frictions or because only a handful of producers make an input appropriate for this particular firm. We could make various assumptions about the price at which the intermediate is available. Because it yields the simplest set of results, we assume Nash bargaining in which the buyer has all the bargaining power, so that the price is pushed down to unit cost.[7]

Let $c_{k,i}^{\min}(j,\omega)$ denote the lowest price available to firm j in country i for an intermediate to perform subtask ω of k. The price it pays to perform this subtask is thus:

$$c_{k,i}(j,\omega) = \min\left\{\frac{w_{k,i}}{q_k(j,\omega)}, c_{k,i}^{\min}(j,\omega)\right\}$$

and the firm's unit cost of delivering a unit of its own output to destination n is:

$$c_{ni}(j;\mathbf{m}(j)) = \frac{d_{ni}}{z(j)} \prod_{k=0}^{K} \left(\left(\sum_{\omega\in\Omega_k(j)} c_{k,i}(j,\omega)^{-(\sigma_k-1)}\right)^{-1/(\sigma_k-1)}\right)^{\beta_k}, \tag{3}$$

[7] An implication is that there are no variable profits. Our model thus cannot accommodate fixed costs, either of market entry as in Melitz (2003) or in accessing markets for inputs, as in Bernard, Moxnes, and Ultveit-Moe (2017) or Antras et al. (2017). An alternative, which would allow for variable profits and hence fixed costs, is Bertrand pricing. While we found this alternative analytically tractable, we deemed the added complexity not worth the benefit.

where $d_{ni} \geq 1$ is the iceberg transport cost of delivering a unit of output from source i to destination n, with $d_{ii} = 1$ for all i.

In order to derive a closed form solution for the distribution of costs in this setting, we impose specific distributions on the parameters of potential producers' technologies.

First, following Eaton, Kortum, and Kramarz (2011), each country has a measure of potential producers. The measure of potential producers in country i with efficiency $z(j) \geq z$ and number of subtasks per task $\mathbf{m}(j) = \mathbf{m}$ is:

$$\mu_i^Z(z; \mathbf{m}) = p(\mathbf{m}) T_i z^{-\theta}, \tag{4}$$

where $T_i \geq 0$ is a parameter reflecting the magnitude of country i's endowments of technology and $\theta \geq 0$ their similarities. We can interpret $p(\mathbf{m})$ as a probability distribution with:

$$\sum_{\mathbf{m} \in \mathbf{\Omega_m}} p(\mathbf{m}) = 1.$$

Second, worker productivity $q_{k,i}(j, \omega)$ performing subtask ω of k for a given producer j is drawn, independently over ω and j, from the probability distribution:

$$H(q) = e^{-q^{-\phi}}, \tag{5}$$

where $\phi \geq 0$ reflects the similarity of labor productivities across tasks and firms. For purposes that will become apparent below we restrict $\phi \leq \theta$.

Our specifications of the heterogeneity in producer efficiency given in (4), the distribution of worker productivity given in (5), and the distribution of numbers of subtasks $p(\mathbf{m})$ are primitives of the model, with T_i, θ, and ϕ exogenous parameters.

Our assumptions about technology, along with the specification of firm-to-firm matching in the next section, imply that the measure of potential producers from i who can produce a good at a unit cost below c is:

$$\mu_{ii}(c) = T_i \Xi_i c^{\theta}, \tag{6}$$

where $\Xi_i \geq 0$. It follows that the measure of potential producers from i who can deliver to n at a cost below c is:

$$\mu_{ni}(c) = \mu_{ii}(c/d_{ni}) = d_{ni}^{-\theta} T_i \Xi_i c^{\theta}. \tag{7}$$

We show below that the distribution of unit costs c given by (6) arises endogenously from our other assumptions, with Ξ_i determined by underlying technology, labor market conditions, and access to intermediates in different countries of the world, as well as to trade barriers between countries.

A potential producer becomes an active firm only if it meets a customer who buys from it. A customer could be a final consumer (a household) or a firm which uses the producer's output as an input. The measure of final consumers in market n is the exogenous measure of households:

$$L_n = \sum_{l=0}^{\mathcal{L}} L_n^l.$$

The measure of *active* producers in market n is determined endogenously by the *potential* producers there that are able to make a sale (either in market n or in some other destination).

3.2 Preferences

Final demand is by households spending their wage income (since there are no profits, saving, or investment in our model). Since we lack data on firm-to-household sales, we keep the consumer side of the model as stripped down as possible. We treat household demand in parallel to firms, except we assume that all households have the same preferences: Households have an integer number $\tilde{K}+1$ tasks, indexed by k', each with a Cobb-Douglas share $\tilde{\beta}_{k'}$. Each task k' can be performed by the set of $\tilde{\Omega}_{k'}$ subtasks labelled by $\tilde{\omega}$, where the number of subtasks for task k' is given by:

$$\tilde{m}_{k'} = \left|\tilde{\Omega}_{k'}\right|.$$

Hence, analogous with a firm's production function (2), household ϖ in any country has preferences given by:

$$U(\varpi) = \prod_{k'=0}^{\tilde{K}} \left(\frac{1}{\tilde{\beta}_{k'}} \left(\sum_{\omega' \in \tilde{\Omega}_{k'}} x_{k'}(\varpi, \omega')^{(\tilde{\sigma}_{k'}-1)/\tilde{\sigma}_{k'}} \right)^{\tilde{\sigma}_{k'}/(\tilde{\sigma}_{k'}-1)} \right)^{\tilde{\beta}_{k'}}$$

where $x_{k'}(\varpi, \omega')$ is the household's consumption of subtask ω' of k' and $\tilde{\sigma}_{k'}$ is the elasticity of substitution across subtasks within task k'.

3.3 Firm-to-Firm Matching

In contrast with standard Walrasian models, we assume that matching between buyers and sellers is random. Even though there are a continuum of possible sellers and buyers, an individual seller matches with only an integer number of potential buyers and, for any subtask, an individual buyer matches with only an integer number of potential sellers.

We first consider firm-to-firm matches between potential producers and buyers that are firms actively engaged in production. We denote the measure of firms buying in market n as F_n, which is determined endogenously in the equilibrium we derive below. We denote the average number of subtasks of k across producers as $\bar{m}_k$, which we derive below and is the same in any market n.

While in our case the measure of potential sellers implied by (6) is unbounded, the measure of sellers with unit cost below c is always bounded. Therefore, we treat the likelihood of a match involving a seller with unit cost c as limited by the measure of sellers with unit cost below c.

We specify the measure of firm-to-firm matches between buyers in n for subtasks of k and sellers from country i with unit cost below c as:

$$M_{k,ni}(c) = \frac{\lambda_k}{1-\gamma} \lambda_{ni} \mu_{ni}(c) \bar{m}_k F_n B_n^{-\varphi} S_n(c)^{-\gamma}. \tag{8}$$

Here, $\mu_{ni}(c)$ is the measure of potential producers from i who can sell in n at cost below c, given by (7), and $\bar{m}_k F_n$ is the number of potential purchases for subtasks of k in market n. The parameter λ_k reflects the ease with which buyers can find a supplier for subtasks of k and λ_{ni} the ease with which sellers from source i can match with buyers in destination n. The final two terms in (8) require a more extensive explanation.

The matching literature (e.g., Mortensen and Pissarides, 1994) typically posits that, as the measure of buyers and sellers in a market increases, the likelihood of a match between any given potential buyer and potential seller is smaller.[8] To capture such a "congestion effect" on the buyers' side we define the presence of buyers in market n as:

$$B_n = \sum_k \lambda_k \bar{m}_k F_n.$$

The parameter $\varphi \geq 0$ captures the extent to which buyers crowd each other out in meeting sellers.

To capture such effects on the sellers' side we define the presence of sellers with unit cost

[8] Matching in our framework can be interpreted literally as coming into contact with each other, but it also could relate to the appropriateness of a seller's product for the buyer's purpose. In this sense we can think of products as differentiated not only by seller, but by buyer as well.

below c in market n as:

$$S_n(c) = \sum_{i'} \lambda_{ni'} \mu_{ni'}(c)$$

which, from (7), we can write as:

$$S_n(c) = \Upsilon_n c^{\theta}$$

where we define:

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i. \tag{9}$$

The parameter $\gamma \geq 0$ captures the extent to which low cost sellers in market n crowd out higher cost ones.

We can write the measure of firm-to-firm matches of buyers in market n for subtasks of k with sellers from anywhere with unit cost below c as:

$$M_{k,n}(c) = \sum_{i'} M_{k,ni'} = \frac{\lambda_k}{1-\gamma} \bar{m}_k F_n B_n^{-\varphi} S_n(c)^{1-\gamma}.$$

We can write the total measure of firm-to-firm matches in market n between buyers and sellers with unit cost below c as:

$$M_n(c) = \sum_k M_{k,n}(c) = \frac{1}{1-\gamma} B_n^{1-\varphi} S_n(c)^{1-\gamma}.$$

a Cobb-Douglas combination of seller and buyer presence.

Consistent with the matching function (8) is a Poisson hazard with which a given buyer in n meets a given seller from i with unit cost c:

$$h_{k,ni}(c) = \lambda_k \lambda_{ni} B_n^{-\varphi} S_n(c)^{-\gamma}. \tag{10}$$

Consider now a buyer in country n seeking the cheapest input for a task. From the Poisson hazard (10), aggregating across potential suppliers from each source i, with different costs of delivering to n, the number of "quotes" below price c that a buyer in n receives for a subtask of k is distributed Poisson with parameter:

$$\rho_{k,n}(c) = \sum_i \int_0^c h_{k,ni}(c')d\mu_{ni}(c') = \frac{M_{k,n}(c)}{\bar{m}_k F_n} = \nu_{k,n} c^{\theta(1-\gamma)}, \tag{11}$$

where[9]:

$$\nu_{k,n} = \frac{\lambda_k}{1-\gamma} B_n^{-\varphi} \Upsilon_n^{1-\gamma}. \tag{12}$$

The probability that no supplier with unit cost below c is available to the buyer in n is $e^{-\rho_{k,n}(c)}$. If its own worker productivity is q, the buyer can also perform the subtask with labor at unit cost $w_{k,n}/q$, which will exceed c with probability $H(w_{k,n}/c)$. Since the two events are independent, the distribution of the lowest cost to fulfill a task is:

$$G_{k,n}(c) = 1 - e^{-\left(w_{k,n}^{-\phi} c^{\phi} + \nu_{k,n} c^{\theta(1-\gamma)}\right)}.$$

To work out the implications of this distribution for the resulting distribution of production costs, we restrict:

$$\phi = \theta\left(1-\gamma\right),$$

so that the parameter governing heterogeneity in the distribution of costs of intermediates becomes the same as the parameter governing heterogeneity in the distribution of worker efficiency (5). The distribution of the cost of fulfilling a task simplifies to:

[9] The steps are as follows:

$$\begin{aligned} \rho_{k,n}(c) &= \sum_i \int_0^c h_{k,ni}(c')d\mu_{ni}(c') = \lambda_k B_n^{-\varphi} \int_0^c S_n(c')^{-\gamma} \sum_i \lambda_{ni} d\mu_{ni}(c') \\ &= \lambda_k B_n^{-\varphi} \int_0^c S_n(c')^{-\gamma} dS_n(c') = \frac{\lambda_k}{1-\gamma} B_n^{-\varphi} S_n(c)^{1-\gamma} = \frac{M_{k,n}(c)}{\bar{m}_k F_n}. \end{aligned}$$

$$G_{k,n}(c) = 1 - e^{-\Phi_{k,n} c^{\theta(1-\gamma)}}, \tag{13}$$

where

$$\Phi_{k,n} = \nu_{k,n} + w_{k,n}^{-\theta(1-\gamma)}. \tag{14}$$

(We drop ϕ from the notation in all that follows.)

3.4 Deriving the Cost Distribution

From (13), (4), and (3), the measure of potential producers from source i with subtasks per task $\mathbf{m}$ that can produce at a unit cost below c is:

$$\begin{aligned}
&\mu_{ii}(c;\mathbf{m}) \\
= \;& p(\mathbf{m})T_i c^{\theta} \prod_{k=0}^{K} \left(\int_0^{\infty} ... \int_0^{\infty} \left(\left(\sum_{\omega=1}^{m_k} c_{k,\omega}^{-(\sigma_k-1)} \right)^{-1/(\sigma_k-1)} \right)^{-\theta\beta_k} dG_{k,i}(c_{k,1})...dG_{k,i}(c_{k,m_k}) \right) \\
= \;& p(\mathbf{m})T_i c^{\theta} \prod_{k=0}^{K} \Psi_{k,i}(m_k),
\end{aligned}$$

where, for $m_k = m$, we have:

$$\begin{aligned}
\Psi_{k,i}(m) &= \int_0^{\infty} ... \int_0^{\infty} \left[\int_0^{\infty} \left(\sum_{\omega=1}^{m} c_{\omega}^{-(\sigma-1)} \right)^{\theta\beta_k/(\sigma_k-1)} dG_{k,i}(c_m) \right] dG_{k,i}(c_{m-1})...dG_{k,i}(c_1) \\
&= \int_0^{\infty} e^{-x_1} ... \int_0^{\infty} e^{-x_{m-1}} \left[\int_0^{\infty} e^{-x_m} \left(\sum_{\omega=1}^{m} \left(\frac{x_\omega}{\Phi_{k,i}} \right)^{-(\sigma_k-1)/(\theta(1-\gamma))} \right)^{\theta\beta_k/(\sigma_k-1)} dx_m \right] dx_{m-1}...dx_1 \\
&= \Phi_{k,i}^{\beta_k/(1-\gamma)} g_k(m),
\end{aligned}$$

where we have changed the variables of integration to $x_\omega = \Phi_{k,i} c_{\omega}^{\theta(1-\gamma)}$ and defined:

$$g_k(m) = \int_0^{\infty} e^{-x_1} ... \int_0^{\infty} e^{-x_{m-1}} \left[\int_0^{\infty} e^{-x_m} \left(\sum_{\omega=1}^{m} x_{\omega}^{-(\sigma_k-1)/(\theta(1-\gamma))} \right)^{\theta\beta_k/(\sigma_k-1)} dx_m \right] dx_{m-1}...dx_1.$$

In the special case $m = 1$:

$$\begin{aligned} g_k(1) &= \int_0^\infty e^{-x} \left(x^{-(\sigma_k-1)/(\theta(1-\gamma))}\right)^{\beta\theta\beta_k/(\sigma_k-1)} dx \\ &= \int_0^\infty e^{-x} x^{-\beta_k/(1-\gamma)} dx \\ &= \Gamma\left(1 - \frac{\beta_k}{1-\gamma}\right), \end{aligned}$$

which is well-behaved only for $\beta_k < 1 - \gamma$.

In the special case of $\theta\beta_k = \sigma_k - 1$:

$$\begin{aligned} g_k(m) &= \int_0^\infty e^{-x_1} ... \int_0^\infty e^{-x_{m-1}} \left[\int_0^\infty e^{-x_m} \left(\sum_{\omega=1}^{m} x_\omega^{-\beta_k/(1-\gamma)}\right) dx_m\right] dx_{m-1}...dx_1 \\ &= m g_k(1). \end{aligned}$$

Returning to the general case and combining the results so far, we have shown that:

$$\mu_{ii}(c; \mathbf{m}) = p(\mathbf{m}) g(\mathbf{m}) T_i c^\theta \prod_{k=0}^{K} \Phi_{k,i}^{\beta_k/(1-\gamma)}.$$

where, recalling that $\mathbf{m} = [m_1, m_2, ...m_K]'$, we have defined:

$$g(\mathbf{m}) = \prod_{k=0}^{K} g_k(m_k)$$

Aggregating over the distribution of m:

$$\begin{aligned} \mu_{ii}(c) &= \sum_{\mathbf{m}\in\Omega_\mathbf{m}} \mu_{ii}(c; \mathbf{m}) \\ &= \bar{g} T_i c^\theta \prod_{k=0}^{K} \Phi_{k,i}^{\beta_k/(1-\gamma)}, \end{aligned}$$

where

$$\bar{g} = \sum_{\mathbf{m}\in\Omega_\mathbf{m}} p(\mathbf{m}) g(\mathbf{m}).$$

Using this term, we can solve for the mean number of subtasks per task k, $\bar{m}_k$, across producers:

$$\bar{m}_k = \sum_{\mathbf{m}\in\Omega_{\mathbf{m}}} \tilde{p}(\mathbf{m}) m_k,$$

where:

$$\tilde{p}(\mathbf{m}) = \frac{p(\mathbf{m})g(\mathbf{m})}{\bar{g}}.$$

We have confirmed our conjecture about the cost distribution in equation (6), that:

$$\mu_{ii}(c) = T_i \Xi_i c^{\theta}$$

with:

$$\Xi_i = \bar{g} \prod_{k=0}^{K} \Phi_{k,i}^{\beta_k/(1-\gamma)}. \tag{15}$$

Combining (15), (14), (12), and (9), we can either solve for the vector of Ξ_i from the system of equations:

$$\Xi_i = \bar{g} \prod_{k=0}^{K} \left(\frac{\lambda_k}{1-\gamma} B_i^{-\varphi} \left(\sum_{i'} \lambda_{ii'} d_{ii'}^{-\theta} T_{i'} \Xi_{i'} \right)^{1-\gamma} + w_{k,i}^{-\theta(1-\gamma)} \right)^{\beta_k/(1-\gamma)}. \tag{16}$$

Or, we can solve for the vector of Υ_n from the system of equations:

$$\Upsilon_n = \sum_i \lambda_{ni} d_{ni}^{-\theta} T_i \bar{g} \prod_{k=0}^{K} \left(\frac{\lambda_k}{1-\gamma} B_i^{-\varphi} \Upsilon_i^{1-\gamma} + w_{k,i}^{-\theta(1-\gamma)} \right)^{\beta_k/(1-\gamma)} \tag{17}$$

for $i, n = 1, 2, ..., \mathcal{N}$. We focus on the second system (17) whose solution, given wages w_i and the measure of active producers F_i, gives us the Υ_n.

A problem can arise in solving the system of equations (17) if it's too easy for producers to find input suppliers for all tasks, as the cost of production can collapse to zero. We need to ensure that labor requirements for production don't vanish. Our strategy for avoiding this problem is to set $\lambda_0 = 0$ (while keeping $\beta_0 > 0$), meaning that subtasks of task 0 can be performed only by labor (of type $l(0)$ at wage $w_{0,i}$). Appendix B shows that this condition suffices to guarantee a unique solution for the Υ_n's and provides an iterative procedure to compute them.

3.5 Firm-to-Household Matching

For household tasks $k' = 0, 1, ..., \tilde{K}$, we treat firm-to-household encounters like firm-to-firm encounters. But since we never observe a household importing from a foreign firm we allow only for matches between households and local firms. In parallel with our specification of firm-to-firm matching, we posit that the measure of matches between households in destination n for task k' and local producers with unit cost below c is:

$$\tilde{M}_{k',n}(c) = \frac{\tilde{\lambda}_{k'}}{1-\gamma} \left(\tilde{m}_{k'} L_n\right) \tilde{B}_n^{-\varphi} \tilde{S}_n(c)^{1-\gamma}.$$

where:

$$\tilde{B}_n = \sum_{k'=1}^{\tilde{K}} \tilde{\lambda}_{k'} \tilde{m}_{k'} L_n$$

and:

$$\tilde{S}_n(c) = \tilde{\lambda}_{nn} \mu_{nn}(c).$$

The parameter $\tilde{\lambda}_{k'}$ reflects the ease with which households can find a supplier for subtasks of task k' and reflects $\tilde{\lambda}_{nn}$ the ease with which households in destination n can find local suppliers. (Our assumption that households can buy only locally implies that $\tilde{\lambda}_{ni} = 0$ for $i \neq n$.). The parameters φ and γ are as above and reflect congestion in matching.

Parallel to our discussion above, the number of producers that a household encounters that can perform a subtask of k' at cost less that c is distributed Poisson with parameter:,

$$\tilde{\rho}_{k'n}(c) = \frac{\tilde{\lambda}_{k'}}{1-\gamma} \tilde{B}_n^{-\varphi} \tilde{S}_n(c)^{1-\gamma} = \tilde{\nu}_{k',n} c^{\theta(1-\gamma)},$$

where:

$$\tilde{\nu}_{k',n} = \frac{\tilde{\lambda}_{k'}}{1-\gamma} \tilde{B}_n^{-\varphi} \tilde{\Upsilon}_n^{1-\gamma},$$

and where:

$$\tilde{\Upsilon}_n = \tilde{\lambda}_{nn} T_n \Xi_n,$$

where Ξ_n is the solution to the set of equations (16) above. Since households represent the end of the line of the production chain their purchases don't feed back into costs.

In parallel to our discussion above, the distribution of lowest cost that a household faces for each subtask of k' is:

$$\tilde{G}_{k',n}(c) = 1 - e^{-\tilde{\nu}_{k',n} c^{\theta(1-\gamma)}}.$$

As with a firm, a household buys only from the lowest cost producer.

For simplicity we don't allow households to substitute goods with labor for tasks $k' = 1, ..., \tilde{K}$. To capture household purchases of nonmanufactures we assume that $\tilde{\lambda}_0 = 0$ and treat task 0 as services which can be provided only by labor of type 0 at a wage w_0 with productivity one.

Depending on the suppliers they encounter, each household will face different prices for each subtask. Defining the consumer price index for task k' as $P_{k'}$ we can derive an expression for:

$$\begin{aligned} E[P_{k',n}^{1-\sigma_{k'}}] &= E\left[\sum_{\omega' \in \Omega_{k'}} c_{k',\omega'}^{-(\sigma_{k'}-1)}\right] \\ &= \tilde{m} \int_0^\infty c^{-(\sigma_{k'}-1)} d\tilde{G}_{k',n}(c) \\ &= \tilde{m}\Gamma\left(1 - \frac{\sigma_{k'}-1}{\theta(1-\gamma)}\right) \tilde{\nu}_{k',n}^{(\sigma_{k'}-1)/(\theta(1-\gamma))}. \end{aligned}$$

showing how the term $\tilde{\nu}_{k',n}$ translates into consumer prices.

3.6 From Potential Producers to Firms

As mentioned at the beginning, a (potential) producer turns into an (active) firm only if it can find a buyer, either a local household or another firm. Consider a potential seller from i with unit cost c in market n. We'll start with its sales to other firms in that destination.

The measure of firm buyers in n is F_n so that, for task k, the measure of subtasks of these buyers is $\bar{m}_k F_n$. The potential seller connects to each with a Poisson hazard $h_{k,ni}(c)$ given by (10). Hence the number of firms that our potential producer from i with unit cost c encounters for a subtask of k is distributed Poisson with parameter:

$$e_{k,ni}(c) = \bar{m}_k F_n h_{k,ni}(c) = \lambda_k \lambda_{ni} \bar{m}_k F_n B_n^{-\varphi} S_n(c)^{-\gamma}.$$

We can also derive this expression directly from the matching function.[10]

But it's not enough for our seller just to encounter a buyer. To make a sale it has to beat out the competition (whether another supplier or labor), which requires that no other seller encountered by the buyer has a lower cost. Using expression (13) for the distribution of the lowest cost for a subtask of k in market n, the probability that our producer is the lowest cost for any buyer is:

$$1 - G_{k,n}(c) = e^{-\Phi_{k,n} c^{\theta(1-\gamma)}}.$$

Combining these results, this producer's number of firm-to-firm transactions in n for subtasks

[10] It is convenient to start by rewriting (8) as:

$$M_{k,ni}(c) = \frac{\lambda_k}{1-\gamma} \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i \bar{m}_k F_n B_n^{-\varphi} \Upsilon_n^{-\gamma} c^{\theta(1-\gamma)}.$$

Taking the derivative with respect to c gives the density of firm-to-firm matches between sellers with cost c and buyers in country n for task k:

$$\partial M_{k,ni}(c)/\partial c = \theta \lambda_k \lambda_{ni} d_{ni}^{-\theta} T_i \Xi_i \bar{m}_k F_n B_n^{-\varphi} \Upsilon_n^{-\gamma} c^{\theta(1-\gamma)-1}.$$

The density of producers from i who can sell in n at cost exactly c is:

$$\partial \mu_{ni}(c)/\partial c = \theta d_{ni}^{-\theta} T_i \Xi_i c^{\theta - 1}.$$

Hence the number of firms in n that our potential producer from i with unit cost c encounters for a subtask of k is distributed Poisson with parameter:

$$\begin{aligned} \frac{\partial M_{k,ni}(c)/\partial c}{\partial \mu_{ni}(c)/\partial c} &= \lambda_k \lambda_{ni} \bar{m}_k F_n B_n^{-\varphi} \Upsilon_n^{-\gamma} c^{-\theta\gamma} \\ &= \lambda_k \lambda_{ni} \bar{m}_k F_n B_n^{-\varphi} S_n(c)^{-\gamma}. \end{aligned}$$

of task k is distributed Poisson with parameter:

$$\begin{aligned}\eta_{k,ni}(c) &= e_{k,ni}(c)e^{-\Phi_{k,n}c^{\theta(1-\gamma)}} \\ &= \lambda_k\lambda_{ni}\bar{m}_k F_n B_n^{-\varphi}\Upsilon_n^{-\gamma}c^{-\theta\gamma}e^{-\Phi_{k,n}c^{\theta(1-\gamma)}}. \end{aligned} \tag{18}$$

which is decreasing in c. Summing across tasks, this producer's number of firm-to-firm transactions in market n is distributed Poisson with parameter:

$$\eta_{ni}(c) = \sum_{k=1}^{K}\eta_{k,ni}(c) = \lambda_{ni}F_n B_n^{-\varphi}\Upsilon_n^{-\gamma}c^{-\theta\gamma}\sum_{k=1}^{K}\lambda_k\bar{m}_k e^{-\Phi_{k,n}c^{\theta(1-\gamma)}}. \tag{19}$$

By the same logic, the number of household buyers that a firm in i with unit cost c encounters is distributed Poisson with parameter:

$$\tilde{\eta}_{ii}(c) = \tilde{\lambda}_{ii}L_i\tilde{B}_i^{-\varphi}\tilde{\Upsilon}_i^{-\gamma}c^{-\theta\gamma}\sum_{k'=1}^{\tilde{K}}\tilde{\lambda}_{k'}\tilde{m}_{k'}e^{-\tilde{\nu}_{k,i}c^{\theta(1-\gamma)}}$$

Aggregating across local households and firms throughout the world, the number of buyers of a firm with unit cost c in its home market i is distributed Poisson with parameter:

$$\eta_i^W(c) = \tilde{\eta}_{ii}(c) + \sum_{n=1}^{\mathcal{N}}\eta_{ni}(cd_{ni}).$$

The probability that the producer has at least 1 buyer, turning it into a firm, is $1 - e^{-\eta_i^W(c)}$, allowing us to write the measure of firms in i as:

$$\begin{aligned}F_i &= \int_0^\infty\left[1 - e^{-\eta_i^W(c)}\right]d\mu_{ii}(c) \\ &= \theta T_i\Xi_i\int_0^\infty\left[1 - e^{-\eta_i^W(c)}\right]c^{\theta-1}dc. \end{aligned} \tag{20}$$

The system of equations (20), along with (17), allows us, for given wages w_i^l around the world, to solve for the Υ_i's and F_i's. The solution gives us the structure of prices as well as firm-to-firm and firm-to-household connections around the world. We turn to the determination of wages in the next section.

4 Aggregate Equilibrium

We now have in place the assumptions we need to solve for the aggregate equilibrium. We begin by solving for labor shares and trade shares, for given wages. We then solve for equilibrium in the production of intermediates, given wages. We conclude by turning to labor-market equilibrium, which determines those wages.

4.1 Labor Shares

From expression (14), with probability $w_{k,n}^{-\theta(1-\gamma)}/\Phi_{k,n}$, a firm hires workers to perform a subtask of k while with probability $\nu_{k,n}/\Phi_{k,n}$ it purchases an intermediate from the lowest-cost supplier. Notice that these probabilities are independent of the unit cost c.[11]

Since there are a continuum of producers, $w_{k,n}^{-\theta(1-\gamma)}/\Phi_{k,n}$ is also the aggregate share of labor in performing task k in country n. The aggregate share of labor of type l in total production costs is consequently:

$$\alpha_n^l = \sum_{k \in \Omega_l} \beta_k w_{k,n}^{-\theta(1-\gamma)}/\Phi_{k,n}$$

and the overall labor share in production costs is:

$$\alpha_n^L = \sum_{l=1}^{\mathcal{L}} \alpha_n^l.$$

Note that, even though our basic technology is Cobb-Douglas across tasks k, the labor share depends on wages and deeper parameters.

[11] For task $k = 0$, the probability that a firm hires labor (of type $l(0)$) is one.

4.2 Trade Shares

From (8), the number of matches between firms in n seeking to fulfill a subtask of $k > 0$ and sellers from i with unit cost c is distributed Poisson with parameter:

$$\frac{\partial M_{k,ni}(c)}{\partial c} = \theta\lambda_k\lambda_{ni}d_{ni}^{-\theta}T_i\Xi_i\bar{m}_k F_n B_n^{-\varphi}\Upsilon_n^{-\gamma}c^{\theta(1-\gamma)-1}$$

These matches result in a sale with probability $e^{-\Phi_{k,n}c^{\theta(1-\gamma)}}$, so that the corresponding number of purchases is distributed Poisson with parameter:

$$s_{k,ni}(c) = \frac{\partial M_{k,ni}(c)}{\partial c}e^{-\Phi_{k,n}c^{\theta(1-\gamma)}} = \theta\lambda_k\lambda_{ni}d_{ni}^{-\theta}T_i\Xi_i\bar{m}_k F_n B_n^{-\varphi}\Upsilon_n^{-\gamma}c^{\theta(1-\gamma)-1}e^{-\Phi_{k,n}c^{\theta(1-\gamma)}}$$

The corresponding number of purchases from anywhere is distributed Poisson with parameter:

$$s_{k,n}(c) = \sum_{i'=1}^{\mathcal{N}} s_{k,ni'}(c).$$

Hence the probability that the purchase is with a seller from i is:

$$\pi_{ni} = \frac{s_{k,ni}(c)}{s_{k,n}(c)} = \frac{\lambda_{ni}d_{ni}^{-\theta}T_i\Xi_i}{\sum_{i'}\lambda_{ni'}d_{ni'}^{-\theta}T_{i'}\Xi_{i'}} = \frac{\lambda_{ni}d_{ni}^{-\theta}T_i\Xi_i}{\Upsilon_n}. \tag{21}$$

Note that this probability is the same for any task $k > 0$ and c. Hence, just as in Eaton and Kortum (2002), with our continuum of producers, in the aggregate π_{ni} is the bilateral trade share of source i in the intermediate purchases of destination n.

4.3 Production Equilibrium

With balanced trade, total final spending in country n, X_n^C, is labor income:

$$X_n^C = \sum_{l=0}^{\mathcal{L}} w_n^l L_n^l = \sum_{k=0}^{K} w_{k,n}L_{k,n}. \tag{22}$$

We think of the producers in our model as applying broadly to manufacturers as well as to retailers and wholesalers of manufactured goods, calling their activities sector M. In contrast, we think of the production that involves only labor (task 0 for households) more narrowly as service activities excluding retail and wholesale, labelling them sector S.

We ignore any input-ouput connections between the two sectors and we ignore trade in services, so that service output in country i, which we denote Y_i^S is simply:

$$Y_i^S = \tilde{\beta}_0 X_i^C$$

Total output of the M sector, Y_i^M, is used to satisfy final demand by local households and to satisfy demand for intermediates by local and foreign firms:

$$Y_i^M = (1 - \tilde{\beta}_0) X_i^C + \sum_{n=1}^{\mathcal{N}} \pi_{ni} (1 - \alpha_n^L) Y_n^M \tag{23}$$

Given wages, and hence final demand X_i^C, labor shares α_n^L, and trade shares π_{ni}, the system of equations (23) determines gross output of the M sector Y_i^M around the world.

4.4 Labor-Market Equilibrium

We assign labor of type 0 to the service sector. For labor of types $l = 1, 2, ..., \mathcal{L}$, labor market equilibrium in country i solves the expression:

$$w_i^l L_i^l = \alpha_i^l Y_i^M. \tag{24}$$

For labor of type 0 labor market equilibrium in country i solves:

$$w_i^0 L_i^0 = \tilde{\beta}_0 X_i^C + \alpha_i^0 Y_i^M. \tag{25}$$

Together these equations determine wages w_i^l around the world.

5 Implications for Observations on Firm-to-Firm Relationships

Having completed our model of firm-to-firm trade we now turn to how it can come to grips with the observations on the interaction of individual French manufacturing firms with their customers in different foreign markets discussed in Section 2. Analogous to the facts described there we consider our model's implications for: (i) the measure N_{ni} of firms from source i selling in different foreign destinations n, (ii) the measure of relationships R_{ni} between sellers from source i and individual buyers in different foreign destinations n, (iii) the average number of buyers per seller from source i in different foreign destinations n (R_{ni}/N_{ni}), conditioning on entry into another foreign market $n' \neq n$; (iv) the entire distribution of the number of buyers per seller from i across foreign destinations n; (v) the number of sellers from source i per buyer across foreign destinations n. In applying these results to our data, of course, we set country i to be France.

5.1 The Measure of Sellers

A firm in source i will sell to destination n if it has at least one customer there. Recall from Section 3 that the number of customers that a seller from i has in foreign destination n, conditional on its unit cost c there, is distributed Poisson with parameter $\eta_{ni}(c)$ given in (19). The probability that it has at least one customer in n is thus $1 - \exp(-\eta_{ni}(c))$.

We can calculate N_{ni}, the measure of firms from i selling in n, by integrating this probability over the distribution of costs in n for firms from i, given in (7):

$$N_{ni} = \int_0^\infty (1 - e^{-\eta_{ni}(c)}) d\mu_{ni}(c) = d_{ni}^{-\theta} \int_0^\infty (1 - e^{-\eta_{ni}(c)}) d\mu_{ii}(c). \tag{26}$$

Expression (26) constitutes our model's theoretical counterpart to Figure 1 in Section 2. Note how the iceberg cost d_{ni} enters directly while the information friction λ_{ni} enters only indirectly, through $\eta_{ni}(c)$. From expression (19), $\eta_{ni}(c)$ is linear in λ_{ni}. Hence the effect of λ_{ni} on N_{ni} is nonlinear. Increasing it from zero has a large effect on N_{ni}. Further increases have a diminishing effect that asymptotes to zero.

5.2 The Measure of Relationships

The measure of relationships between sellers from source i and buyers in foreign destination n, R_{ni}, is simply the expected number of customers for a firm with unit cost c there, given by the Poisson parameter $\eta_{ni}(c)$, integrated over the distribution of costs c of firms from i in n:

$$\begin{aligned} R_{ni} &= \int_0^\infty \eta_{ni}(c) d\mu_{ni}(c) = T_i \Xi_i d_{ni}^{-\theta} \int_0^\infty \eta_{ni}(c) \theta c^{\theta-1} dc & (27) \\ &= \pi_{ni} \Upsilon_n F_n B_n^{-\varphi} \Upsilon_n^{-\gamma} \int_0^\infty \sum_{k=1}^K \bar{m}_k \lambda_k c^{-\gamma\theta} e^{-\Phi_{k,n} c^{\theta(1-\gamma)}} \theta c^{\theta-1} dc \\ &= \pi_{ni} F_n (1-\gamma) \sum_{k=1}^K \nu_{k,n} \bar{m}_k \int_0^\infty e^{-\Phi_{k,n} c^{\theta(1-\gamma)}} \theta c^{\theta(1-\gamma)-1} dc \\ &= \pi_{ni} F_n \sum_{k=1}^K \nu_{k,n} \bar{m}_k \left. \frac{-1}{\Phi_{k,n}} e^{-\Phi_{k,n} c^{\theta(1-\gamma)}} \right|_0^\infty \\ &= \pi_{ni} F_n \sum_{k=1}^K \bar{m}_k \frac{\nu_{k,n}}{\Phi_{k,n}}. & (28) \end{aligned}$$

Expression (28) constitutes our model's theoretical counterpart to Figure 2 in Section 2. Note how the measure of relationships between sellers from i and buyers from n is proportional to i's trade share in n and a term that depends only on destination n magnitudes. Recall from

(21) that π_{ni} is proportional to $\lambda_{ni} d_{ni}^{-\theta}$, so that relationships R_{ni} are affected by these two frictions distinctly from how they affect exporters N_{ni}: An increase in λ_{ni} has a linear effect on relationships but a diminishing effect on the number of exporters. The distinction allows us to identify the separate roles of iceberg costs and match frictions in trade shares.

5.3 Buyers per Seller

Dividing the measure of relationships between sellers in i and buyers in n by the measure of firms from i selling in foreign destination n, we get the mean number of buyers per seller (customers per firm):

$$\bar{B}_{ni} = \frac{R_{ni}}{N_{ni}}.$$

It represents the mean of the integer-valued random number B_{ni} of customers in n buying from a firm from i.

Consider a firm from i selling in n at cost c. Its number of customers is distributed Poisson with parameter $\eta_{ni}(c)$ given in (19). The probability that it has x buyers there is thus:

$$p_{ni}(x;c) = \Pr[B_{ni}(c) = x] = \frac{e^{-\eta_{ni}(c)} \left[\eta_{ni}(c)\right]^x}{x!},$$

for $x = 0, 1, ...$ The mean of $B_{ni}(c)$ is simply:

$$\bar{B}_{ni}(c) = \sum_{x=1}^{\infty} x p_{ni}(x;c) = \sum_{x=1}^{\infty} x \frac{e^{-\eta_{ni}(c)} \left[\eta_{ni}(c)\right]^x}{x!} = \eta_{ni}(c). \tag{29}$$

Integrating this term over the distribution of costs in n for firms from i that find at least one customer there, we come full circle to:

$$\bar{B}_{ni} = \int_0^{\infty} \bar{B}_{ni}(c) \frac{d\mu_{ni}(c)}{\int_0^{\infty}(1 - e^{-\eta_{ni}(x)}) d\mu_{ni}(x)} = \frac{1}{N_{ni}} \int_0^{\infty} \eta_{ni}(c) d\mu_{ni}(c) = \frac{R_{ni}}{N_{ni}}. \tag{30}$$

Expression (30) constitutes our model's theoretical counterpart to Figure 3a in Section 2.

For any $x \geq 1$, we can integrate over the cost measure to obtain the measure of firms from i with x buyers in n:

$$N_{ni}(x) = \int_0^\infty p_{ni}(x;c)d\mu_{ni}(c) = \int_0^\infty \frac{e^{-\eta_{ni}(c)}\left[\eta_{ni}(c)\right]^x}{x!}d\mu_{ni}(c) \tag{31}$$

Thus, the fraction of firms from i selling in n who have x buyers is:

$$p_{ni}(x) = \frac{N_{ni}(x)}{N_{ni}},$$

for $x = 1, 2, ...$[12] Expression (31) constitutes our model's theoretical counterpart to Figure 3b and 3c in Section 2.

5.4 Buyers per Seller, Conditional on Selling Elsewhere

By analogy to moments in Eaton, Kortum, and Kramarz (2011), we can calculate the expected number of buyers $B_{ni|n'}$ in foreign destination n per seller from i, conditional on the firm also selling in some third country n'. The firm's ability to sell in n' indicates that it is likely to

[12] An alternative way to express the measure of firms from i with at least one firm buyer in n is:

$$N_{ni} = \sum_{x=1}^{\infty} N_{ni}(x).$$

An alternative for the number of relationships is:

$$R_{ni} = \sum_{x=1}^{\infty} xN_{ni}(x).$$

Thus a third way to think of mean buyers per seller is

$$\bar{B}_{ni} = \sum_{x=1}^{\infty} x\frac{N_{ni}(x)}{N_{ni}} = \sum_{x=1}^{\infty} xp_{ni}(x).$$

have a lower cost in n than the typical firm from i that sells there. Hence such a firm should have more buyers in n, on average, than those not selling in n'.

We first compute the measure of relationships in n for firms from i that also sell in n':

$$R_{ni,n'} = \int_0^\infty \eta_{ni}(cd_{ni}) \left(1 - e^{-\eta_{n'i}(cd_{n'i})}\right) d\mu_{ii}(c). \tag{32}$$

In terms of buyers per firm, conditional on a firm from i selling in both n and n', we have:

$$B_{ni|n'} = \frac{R_{ni,n'}}{N_{(nn')i}},$$

where the measure of firms from i selling in both n and n' is:

$$N_{(nn')i} = \int_0^\infty (1 - e^{-\eta_{ni}(cd_{ni})})(1 - e^{-\eta_{n'i}(cd_{n'i})}) d\mu_{ii}(c). \tag{33}$$

We have exploited the fact that, given the firm's cost at home c, having at least one buyer in n is independent of having at least one buyer in n'.

Taking the ratio of (32) and (33) we get the average number of buyers in n for firms from i selling in both n and n':

$$\bar{B}_{ni,n'} = \frac{R_{ni,n'}}{N_{(nn')i}} \tag{34}$$

Expression (34) constitutes our model's theoretical counterpart to Figure 3d in Section 2.

5.5 Sellers Per Buyer

A firm seeking inputs has an average of $\bar{m}_k$ subtasks to fulfill for each task k. For a firm in country n, the probability of outsourcing a subtask of k is $\nu_{k,n}/\Xi_{k,n}$. The probability that the supplier comes from country i is π_{ni}. Let S_{ni} be the random number of suppliers from country i for a firm in country n. The expected number of such suppliers is:

$$E\left[S_{ni}\right] = \pi_{ni} \sum_{k=1}^{K} \bar{m}_k \frac{\nu_{k,n}}{\Xi_{k,n}}.$$

What we observe, however, is the expected number of suppliers from i for a firm in n, conditional on the firm having at least one supplier from i.

The probability of a firm in n having at least one supplier from i is:

$$\Pr\left[S_{ni} \geq 1\right] = 1 - \prod_{k=1}^{K} \left(\sum_{m_k=1}^{\infty} \tilde{p}(m_k) \left(1 - \pi_{ni} \frac{\nu_{k,n}}{\Xi_{k,n}}\right)^{m_k} \right).$$

Under the assumption that the number of subtasks for task k is a the same for all firms, this equation simplifies to:

$$\Pr\left[S_{ni} \geq 1\right] = 1 - \prod_{k=1}^{K} \left(1 - \pi_{ni} \frac{\nu_{k,n}}{\Xi_{k,n}}\right)^{m_k}.$$

In that case, the theoretical analog of the observable moment is:

$$E\left[S_{ni} | S_{ni} \geq 1\right] = \frac{\pi_{nF} \sum_{k=1}^{K} \frac{\nu_{k,n}}{\Xi_{k,n}}}{1 - \prod_{k=1}^{K} \left(1 - \pi_{ni} \frac{\nu_{k,n}}{\Xi_{k,n}}\right)^{m_k}}. \tag{35}$$

In the appendix we provide a more general formula, removing the assumption that all firms have the same number of subtasks per task. That generalized version of expression (35) constitutes our model's theoretical counterpart to Figure 4a in Section 2. The appendix also derives the closely related theoretical counterpart of Figure 4b in Section 2.

6 Estimation

We now describe our procedure for estimating the model and present preliminary results. Our strategy is to search for a vector of parameters that minimizes deviations between the moments shown in Section 2 and the model's predictions for these moments. (At this point we do not attempt to match the moments on sales per buyer.) Recall that our data are for French exporters, $i = F$, and buyers in 24 EU countries n.

To eliminate parameters that would not be well identified from the moments in Section 2, we estimate a restricted version of the model. These restrictions impose symmetry across tasks:

- We assume that λ_k, σ_k, β_k, w_k, and $\bar{m}_k$ are the same for tasks $k = 1, ..., K$. As a result we have: $\nu_{k,n} = \nu_n$ and $\Phi_{k,n} = \Phi_n$.
- To streamline the notation we rename $\beta_0 = \beta$ so that $\beta_k = (1 - \beta)/K$.
- As mentioned in Section 3.4 we set $\lambda_0 = 0$. We can then normalize $\lambda_k = 1$ for $k = 1, ..., K$.
- In this restricted model, the measure of buyers in country n simplifies to:

  $$B_n = \bar{m} K F_n$$

  and the Poisson parameter, for the number of buyers in country n of a French firm delivering to n at cost c, simplies to:

  $$\eta_{nF}(c) = \lambda_{nF} B_n^{1-\varphi} \Upsilon_n^{-\gamma} c^{-\theta\gamma} e^{-\Phi_n c^{\theta(1-\gamma)}}.$$

In linking the model to the data, we condition on three sets of outcomes. A consequence is that the model will fit these outcomes exactly. The purpose of conditioning is to absorb parameters so that we reduce the dimension of the unknown parameter space over which we need to search. Appendix C describes the details:

- To absorb the iceberg parameters d_{nF}, we condition on French market share in each destination:

  $$\frac{X_{nF}}{X_n} = \pi_{nF}.$$

- To absorb the parameters of the availability of suppliers Υ_n we condition on the intermediate share of manufacturing production β_n^M, which we define as 1 minus the value added share of gross production in country n:

$$\beta_n^M = (1-\beta)\frac{\nu_n}{\Phi_n}.$$

- To absorb the measure of buyers in each destination, we condition on relationships of French exporters:

$$R_{nF} = \pi_{nF} B_n \frac{\nu_n}{\Phi_n}.$$

After restricting the model and conditioning on these three sets of outcomes, we vastly reduce the number of parameters to estimate. Since the number of tasks K is only weakly identified by our current moments, we fix $K = 3$. Note that θ and σ are not separately identified, but we do identify:

$$\tilde{\sigma} = \frac{\sigma - 1}{\theta}.$$

We are left with the following vector of parameters:

$$\Theta = \left\{\gamma, \varphi, \beta, \tilde{\sigma}, \{\lambda_{nF}\}_{n=1}^{24}, \{\tilde{p}(m) \,|m = 2^x, x = 0, 1, ..., 8\}\right\}$$

Note that we restrict the number of subtasks per task m to be in the set $\Omega_m = \{1, 2, 4, ..., 256\}$. Details of the estimation algorithm are presented in Appendix C.

The best fitting set of parameters are: $\gamma = 0.46$, $\varphi = 0.37$, $\beta = 0.16$, $\tilde{\sigma} = 0.41$, together with the $\{\lambda_{nF}\}$ plotted in Figure 6 and the $\{\tilde{p}(m)\}$ plotted in Figure 7.

As shown in Figure 7, the modal number of subtasks per task is 4. Multiplying by the number of tasks means that the modal firm has 12 production subtasks. There is a long right tail to this distribution.

The estimates of $\gamma = 0.46$ and $\varphi = 0.37$ together imply increasing returns in the matching function. A doubling in the number of buyers and in the number of sellers (with cost below c) leads to a 17 percent increase in matches.

Returning to Figures 1-5, we see the ability of the model to fit the observations in Section 2. The fit is perfect in Figure 2 since we condition on relationships, but even in the other figures the deviations between data and model are modest.

While Figure 8 was not used in the estimation, we can still see how well the model and parameters capture the relationship in the data. Apparently we understate the effect of selection on sales per buyer in Germany. That contrasts with Figure 3d, showing that we overstate the effect of selection on buyers per French exporter in Germany.

7 Conclusion

Taking into account the granularity of individual buyer-seller relationships expands the scope for firm heterogeneity in a number of dimensions. Aside from differences in raw efficiency, firms experience different luck in finding cheap inputs. These two sources of heterogeneity combine to create differences in a firm's cost to deliver to different markets around the world. But within each market firms have different degrees of luck in connecting with buyers. We can thus explain why a firm may happen to sell in a small, remote market while skipping over a large one close by. It also explains why one firm may appear very successful in one market and sell very little in another, while another firm does just the opposite.

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8 Appendix A: Data Source

The empirical analysis is conducted using detailed export data covering the universe of French exporting firms. The data have been provided by the French Customs, and have been used by Kramarz, Martin, and Mejean (2014). The full data set covers all export transactions that involve a French exporter and an importing firm located in the European Union. In this paper, we use only the data for the year 2005.

Many researchers before us have used individual trade data from the French Customs. Typically, the data used in such empirical analyses are annual measures disaggregated at the level of the exporting firm, as in Eaton, Kortum, and Kramarz (2011) among others. Some papers, such as Biscourp and Kramarz (2007) and Blaum, Lelarge, and Peters (2014), also use data at the level of the importer. An exception is Bricongne, Fontagné, Gaulier, Taglioni, and Vicard (2012) who use data that record, for each exporting firm, each transaction in each month, although not identifying the exact buyer. In this respect, the data we use are more precise since they not only record the transaction but also the exact identity of the buyer. For each transaction, the dataset gives us the identity of the exporting firm (its name and its SIREN identifier), the identification number of the importer (an anonymized version of its VAT number), the date of the transaction (month and year), the product category of the transaction (at the 8-digit level of the combined nomenclature), the value and the quantity of the shipment. For the analysis here, records will be aggregated across transactions within a year, for each exporter-importer-product triplet. Such measurement is possible because, whereas goods are perfectly free to move across countries within the European Union, firms selling goods outside France are still compelled to fill a Customs form. Such forms are used to repay VAT for transactions on intermediate consumptions. Hence, our data are exhaustive. However, small exporters are allowed to fill a "simplified" form that does not require the product category of exported goods. The "simplified" regime can be used by firms with total exports in the EU below 100,000 euros in 2005 (and 150,000 euros thereafter). In 2005, we have data for 46,928 French firms exporting 7,807 8-digit products to 571,149 buyers located in the EU. Total exports by these firms amounts to 207 billions of euros. Such exports account for 58 percent of French total exports. The total number of observations is 3,983,909.

9 Appendix B: Computing Υ

[***THIS APPENDIX IS NOT UPDATED***] We derive conditions under which there is a unique solution for Υ, given wages, that can be computed by simple iteration. To ensure a solution it helps to have a sufficient share of tasks in which outsourcing is not possible ($\lambda_k = 0$). Denote the set of such tasks as Ω^0 and its complement (among the set of all tasks $\{1, 2, ..., K\}$) as Ω^P. We require:

$$\beta^P = \sum_{k \in \Omega^P} \beta_k < 1.$$

As a warm-up exercise, we start with the case of a single country ($\mathcal{N} = 1$), so that Υ is a scalar. We then turn to the general case with multiple countries, in which Υ is an $\mathcal{N} \times 1$ vector.

9.1 The Case of a Single Country

With a single country, the solution for Υ is a fixed point

$$\Upsilon = f(\Upsilon)$$

of the function f defined as:

$$f(x) = T \prod_{k=1}^{K} \left(\frac{\theta}{\phi} \lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k}.$$

Employing our assumption that $\lambda_k = 0$ for all tasks $k \in \Omega^0$, we can write:

$$f(x) = T \left(\prod_{k \in \Omega^0} (w_k)^{-\theta\beta_k} \right) \prod_{k \in \Omega^P} \left(\frac{\theta}{\phi} \lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k}.$$

It is convenient to work in logs. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the function:

$$F(y) = A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right),$$

where

$$A = \ln T - \sum_{k \in \Omega^0} \theta \beta_k \ln w_k,$$

and

$$u_k = \frac{\theta}{\phi} \lambda_k$$

There exists a unique fixed point of F if it is a contraction. To show that it is, we can check Blackwell's sufficient conditions, monotonicity and discounting. For monotonicity, note that $x \leq y$ implies:

$$F(x) = A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} x} + w_k^{-\phi} \right) \leq A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right) = F(y).$$

For discounting, $a > 0$ implies:

$$\begin{aligned} F(y+a) &= A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta}(y+a)} + w_k^{-\phi} \right) = A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(e^{\frac{\phi}{\theta} a} u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right) \\ &= A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \left[\frac{\phi}{\theta} a + \ln \left(u_k e^{\frac{\phi}{\theta} y} + e^{-\frac{\phi}{\theta} a} w_k^{-\phi} \right) \right] = A + \beta^P a + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + e^{-\frac{\phi}{\theta} a} w_k^{-\phi} \right) \\ &\leq A + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_k e^{\frac{\phi}{\theta} y} + w_k^{-\phi} \right) + \beta^P a = F(y) + \beta^P a. \end{aligned}$$

We can thus compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with $y^{(0)} = 0$. This method is justified, since the contraction mapping theorem guarantees that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that $\ln \Upsilon$ is increasing in technology T, decreasing in any task-specific wage w_k, and increasing in any task-specific arrival of price quotes λ_k.

9.2 Multiple Countries

Consider generalizing the argument above to a world of many countries, trading intermediates and final goods with each other. Now Υ is an $\mathcal{N} \times 1$ vector satisfying

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left(\frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i^{\frac{\phi}{\theta}} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k},$$

for $n = 1, ..., \mathcal{N}$.

Let $\ln \Upsilon$ be the corresponding vector with $\ln \Upsilon_n$ in place of Υ_n for $n = 1, ..., \mathcal{N}$. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the mapping F, whose n'th element is:

$$\begin{aligned} F_n(y) &= \ln \left[\sum_i \exp \left(\ln \left(T_i d_{ni}^{-\theta} \right) + \sum_k \frac{\theta}{\phi} \beta_k \ln \left(u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right] \\ & \ln \left[\sum_i \exp \left(A_{ni} + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left(u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right], \end{aligned}$$

where

$$A_{ni} = \ln\left(T_i d_{ni}^{-\theta}\right) - \sum_{k\in\Omega^0} \theta\beta_k \ln w_{k,i}$$

and

$$u_{k,i} = \frac{\theta}{\phi}\lambda_{k,i}.$$

We can check Blackwell's conditions again. For monotonicity, it is readily apparent that for a vector $x \leq y$ we have $F_n(x) \leq F_n(y)$ for all $n = 1, ..., \mathcal{N}$. For discounting, consider $a > 0$ so that

$$\begin{aligned}
F_n(y+a) &= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}(y_i+a)} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \left[\frac{\phi}{\theta}a + \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right]\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right)\right] \\
&\leq \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] + \beta^P a \\
&= F_n(y) + \beta^P a.
\end{aligned}$$

Thus, even with multiple countries, we can still compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with an $\mathcal{N} \times 1$ vector $y^{(0)}$ (which could simply be a vector of zeros). This method is justified, since the contraction mapping theorem guarantees (just as in the scalar case) that:

$$\lim_{t\to\infty} y^{(t)} = \ln\Upsilon.$$

This result also give us the comparative statics. We see directly that each element of $\ln \Upsilon$ is increasing in technology anywhere T_i, decreasing in any task-specific wage $w_{k,i}$ in any country, and increasing in any task-specific arrival of price quotes $\lambda_{k,i}$ in any country. An important caveat, however, is that these comparative statics take task-specific wages as given, so do not predict general-equilibrium outcomes.

10 Appendix C: Estimation Algorithm

Here we describe our algorithm to generate prediction of the model, given parameters. This algorithm is the basis of our method of moments estimation procedure.

The estimation algorithm can be decomposed into three separate modules. These modules can be thought of as identifying different pieces of the parameter vector:

$$\Theta = \{\Theta_1, \Theta_2, \Theta_3\}.$$

While we estimate the entire parameter vector Θ jointly, it is more transparent to describe the three modules as being distinct, each one building on those that precede it. Recall that we have fixed $K = 3$.

1. The first module involves the moments of French firm entry and relationships by destination market. This module most strongly identifies the parameters:

$$\Theta_1 = \left\{\gamma, \varphi, \{\lambda_{nF}\}_{n=1}^{24}\right\}.$$

2. The second module concerns the moments of the distribution of French suppliers per customer, by destination market. This module most strongly identifies the parameters

of the distribution of sub-tasks per task across firms:

$$\Theta_2 = \{\tilde{p}(2^x)\}_{x=0}^{8}.$$

3. The third module concerns the distribution of labor's share, which we measure for French manufactures. This module most strongly identifies the parameters:

$$\Theta_3 = \{\beta, \tilde{\sigma}\}.$$

We describe the algorithm for each module in turn.

10.1 Algorithm for Module 1

We present the algorithm as a series of steps:

1. Exploiting our measure of intermediates as a share of total costs (noting that a share β of total costs are administrative labor):

$$\beta_n^M = (1-\beta)\frac{\nu_n}{\Phi_n},$$

or:

$$\frac{\nu_n}{\Phi_n} = \frac{\beta_n^M}{1-\beta}.$$

Linking back to fundamental parameters of the model:

$$\Phi_n = \frac{(1-\beta)\,\nu_n}{\beta_n^M} = \frac{1-\beta}{1-\gamma}\frac{B_n^{-\varphi}}{\beta_n^M}\Upsilon_n^{1-\gamma}.$$

While we don't know Υ_n, on the right hand side of this expression, its value will cancel out in what follows.

2. Invert the expression for relationships:

$$R_{nF} = \pi_{nF} B_n \frac{\nu_n}{\Phi_n} = \pi_{nF} B_n \frac{\beta_n^M}{1-\beta},$$

to get the measure of buyers (multiplied by the total subtasks per buyer):

$$B_n = \frac{(1-\beta)\, R_{nF}}{\beta_n^M \pi_{nF}}. \tag{36}$$

3. Calculate a transformed version of $\eta_{nF}(c)$, taking account of the transport cost in delivering to destination n:

$$\begin{aligned} \eta_{nF}(cd_{nF}) &= \lambda_{nF} B_n^{1-\varphi} \left(\Upsilon_n c^\theta d_{nF}^\theta\right)^{-\gamma} e^{-\Phi_n (cd_{nF})^{\theta(1-\gamma)}} \\ &= \lambda_{nF} B_n^{1-\varphi} \left(\Upsilon_n c^\theta d_{nF}^\theta\right)^{-\gamma} e^{-\frac{1-\beta}{1-\gamma}\frac{B_n^{-\varphi}}{\beta_n^M}\left(\Upsilon_n c^\theta d_{nF}^\theta\right)^{1-\gamma}}. \end{aligned}$$

Note that B_n cancels. To cancel Υ_n as well, make the change of variable $y = T_F \Xi_F c^\theta$ and construct:

$$\begin{aligned} \tilde{\eta}_{nF}(y) &= \eta_{nF}\left(\left(\frac{y}{T_F \Xi_F}\right)^{1/\theta} d_{nF}\right) \\ &= \lambda_{nF} B_n^{1-\varphi} \left(\frac{\lambda_{nF}}{\pi_{nF}} y\right)^{-\gamma} e^{-\frac{1-\beta}{1-\gamma}\frac{B_n^{-\varphi}}{\beta_n^M}\left(\frac{\lambda_{nF}}{\pi_{nF}} y\right)^{1-\gamma}}. \end{aligned} \tag{37}$$

All terms on the right-hand-side (with the exception of y) are either parameters or data. We now propose a set of moments that will allow us to identify the parameters. These moments will be constructed for the sample of 24 EU countries. They all involve integals of (37).

4. The first of these moments is entry, by destination, of French firms:

$$N_{nF} = T_F \Xi_F \int_0^\infty (1 - e^{-\eta_{nF}(cd_{nF})}) \theta c^{\theta-1} dc.$$

Applying the change of variables $y = T_F \Xi_F c^\theta$, so that $dy = \theta T_F \Xi_F c^{\theta-1} dc$, and substituting in (37):

$$N_{nF} = \int_0^\infty \left(1 - e^{-\tilde{\eta}_{nF}(y)}\right) dy,$$

which depends only on parameters and data.

5. The second of these moments is the distribution, by destination, of the number of buyers per French firm. In particular, the fraction of French entrants into n who have b buyers is:

$$\begin{aligned} \frac{N_{nF}(b)}{N_{nF}} &= \frac{T_F \Xi_F}{N_{nF}} \int_0^\infty \frac{e^{-\eta_{nF}(cd_{nF})}}{b!} \left[\eta_{nF}(cd_{nF})\right]^b \theta c^{\theta-1} dc \\ &= \frac{1}{N_{nF}} \int_0^\infty \frac{e^{-\tilde{\eta}_{nF}(y)}}{b!} \left[\tilde{\eta}_{nF}(y)\right]^b dy \end{aligned}$$

for $b = 1, 2, ...$

6. The third of these moments is the expected number of buyers in n per French firm exporting to both n and n':

$$\begin{aligned} \bar{b}_{nF|n'} &= \frac{R_{nF,n'}}{N_{(nn')F}} = \frac{T_F \Xi_F}{N_{(nn')F}} \int_0^\infty \eta_{nF}(cd_{nF}) \left(1 - e^{-\eta_{n'F}(cd_{n'F})}\right) \theta c^{\theta-1} dc \\ &= \frac{1}{N_{(nn')F}} \int_0^\infty \tilde{\eta}_{nF}(y) \left(1 - e^{-\tilde{\eta}_{n'F}(y)}\right) dy, \end{aligned}$$

where:

$$\begin{aligned} N_{(nn')F} &= T_F \Xi_F \int_0^\infty (1 - e^{-\eta_{nF}(cd_{nF})})(1 - e^{-\eta_{n'F}(cd_{n'F})}) \theta c^{\theta-1} dc \\ &= \int_0^\infty \left(1 - e^{-\tilde{\eta}_{nF}(y)}\right) \left(1 - e^{-\tilde{\eta}_{n'F}(y)}\right) dy. \end{aligned}$$

We can calculate this moment across both $n \neq F$ and for all n' (including $n' = F$).

10.2 Algorithm for Module 2

We now describe how data on the distribution of French suppliers per buyer allow us to uncover the distribution of the number of sub-tasks per task $\tilde{p}(m)$ (and hence $\bar{m}$). Fortunately, we can do so independently of the parameters identified in the first algorithm.

The probability of a firm in n outsourcing a sub-task to a French firm is:

$$p_{nF} = \pi_{nF}\frac{\nu_n}{\Phi_n} = \pi_{nF}\frac{\beta_n^M}{1-\beta}.$$

Hence these probabilities are as good as known. Let s_{nF} denote the number of French suppliers of the firm. Conditional on m sub-tasks per task there are a total of mK sub-tasks per firm. Hence, for $s = 0, 1, ..., m$:

$$\Pr\left[s_{nF} = s|m, K\right] = \binom{mK}{s}\left(p_{nF}\right)^s\left(1-p_{nF}\right)^{mK-s},$$

where, for $s > mK$:

$$\Pr\left[s_{nF} = s|m, K\right] = 0.$$

Unconditionally, allowing for any value of $s = 0, 1, 2, ...$:

$$\Pr\left[s_{nF} = s\right] = \sum_{m\in\Omega_m}\tilde{p}(m)\Pr\left[s_{nF} = s|m, K\right].$$

In the data we only observe a firm in n if it has at least one French supplier. For those firms we observe the fraction with $s = 1, 2, ...$ French suppliers. In fact, we observe such fractions separately for firms in each destination n. The analog to these empirical moments is the conditional probability:

$$\begin{aligned}\Pr\left[s_{nF} = s|s_{nF} \geq 1\right] &= \sum_{m\in\Omega_m}\frac{\tilde{p}(m)}{1-\left(1-p_{nF}\right)^{mK}}\Pr\left[s_{nF} = s|m, K\right] \\ &= \sum_{m\in\Omega_m}\frac{\tilde{p}(m)}{1-\left(1-p_{nF}\right)^{mK}}\binom{mK}{s}\left(p_{nF}\right)^s\left(1-p_{nF}\right)^{mK-s}.\end{aligned}$$

for $s = 1, 2, ...$ For any destination n, equating the population moments to the analog observed fractions, we should be able to back out $\tilde{p}(m)$, for all $m \in \Omega_m$.

10.3 Algorithm for Module 3

Now consider the distribution of production labor shares across French firms. We start by defining the probability that a French firm does not perform a given sub-task with its own workers (hence outsources that sub-task to another firm, located anywhere):

$$p_F = \frac{\nu_F}{\Phi_F} = \frac{\beta_F^M}{1-\beta}.$$

To simulate the distribution of the labor share across French firms, we need to carry out four steps:

1. Draw the number of sub-tasks per task for the firm from the distribution, $\tilde{p}(m)$, for $m \in \Omega_m$. This value of m will apply to each of the firm's K tasks.

2. For each task, independently simulate the fraction of spending devoted to each of its m sub-tasks. All we need are the fractions, since we know that the overall production spending share on a task is $(1-\beta)/K$.

3. For each task, independently take Bernoulli trials, with probability of success $1-p_F$, to determine which sub-tasks are carried out with the firm's own workers.

4. Aggregate across the sub-tasks to obtain the production labor share of each task.

5. Average across the K tasks to obtain the overall production labor share of the firm.

Four of these five steps are obvious. Step 2 requires further explanation.

To simulate the vector of spending shares across sub-tasks, consider a firm with m sub-tasks per task. If we knew the costs $c(\omega)$, for $\omega = 1, 2, ..., m$, the spending shares per task would be:

$$\pi(\omega) = \frac{c(\omega)^{-(\sigma-1)}}{\sum_{\omega'=1}^{m} c(\omega')^{-(\sigma-1)}}.$$

We don't know these costs, but we do know that they are drawn independently from the distribution:

$$G_F(c) = 1 - e^{-\Phi_F c^{\theta(1-\gamma)}},$$

where the F subscript denotes France.

Under the transformation:

$$x_\omega = \Phi_F c_\omega^{\theta(1-\gamma)},$$

the x_ω can be drawn independently from unit exponential distribution since:

$$\begin{aligned}
\Pr\left[x_\omega \leq x\right] &= \Pr\left[\Phi_F c_\omega^{\theta(1-\gamma)} \leq x\right] \\
&= \Pr\left[c_\omega \leq \left(\frac{x}{\Phi_F}\right)^{\frac{1}{\theta(1-\gamma)}}\right] \\
& \quad 1 - e^{-x}.
\end{aligned}$$

We can therefore simulate π_ω using random variables x_ω since:

$$\pi_\omega(m) = \frac{c_\omega^{-(\sigma-1)}}{\sum_{\omega'=1}^{m} c_{\omega'}^{-(\sigma-1)}} = \frac{\left(\frac{x_\omega}{\Phi_F}\right)^{-\frac{\sigma-1}{\theta(1-\gamma)}}}{\sum_{\omega'=1}^{m}\left(\frac{x_{\omega'}}{\Phi_F}\right)^{-\frac{\sigma-1}{\theta(1-\gamma)}}} = \frac{x_\omega^{-\tilde{\sigma}/(1-\gamma)}}{\sum_{\omega'=1}^{m} x_{\omega'}^{-\tilde{\sigma}/(1-\gamma)}}.$$

for $\omega = 1, 2, ..., m$. Notice that Φ_F drops out.

Table 1

Regressions

- Number of French exporters (ignore constants)

$$\ln N_{nF} = \underset{(0.04)}{0.50} \ln X_n + \underset{(0.11)}{0.68} \ln \pi_{nF}$$

- Number of relationships

$$\ln R_{nF} = \underset{(0.06)}{0.83} \ln X_n + \underset{(0.16)}{1.04} \ln \pi_{nF}$$

- Mean number of buyers per French exporter

$$\ln \bar{B}_{nF} = \underset{(0.03)}{0.33} \ln X_n + \underset{(0.08)}{0.36} \ln \pi_{nF}$$

Figure 1

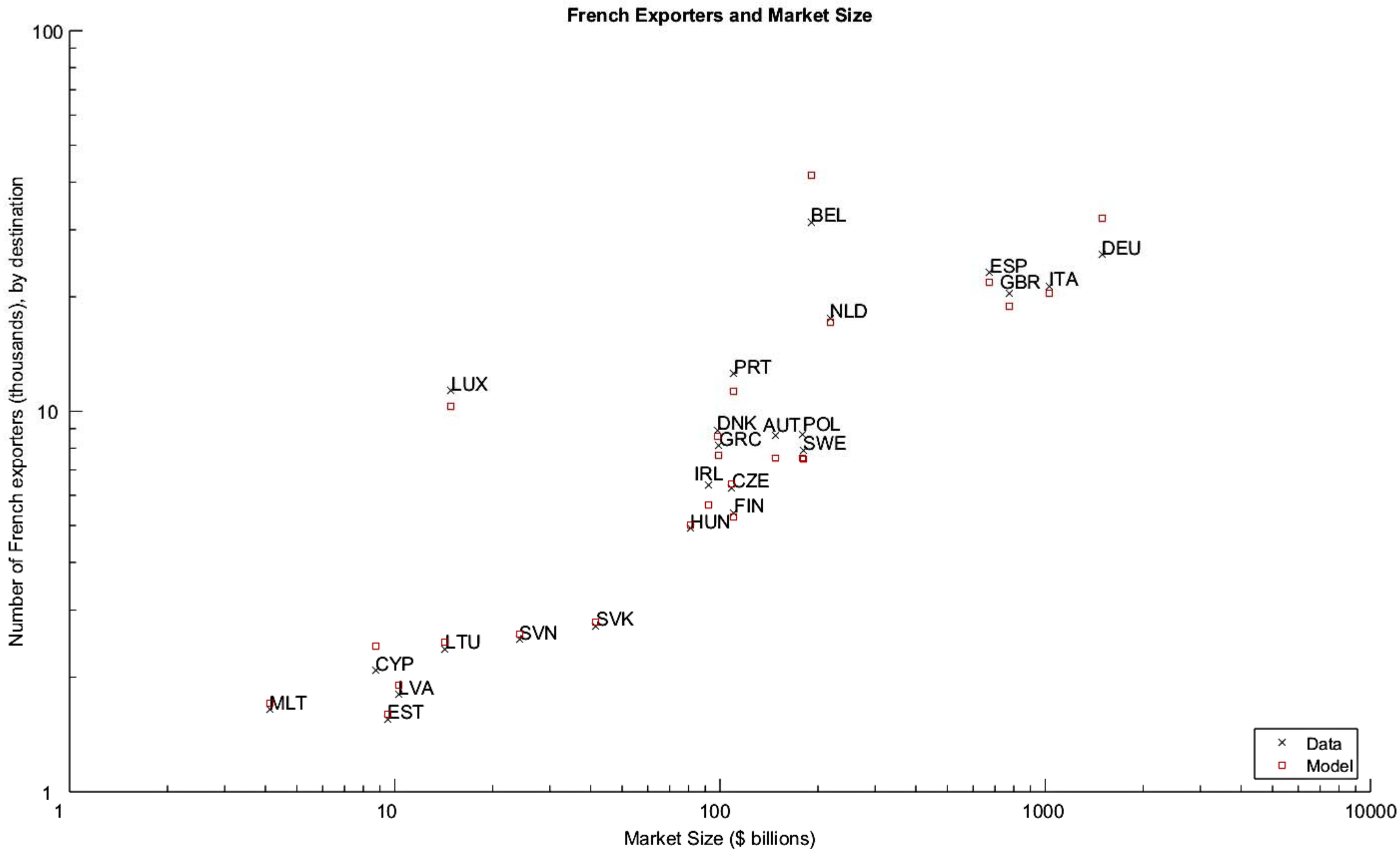

French Exporters and Market Size
Number of French exporters (thousands), by destination
Market Size ($ billions)
100
10
1
1
10
100
1000
10000
BEL
DEU
ESP
GBR
ITA
NLD
PRT
LUX
DNK
AUT
POL
GRC
SWE
IRL
CZE
FIN
HUN
SVK
SVN
LTU
CYP
LVA
MLT
EST
Data
Model

Figure 2

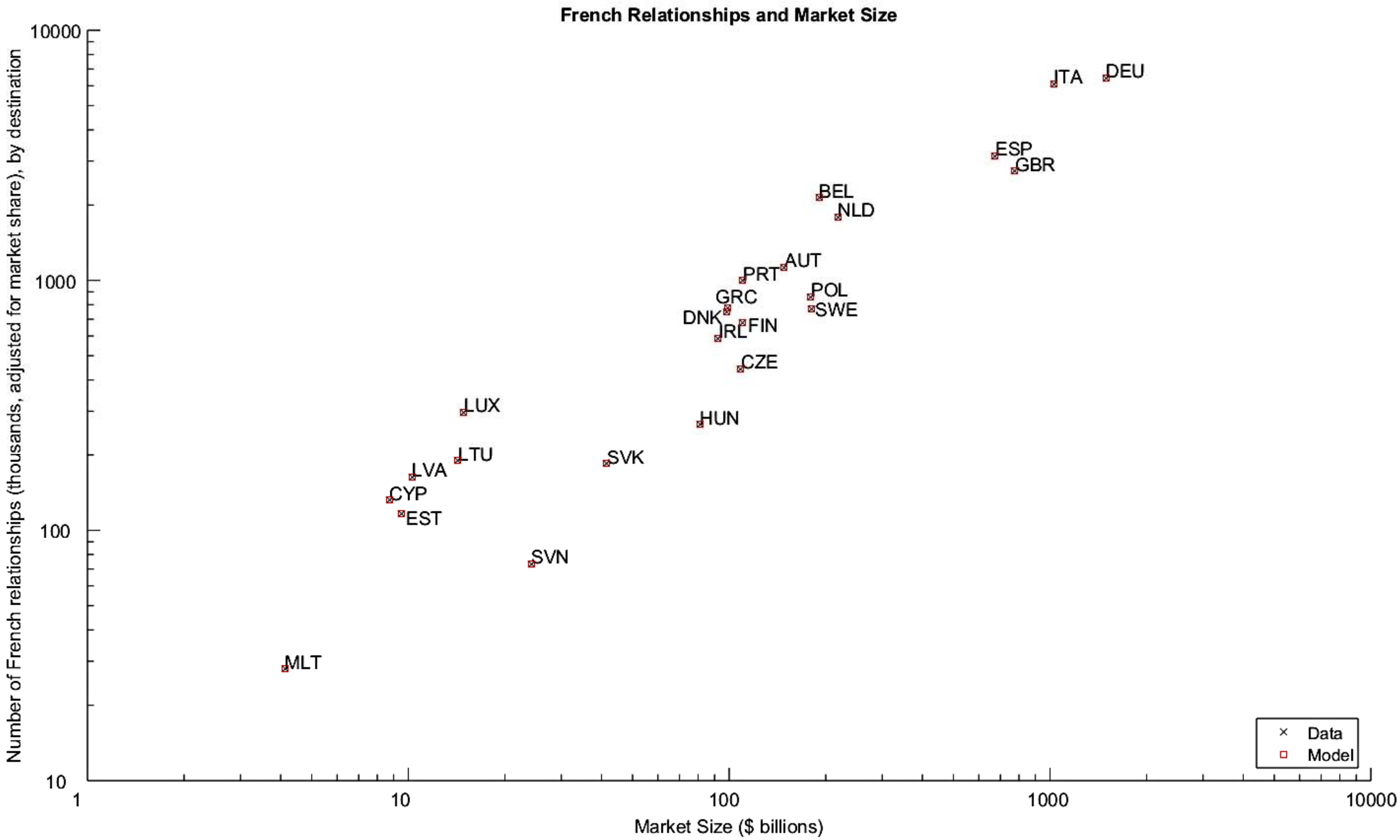

French Relationships and Market Size
Number of French relationships (thousands, adjusted for market share), by destination
Market Size ($ billions)
10000
1000
100
10
1
10
100
1000
10000
ITA
DEU
ESP
GBR
BEL
NLD
AUT
PRT
POL
GRC
SWE
DNK
FIN
IRL
CZE
LUX
HUN
LTU
SVK
LVA
CYP
EST
SVN
MLT
Data
Model

Figure 3a

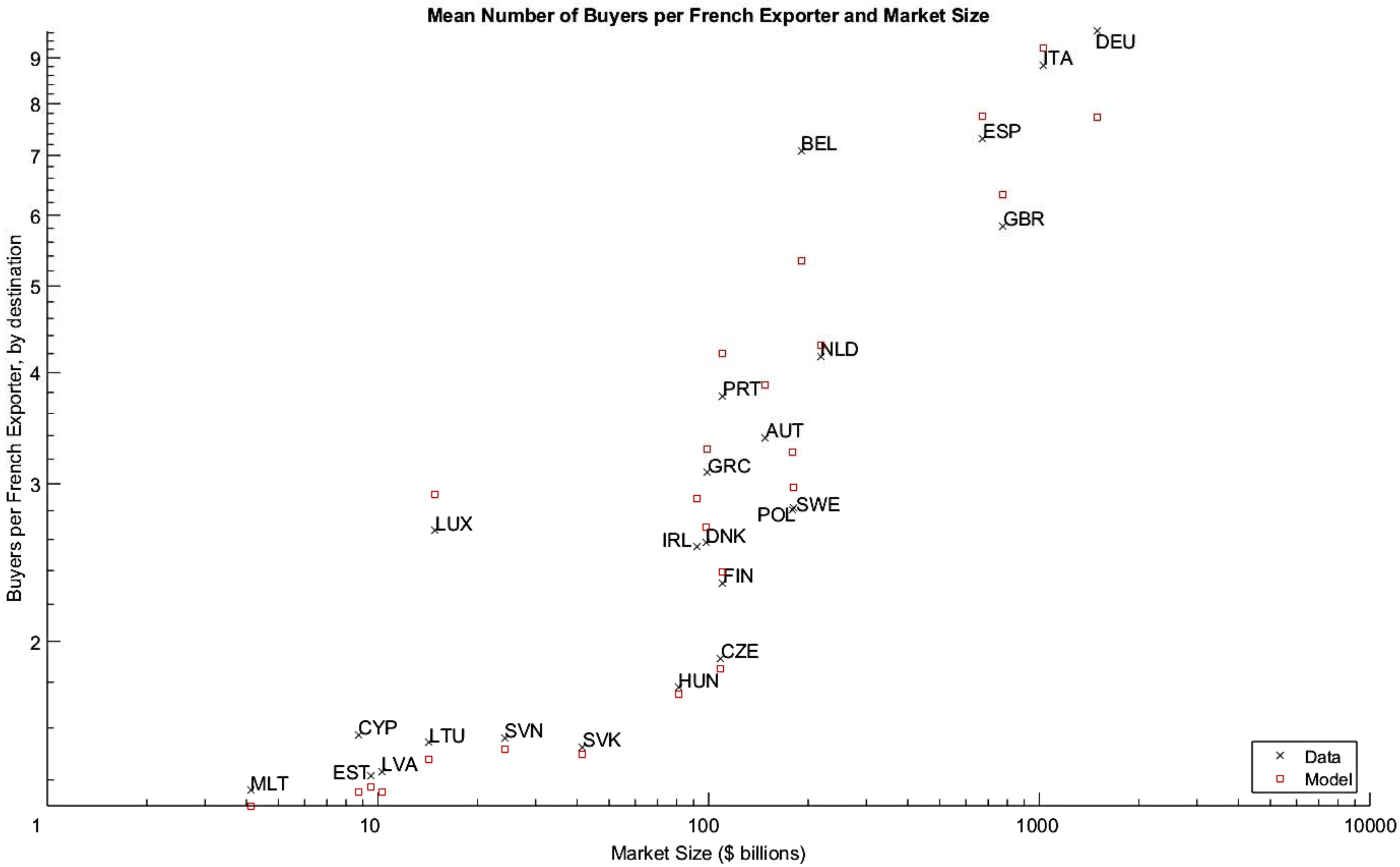

Mean Number of Buyers per French Exporter and Market Size
Buyers per French Exporter, by destination
Market Size ($ billions)
9
8
7
6
5
4
3
2
1
10
100
1000
10000
DEU
ITA
ESP
BEL
GBR
NLD
PRT
AUT
GRC
SWE
POL
LUX
IRL
DNK
FIN
CZE
HUN
CYP
LTU
SVN
SVK
EST
LVA
MLT
Data
Model

Figure 3b

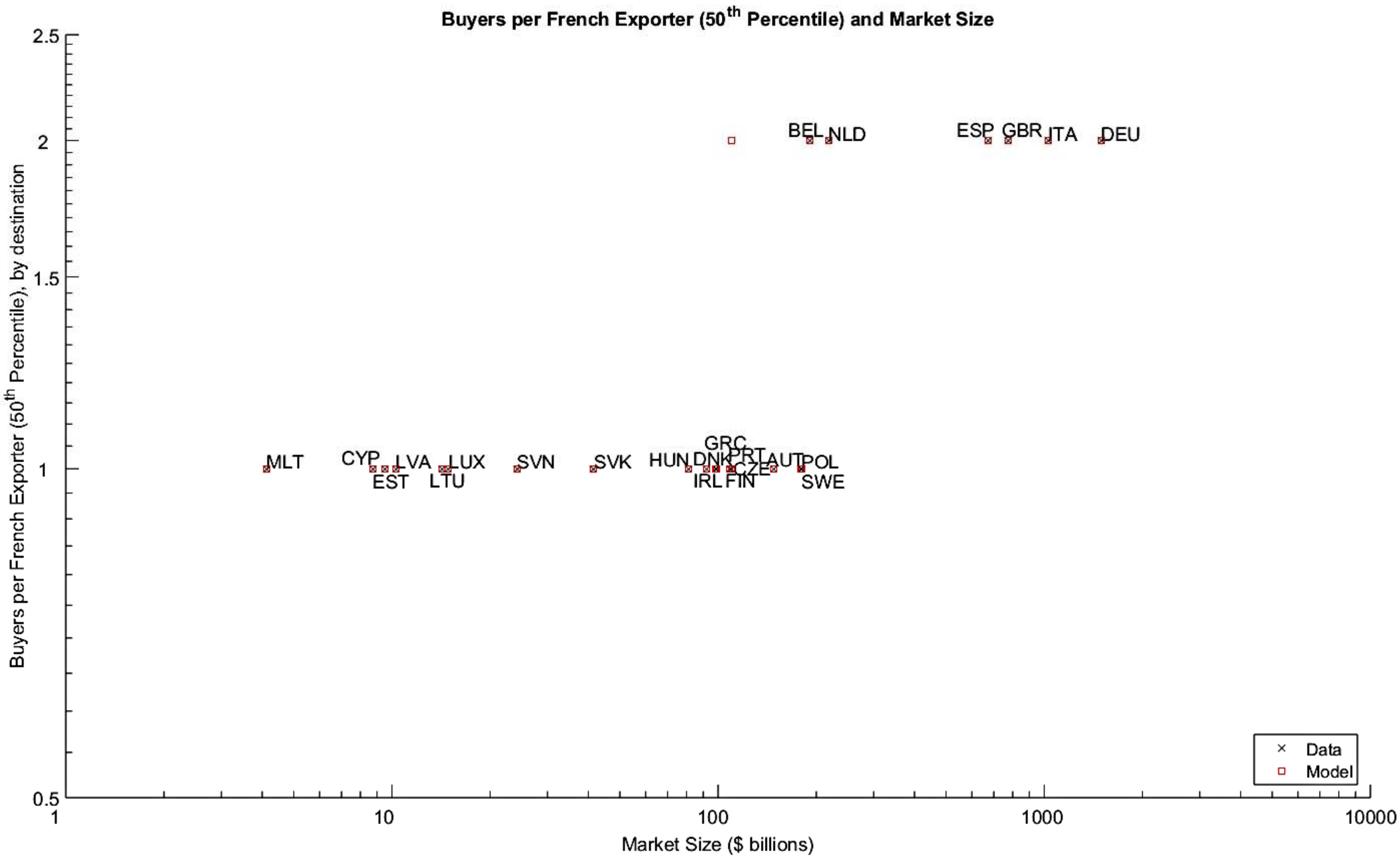
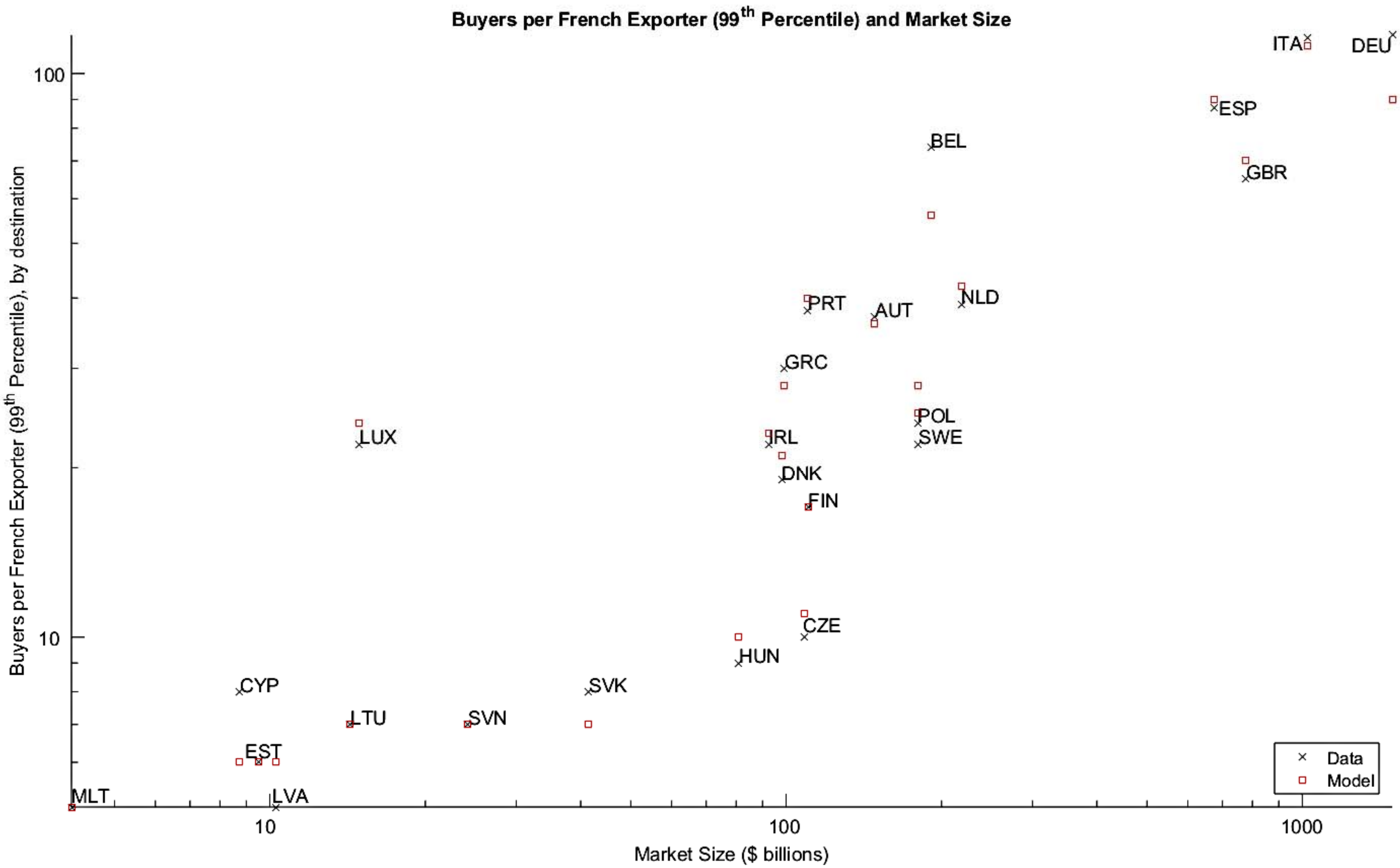
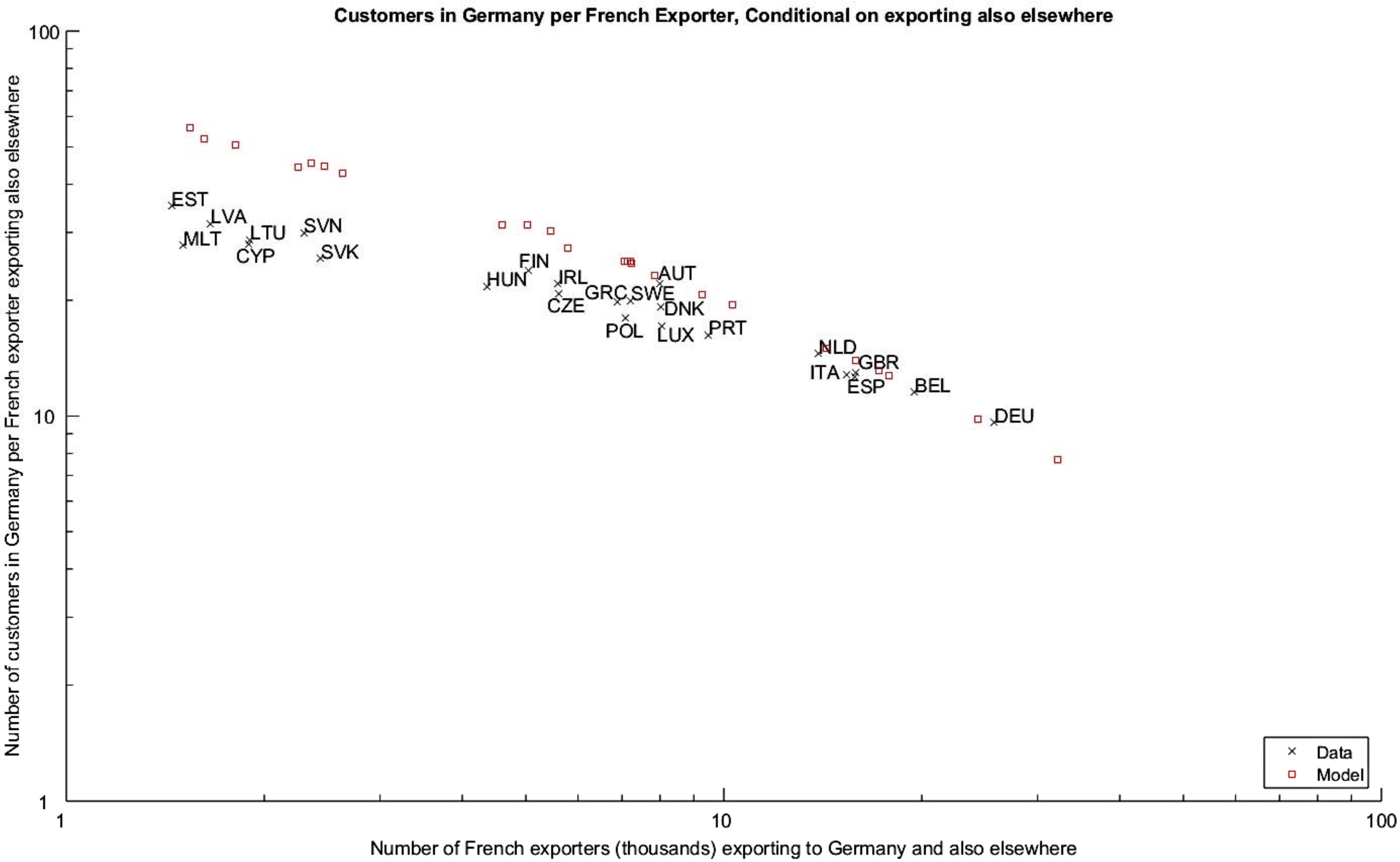

Buyers per French Exporter (50th Percentile) and Market Size
Buyers per French Exporter (50th Percentile), by destination
Market Size ($ billions)
2.5
2
1.5
1
0.5
1
10
100
1000
10000
MLT
CYP
EST
LVA
LTU
LUX
SVN
SVK
HUN
DNK
IRL
GRC
PRT
CZE
FIN
AUT
POL
SWE
BEL
NLD
ESP
GBR
ITA
DEU
Data
Model

Figure 3c



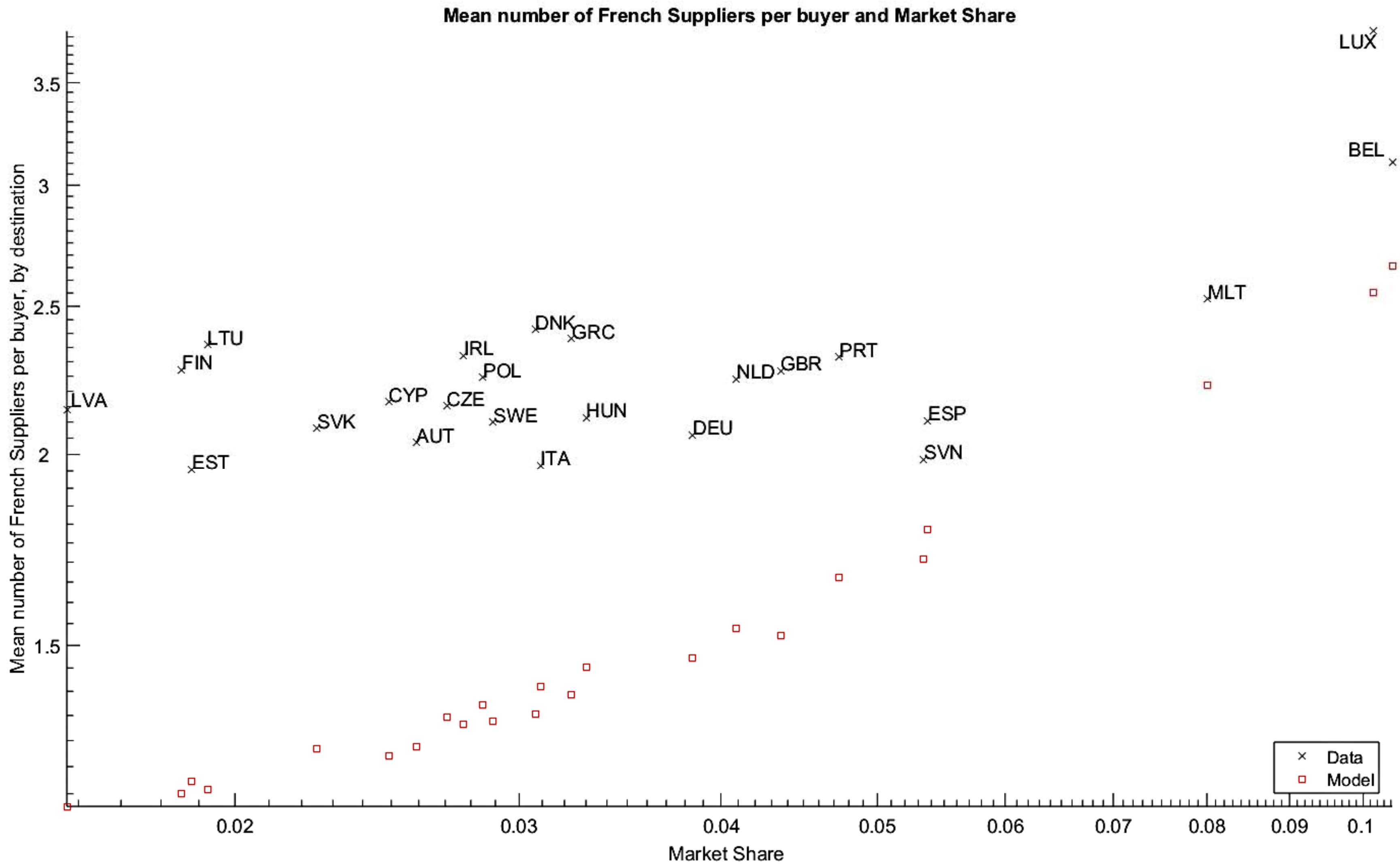
Buyers per French Exporter (99th Percentile) and Market Size
Buyers per French Exporter (99th Percentile), by destination
100
10
10
100
1000
Market Size ($ billions)
ITA
DEU
ESP
BEL
GBR
PRT
AUT
NLD
GRC
POL
LUX
IRL
SWE
DNK
FIN
CZE
HUN
CYP
SVK
LTU
SVN
EST
MLT
LVA
Data
Model

Figure 3d



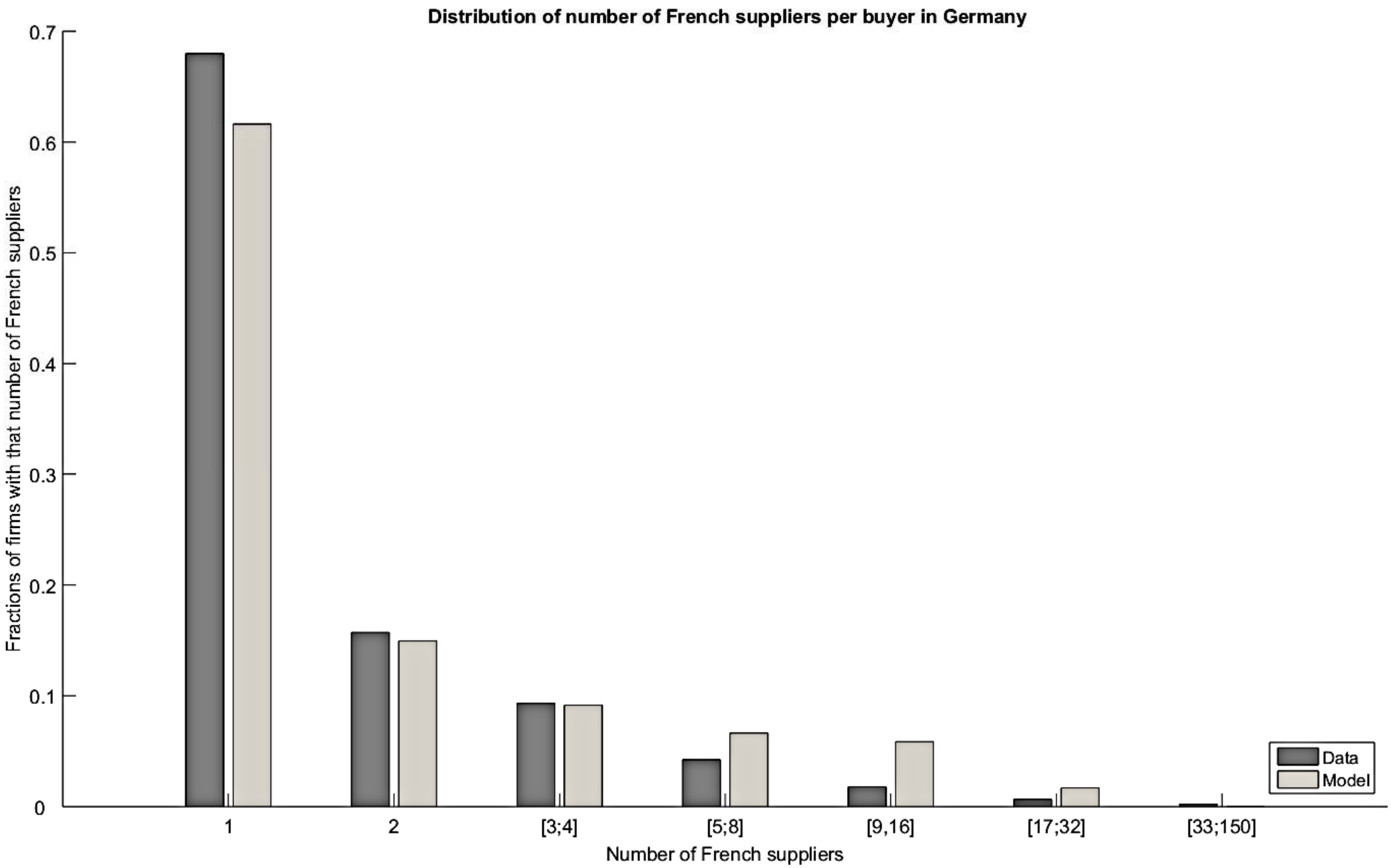
Customers in Germany per French Exporter, Conditional on exporting also elsewhere
Number of customers in Germany per French exporter exporting also elsewhere
Number of French exporters (thousands) exporting to Germany and also elsewhere
100
10
1
1
10
100
EST
LVA
MLT
LTU
CYP
SVN
SVK
FIN
HUN
IRL
CZE
GRC
SWE
AUT
DNK
POL
LUX
PRT
NLD
ITA
GBR
ESP
BEL
DEU
Data
Model

Figure 4a



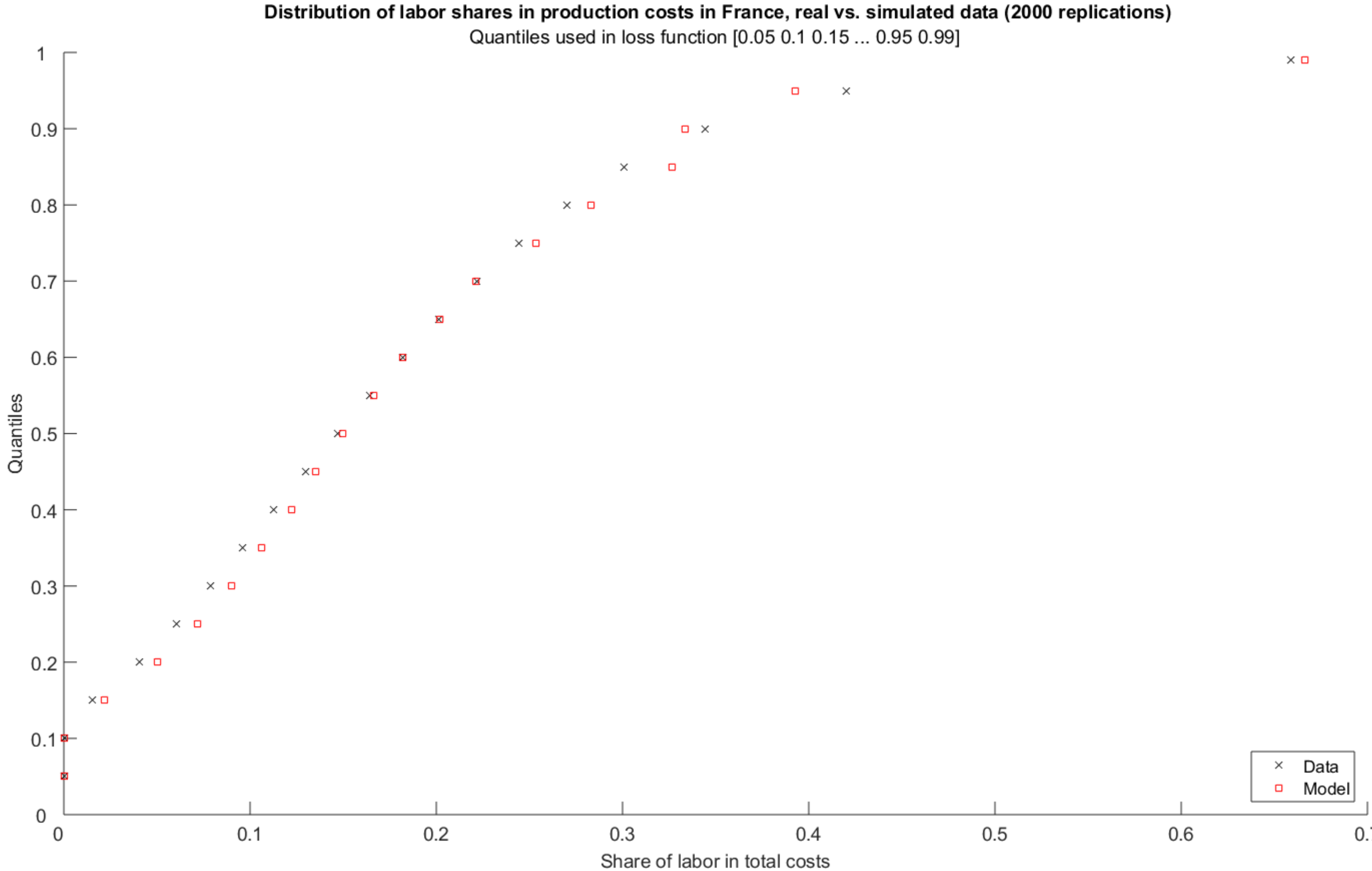
Mean number of French Suppliers per buyer and Market Share
Mean number of French Suppliers per buyer, by destination
Market Share
3.5
3
2.5
2
1.5
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
LUX
BEL
MLT
DNK
GRC
LTU
IRL
FIN
POL
PRT
GBR
NLD
LVA
CYP
CZE
SVK
SWE
HUN
ESP
DEU
AUT
EST
ITA
SVN
Data
Model

Figure 4b



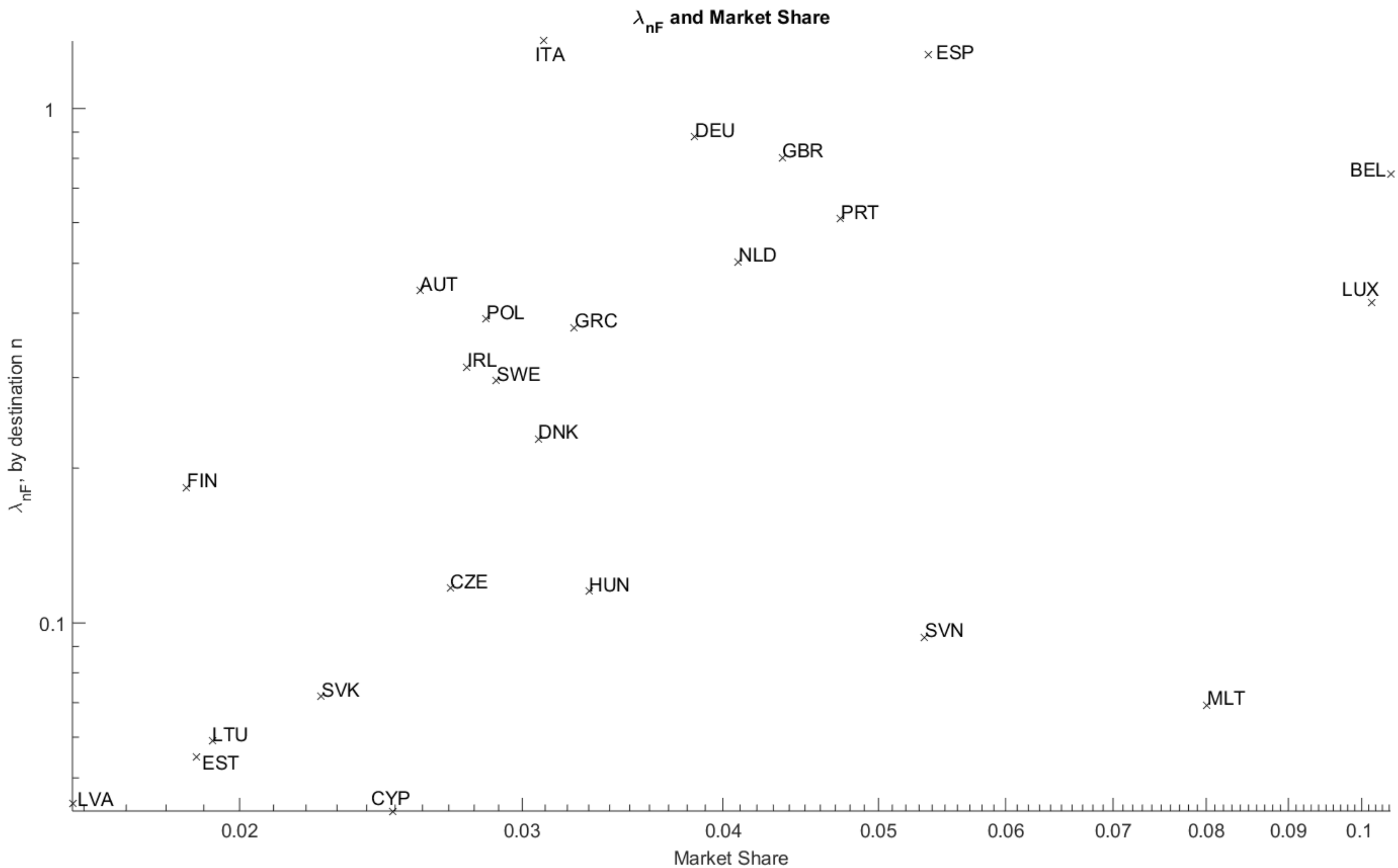
Distribution of number of French suppliers per buyer in Germany
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Fractions of firms with that number of French suppliers
1
2
[3;4]
[5;8]
[9,16]
[17;32]
[33;150]
Number of French suppliers
Data
Model

Figure 5



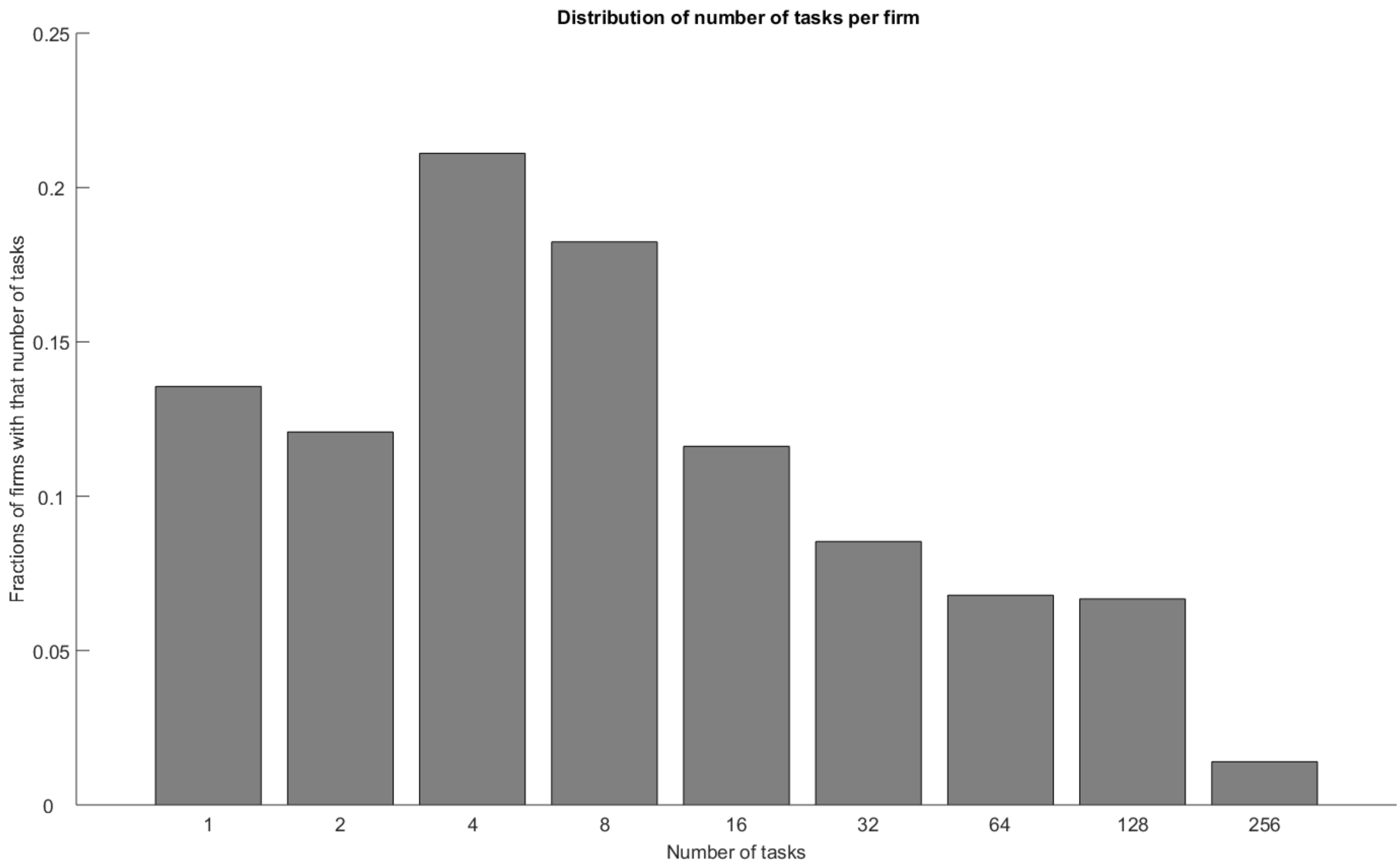
Distribution of labor shares in production costs in France, real vs. simulated data (2000 replications)
Quantiles used in loss function [0.05 0.1 0.15 ... 0.95 0.99]
Quantiles
Share of labor in total costs
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Data
Model

Figure 6



λ_{nF} and Market Share
ITA
ESP
1
DEU
GBR
BEL
PRT
NLD
AUT
LUX
POL
GRC
IRL
SWE
λ_{nF}, by destination n
DNK
FIN
CZE
HUN
0.1
SVN
SVK
MLT
LTU
EST
LVA
CYP
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Market Share

Figure 7



Distribution of number of tasks per firm
0.25
0.2
0.15
0.1
0.05
0
Fractions of firms with that number of tasks
1
2
4
8
16
32
64
128
256
Number of tasks

Figure 8

Sales ratio $(\bar{x}_{nF|n'}/\bar{x}_{nF})$ in Germany
SVN
MLT
SVK
HUN
CYP
CZE
LVA
EST
FIN
POL
AUT
SWE
LTU
IRL
GRC
DNK
PRT
ESP
ITA
GBR
NLD
DEU
BEL
LUX
Data
Model
2.2
2
1.8
1.6
1.4
1.2
1
0.8
1
10
100
Sales ratio, by destination n'
Number of French exporters (thousands) exporting to Germany and also elsewhere